Image analysis of hyperspectral data using mathematical morphology

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Tutorial WHISPERS 2014

24-27 June 2014, Lausanne, Switzerland

History

2001 M. Pesaresi & J.A. Benediktsson: **Morphological Profile**

2003 J.A. Benediktsson, M. Pesaresi & K. Arnason: **Morphological Profile**

2005 J.A. Benediktsson, J. A. Palmason & J. Sveinsson: **Extended MP**

2009 M. Fauvel, J. Chanussot & J.A. Benediktsson: **Kernel PCA based EMP**

2010 M. Dalla Mura, J.A. Benediktsson, Waske & L. Bruzzone: **Attribute Profile**

2011 M. Dalla Mura, A. Villa, J.A. Benediktsson & L. Bruzzone: **ICA based AP**

2014 WHISPERS Tutorial!

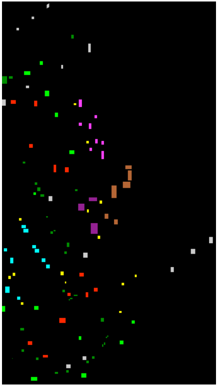
VHR Hyperspectral Images: Urban Area

ROSIS: 1.5 m/pixel - 115 spectral channels - 430 nm, 860 nm.

- Spatial resolution: Lower than 2.5 meter by pixel
- Spectral resolution: More than 100 spectral channels (up to 4096!)

Classification

Original Data



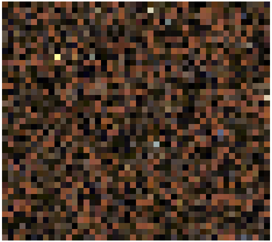
Ground-truth

Thematic Map

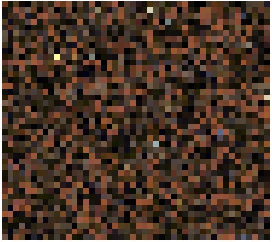
Classification: A pattern recognition approach

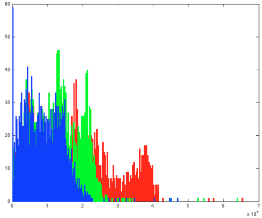
1. Feature extraction: One vector of attributes extracted for every pixels
2. Pattern recognition algorithms: Support Vectors Machines ...

Why spatial information? 1/2



Why spatial information? 1/2





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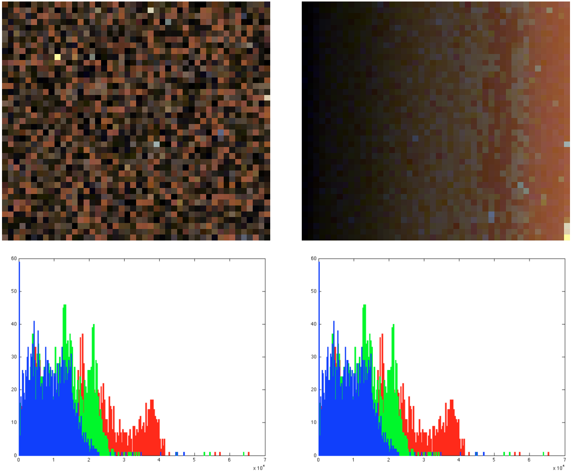

