Stellar magnetic equilibria with the Pencil code

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OBA Stars: Variability and Magnetic Fields (STARS-2021)

keywords: Magnetic field in stars, MHD-simulations, Stability of magnetic fields

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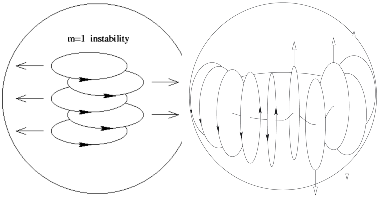
Stars with strong large-scaled and long-lived magnetic fields

	%	Mass	Radius	$\|\vec{B}\|$
Ap/Bp stars				
(Donati and Landstreet, 2009)	5-10%	$1.5-15\ M_\odot$	$3\ R_\odot$	3×10^4 G
O-stars				
(Grunhut and Neiner, 2015)	7%	$15-50\ M_\odot$	$10\ R_\odot$	10^3 G
White dwarfs				
(Ferrario et al., 2015, García-Berro et al., 2016)	10%	$0.93\ M_\odot$	10^4 Km	10^3- 10^9 G
Neutron Stars	100%	$1.4\ M_\odot$	10 Km	$< 10^9$ G (Millisecond pulsars) $10^{11}-10^{13}$ G (classical radio pulsars) $10^{13}-10^{15}$ G (SGRs-AXPs)

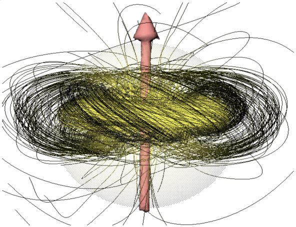
- Origin of the magnetic field: core dynamo or fossil field?
- $\phi_{\rm max} = \pi^2 R^2 B_{\rm max} \approx 10^{27.5}\, G{\rm cm}^{-2}$ (Reisenegger, 2009)

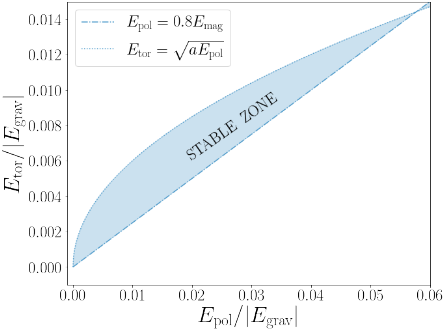
Stability of the magnetic field inside the star

Purely toroidal and purely poloidal magnetic fields configurations are unstable (Tayler (1973), Markey and Tayler (1973) and Wright (1973))



Some mixed poloidal-toroidal magnetic configuration are stable (Braithwaite and Spruit (2006)).





- **Akgün et al. (2013), Braithwaite (2009) :**

$$\frac{1}{4} E_{\rm pol} < E_{\rm tor} < \sqrt{\frac{(\Gamma/\gamma - 1)}{3.7} E_{\rm grav} E_{\rm pol}}$$

$$\Gamma = \left(\frac{d\ln \mathrm{P}}{d\ln \rho}\right)_s \;;\; \gamma = \left(\frac{d\ln \mathrm{P}_0}{d\ln \rho_0}\right)$$

Numerical simulation

Pencil Code *(http://www.nordita.org/software/pencil-code/)*

We integrate the magneto-hydrodynamical equations (with $\mu_0 = 1$):

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\rho \vec{\nabla} \cdot \vec{u} - \vec{u} \cdot \vec{\nabla}\, \rho \\
\frac{\partial \vec{u}}{\partial t} &= -\vec{u} \cdot \vec{\nabla} \vec{u} - \frac{\vec{\nabla} p}{\rho} - \vec{\nabla} \Phi_{\rm grav} + \frac{\vec{j} \times \vec{B}}{\rho} + \vec{F}^{\rm visc}_{\rm hyp} \\
\frac{\partial \vec{A}}{\partial t} &= \vec{u} \times \vec{B} - \eta \vec{j} + \eta_3 \vec{\nabla}^6 \vec{A} \\
\frac{\partial s}{\partial t} &= -\vec{u} \cdot \vec{\nabla} s + \frac{\eta |\vec{j}|^2}{\rho T} + \chi_3 \vec{\nabla}^6 s \,,
\end{aligned}
$$