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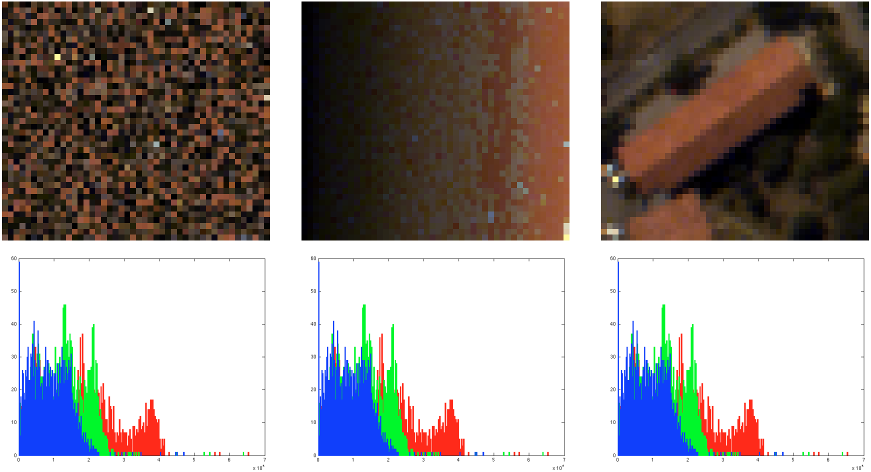
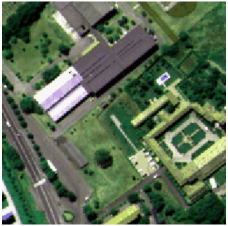
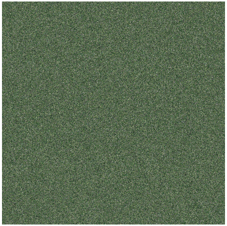


Why spatial information? 2/2



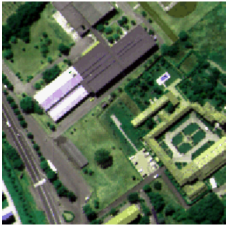
Random permutation
of the spatial position
of the pixels



Spectral classification → Same classification !! ← Spectral classification

Need to incorporate information from the spatial domain

Why spatial information? 2/2



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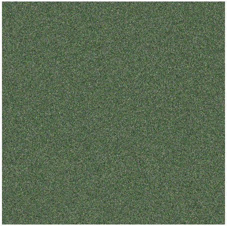

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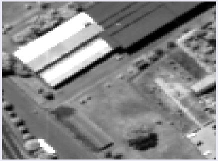
Need to incorporate information from the spatial domain

Dilation and Erosion

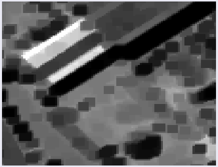
Definition (Image)

$$f : \mathcal{D}_f \subset \mathbb{Z}^d \rightarrow \{0, \ldots, f_{max}\}$$
$$f^c = f_{max} - f$$



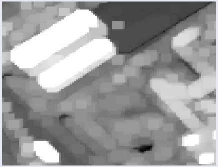
Definition (Erosion)

$$\epsilon_B(f) = \bigwedge_{b \in B} f_{-b}$$

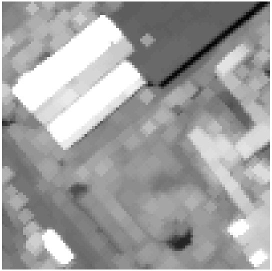


Definition (Dilation)

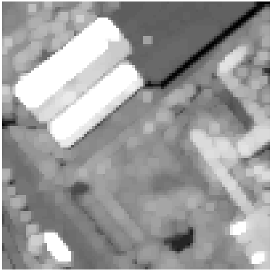
$$\delta_B(f) = \bigvee_{b \in B} f_{-b}$$



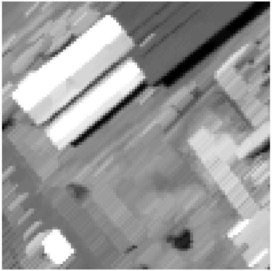
Effect of the structuring element B



0	0	1	0	0
0	1	1	1	0
1	1	1	1	1
0	1	1	1	0
0	0	1	0	0



1	1	1
1	1	1
1	1	1

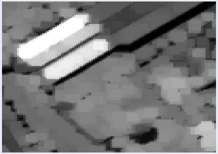


0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	1	0	0	0
1	0	0	0	0

Opening and Closing

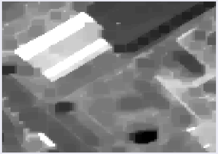
Definition (Opening)

$$\gamma_B(f) = \delta_B \circ \epsilon_B(f)$$



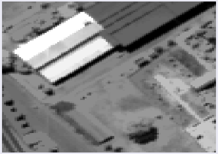
Definition (Closing)

$$\phi_B(f) = \epsilon_B \circ \delta_B(f)$$



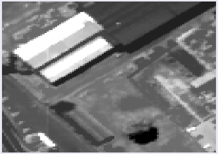
Definition (Opening by reconstruction)

$$\gamma_B^r(f) = Rec_f \circ \epsilon_B(f)$$



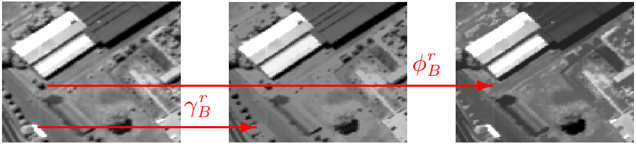
Definition (Closing by reconstruction)

$$\phi_B^r(f) = Rec_f^* \circ \delta_B(f)$$



Opening and Closing Profile

- For a given B, $\gamma_B^r(f)$ (resp. $\phi_B^r(f)$) indicates which clear (dark) objects fit B.



- Applying γ_{B_i} with a set of $\left\{B_i | B_i \subset B_{i+1}, i \in [1, \ldots, n]\right\}$: Opening Profile
- Applying ϕ_{B_i} with a set of $\left\{B_i | B_i \subset B_{i+1}, i \in [1, \ldots, n]\right\}$: Closing Profile

Definition (Granulometry)

A granulometry is a transformation having a size parameter n, and with the following properties: *Anti-extensivity*, *Increasingness* and *Absorption*.

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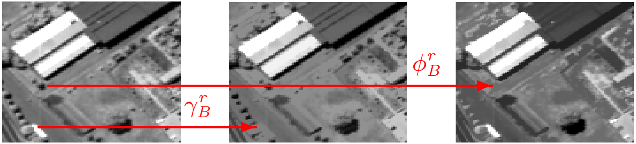

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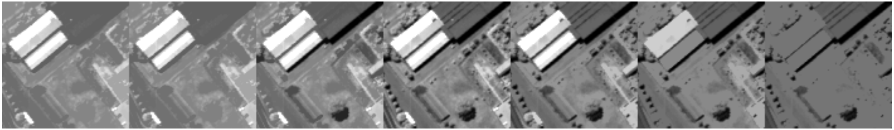
Morphological Profile

Definition (Morphological profile)

The Morphological Profile of size n is a $(2n+1)$-dimensional vector such as:

$$\mathsf{MP}(\mathbf{x}) = \Big[\mathsf{CP}_n(\mathbf{x}), f(\mathbf{x}), \mathsf{OP}_n(\mathbf{x})\Big].$$



$\mathsf{CP}(\mathbf{x}) \longleftarrow \quad f(\mathbf{x}) \quad \longrightarrow \mathsf{OP}(\mathbf{x})$

For a given pixel $\mathbf{x}$, information include in the $\mathsf{MP}(\mathbf{x})$ are:

- Contrast: Is the structure to which the pixel belongs to darker or lighter than his surrounding neighbors?
- Size: Is the structure to which the pixel belongs to small or big compared to B_i?

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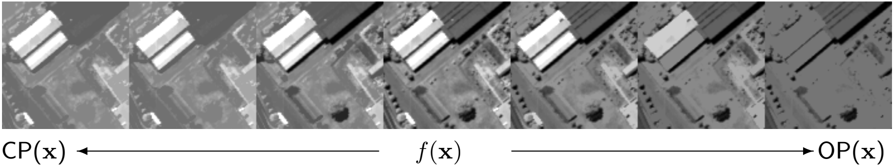

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Derivative of the MP

Definition (Derivative of the Morphological profile)