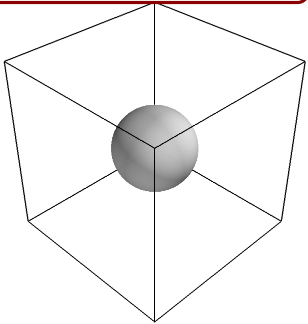
with

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad ; \quad \vec{j} = \vec{\nabla} \times \vec{B}$$

and assuming an ideal equation of state:

$$p(s, \rho) = (\mathcal{R}/\mu)\, \rho\, T(s, \rho)$$

- Cartesian grid: Star in a box
- Periodic boundary condition



$$\tau_{\rm alfven} = \frac{\sqrt{\rho_{\rm rms}}}{B_{\rm rms}} \quad ; \quad \tau_{\rm sound} = \frac{1}{c_{\rm s,rms}} \quad ; \quad \tau_{\rm diff} = \frac{R_s^2}{\eta}$$

$$\tau_{\rm sound} < \tau_{\rm alfven} < \tau_{\rm diff}$$

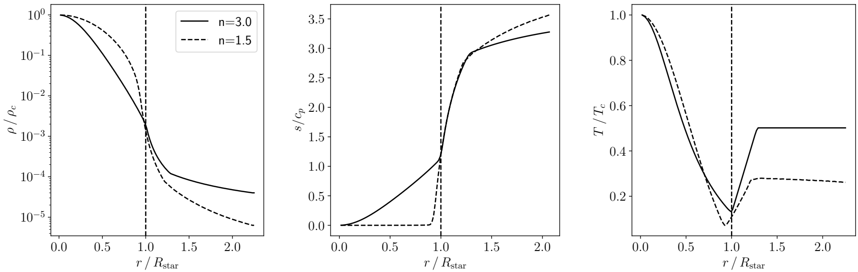
Initial conditions: polytropic star

Non-magnetic hydrodynamic equilibrium:

$$\begin{aligned}\vec{\nabla} p &= -\rho \vec{\nabla} \Phi_{\rm grav} \\ \nabla^2 \Phi_{\rm grav} &= 4\pi\rho \\ p &= K\rho^{1+1/n}\end{aligned}$$

$$\gamma = \left(\frac{d\ln \mathrm{p}}{d\ln\rho}\right) = 1 + \frac{1}{n}$$

$$\Gamma = \frac{5}{3}$$



Initial conditions: random magnetic field

The initial magnetic vector potential is:

$$\vec{A} = \sum_k \vec{A}_{\vec{k}}\, \mathrm{e}^{i\vec{k}\cdot\vec{r}}$$

For $k_{\text{min}} < k < k_{\text{max}}$:

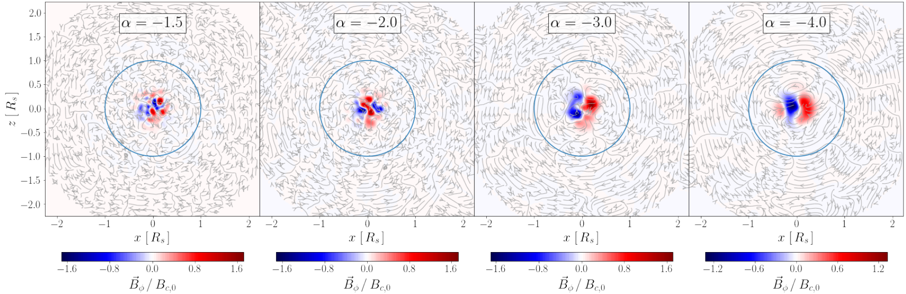
$$\left(A_{\vec{k}}\right)_i = (\cos\phi_1 + i\sin\phi_2)\, k^{\alpha}$$

with ϕ_1 and ϕ_2 random numbers.
Finally:

$$\vec{A} \rightarrow \vec{A} e^{-r^2/r_m^2}$$

Low electrical conductivity atmosphere:

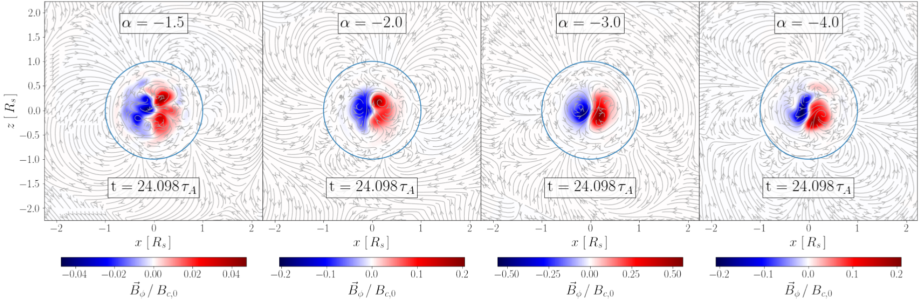
$$\eta(r) = \begin{cases} 0, & r < R_i \\ \dfrac{\eta_{\text{ext}}}{\Delta_r}(r - R_i) + \eta_{\text{ext}}, & R_i \le r < R_{\text{atm}} \\ \eta_{\text{ext}}, & r \ge R_{\text{atm}} \end{cases}$$



Simulations with $\Gamma = \frac{5}{3}$ *and* $\gamma = \frac{4}{3}$

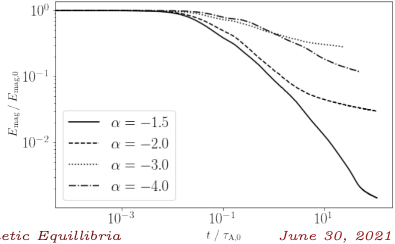
$N_x \times N_y \times N_z = 128 \times 128 \times 128$

$$E_{\text{mag}} = 1.2 \times 10^{-3} |E_{\text{grav}}| \quad ; \quad E_{\text{thermal}} = 0.5 |E_{\text{grav}}|$$



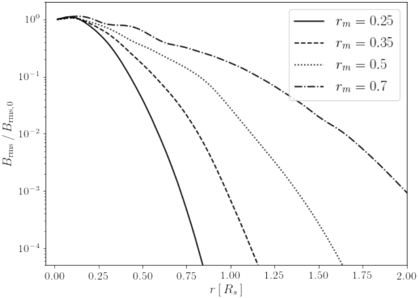
Magnetic axis:

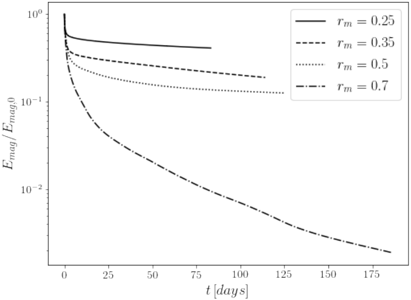
$$\vec{M} = \int_{\text{box}} \vec{B} \times \vec{r}\, dV$$

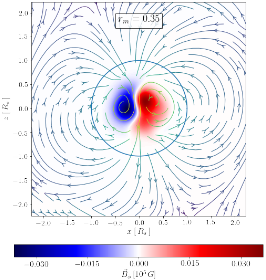


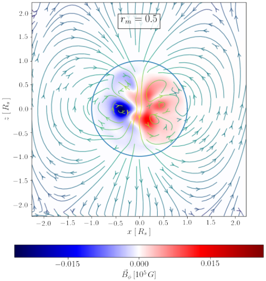
Magnetic field concentration

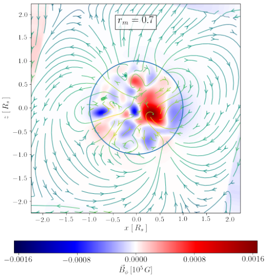
$$\vec{A} \rightarrow \vec{A}e^{-r^2/r_m^2}$$





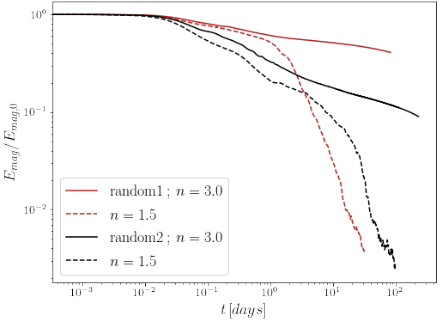




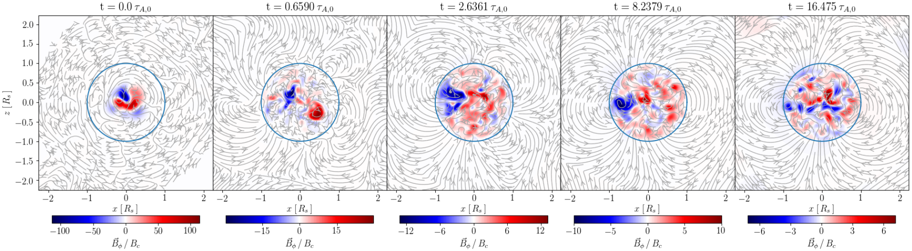


Barotropic star (isentropic star)

$\Gamma = \frac{5}{3}$; $\gamma = \frac{5}{3}$

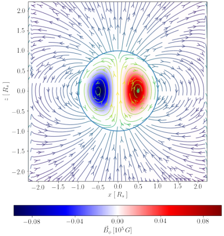


In Barotropic stars, the magnetic field doesn't evolve to a stable equilibrium configuration