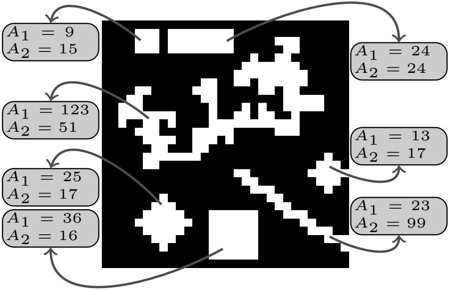
The Derivative of the Morphological Profile of size n is a $(2n)$-dimensional vector such as:

$$\mathrm{DMP}(\mathbf{x}) = \Big[|\phi_n(\mathbf{x}) - \phi_{n-1}(\mathbf{x})|, \ldots, |\gamma_{n-1}(\mathbf{x}) - \gamma_n(\mathbf{x})|\Big].$$

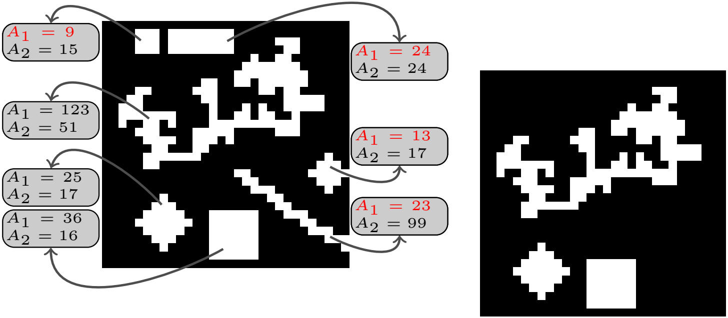
Attribute filters 1/3

- Objects are filtered according to a given attribute (no SE anymore):
  - Area
  - Perimeter
  - Standard deviation
  - Moment of inertia
  - ...
- Combination of attributes are possible !
- Filtering procedure: Attribute filters are based on the following operations:
  - Compute *attribute* for each connected component in the image
  - Keep the components that satisfy the *criterion*.

Attribute filters 2/3

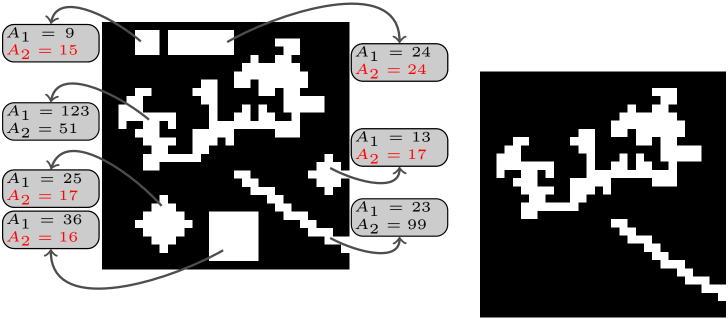


Attribute filters 2/3



$$T_1 = A_1 \geq 25?$$

Attribute filters 2/3

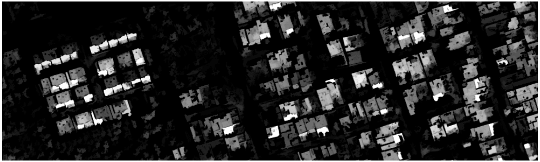


$$T_1 = A_2 > 30?$$

Attribute filters 3/3

Panchromatic image (f)

Filter by rec. $f - \gamma_R^B(f)$ (B: disk radius 7 pixels)

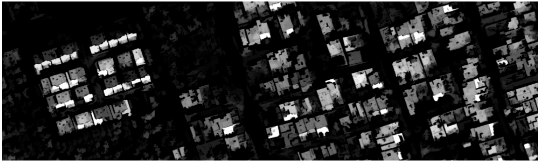


Attribute filter $\gamma^T(f)$ with $T = (R > 0.3) \wedge (I < 0.5) \wedge (50 < A < 5000)$

Attribute filters 3/3

Panchromatic image (f)

Filter by rec. $f - \gamma_R^B(f)$ (B: disk radius 11 pixels)