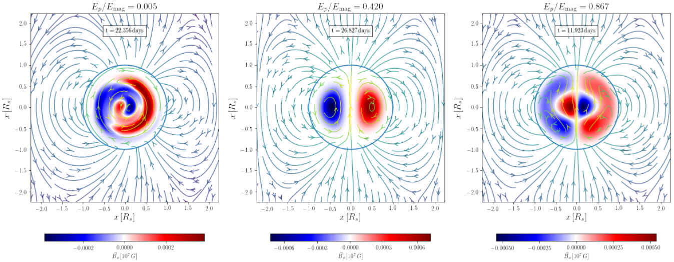
Magnetic field stability

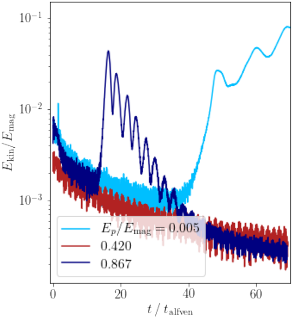
$$\vec{B} = \vec{B}_{\mathrm{pol}} + \vec{B}_{\mathrm{tor}}$$

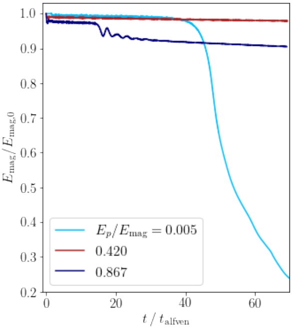


$$\vec{B} = \vec{\nabla}\alpha(r,\theta) \times \vec{\nabla}\phi$$

$$+\beta(r,\theta)\vec{\nabla}\phi$$





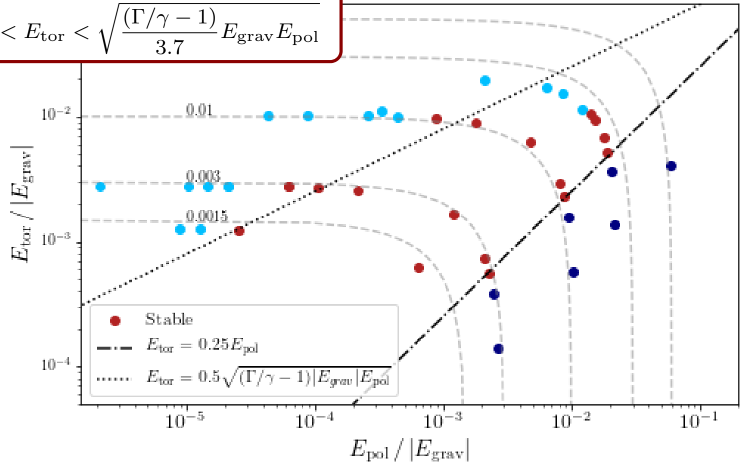


Magnetic field stability

Strength of the magnetic field.

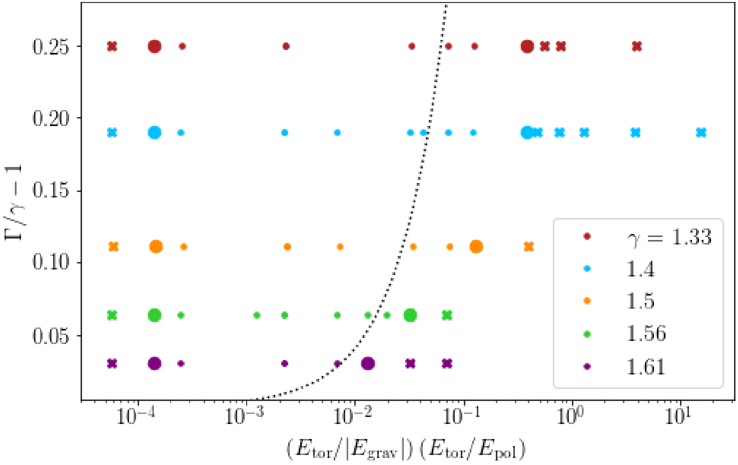
Akgün et al. (2013), Braithwaite (2009)

$$\frac{1}{4}E_{\rm pol} < E_{\rm tor} < \sqrt{\frac{(\Gamma/\gamma - 1)}{3.7}E_{\rm grav}E_{\rm pol}}$$



Magnetic field stability

Stellar stratification.



Summary

- We have evolved random magnetic field configuration in the interior of stratified and barotropic stars with the PENCIL CODE.
- We have reproduced the main results of the works of Braithwaite 2006.
- We have proved the stability of magnetic field configurations with different fraction between the magnetic energy in the poloidal component and the total magnetic energy.

THANK YOU