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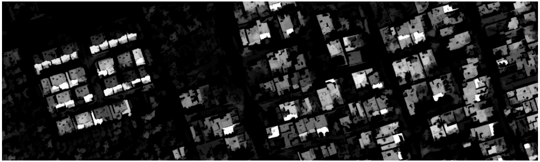
Attribute filters 3/3

Panchromatic image (f)

Filter by rec. $f - \gamma_R^B(f)$ (B: disk radius 15 pixels)

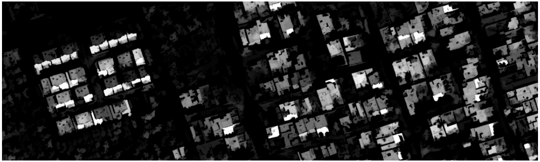


Attribute filter $\gamma^T(f)$ with $T = (R > 0.3) \wedge (I < 0.5) \wedge (50 < A < 5000)$

Attribute filters 3/3

Panchromatic image (f)

Filter by rec. $f - \gamma_R^B(f)$ (B: disk radius 19 pixels)



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1. Introduction
2. Spatial information from MM
   - Basic tools
   - Remote sensing tools
   - Advanced tools
   - Extension to multivalued images
3. Classification
   - Support Vectors Machines
   - Classification of spectral-spatial features
4. Experiments
   - Data
   - Results-ROSIS-03
5. Conclusions

Ordering relation

- MM is based on $\inf$ and $\sup$ operators
- No unambiguous $\inf/\sup$ for pixel/vector:

$$\begin{bmatrix}1\\5\\2\end{bmatrix} \overset{?}{\lesseqgtr} \begin{bmatrix}0\\6\\1\end{bmatrix}$$

- Marginal ordering $\Rightarrow$ by band filtering
- Reduced ordering $\Rightarrow h : \mathbb{R}^d \rightarrow \mathbb{R}$

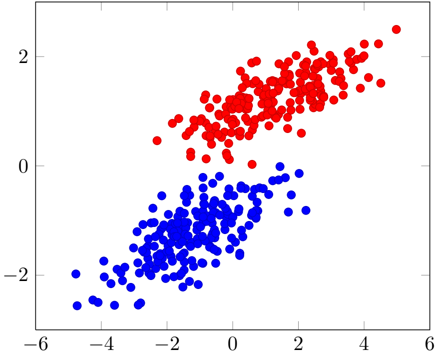
$$\mathbf{x} \mapsto h(x)$$

Principal Component Analysis

- $h(\mathbf{x}) = \langle \mathbf{x}, \mathbf{v}_m \rangle$ such as $\mathbf{v}_m$ is the m^{th} principal component of the spectral data
- $\mathbf{v}_m$ is computed as the eigenvector corresponding to the m^{th} largest eigenvalue of the covariance matrix of the spectral data

$$\lambda_m \mathbf{v}_m = \mathbf{\Sigma} \mathbf{v}_m$$

- The number of retained PCs correspond to a certain percentange of the cummulative variance



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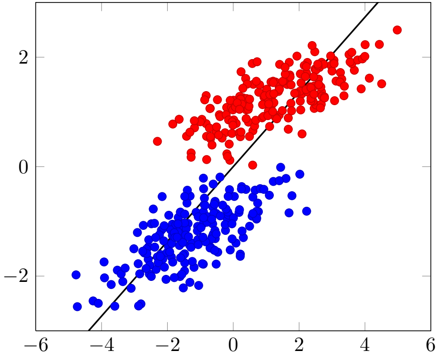
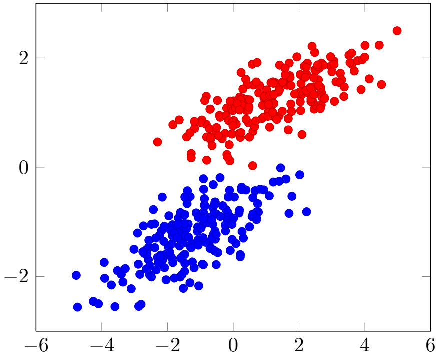


Linear Discriminant Analysis

- $h(\mathbf{x}) = \langle \mathbf{x}, \mathbf{v}_l \rangle$ such as $\mathbf{v}_l$ maximizes the separability of the classes
- $\mathbf{v}_l$ is computed by solving the generalized eigenvalue problem

$$\lambda_l \mathbf{S}_w \mathbf{v}_l = \mathbf{S}_b \mathbf{v}_l$$

where $\mathbf{S}_w = \sum \mathbf{\Sigma}_c$ and $\mathbf{S}_b = \sum (\boldsymbol{\mu}_c - \boldsymbol{\mu})(\boldsymbol{\mu}_c - \boldsymbol{\mu})^T$



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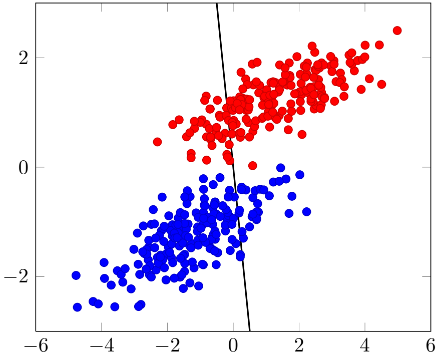
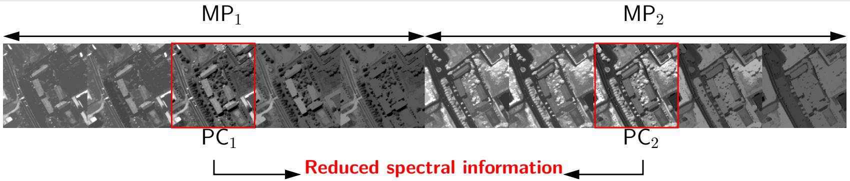


Extended Morphological Profile

Definition (Extended Morphological Profile - EMP)

The EMP of size $n \times p$ is a $(2n+1)p$-dimensional vector made of the MP build with the p first principal components:

$$\text{EMP}(\mathbf{x}) = \Big[\text{MP}_1(\mathbf{x}), \dots, \text{MP}_p(\mathbf{x})\Big].$$

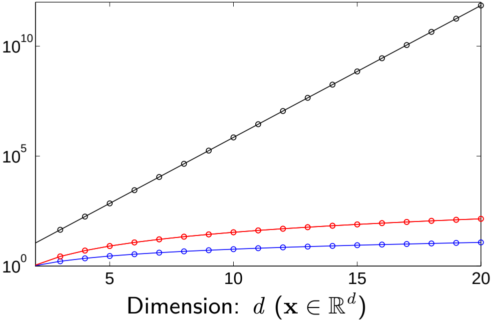


- Fusion of morphological and spectral features

High dimensional data

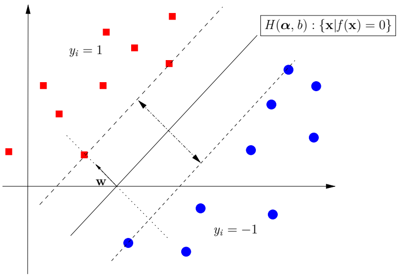
- Number of samples for *accurate* classification:

Linear	$\gg$	d
Quadratic	$\gg$	d^2
Non-parametric	$\gg$	$\exp(d)$



- Hyperspectral data: $d > 100$

Principle

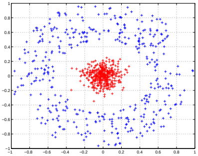


- Training set: $\mathcal{S} = \left\{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_\ell, y_\ell)\right\} \in \mathbb{R}^n \times \{-1; 1\}$
- Learn a function $f(\mathbf{z}) = \text{sgn}\left(\sum_{i=0}^{\ell} \alpha_i k(\mathbf{z}, \mathbf{x}_i) + b\right)$
- Solve:

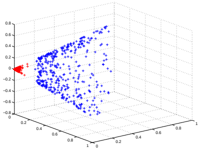
$$\begin{aligned} \max_{\alpha} \; g(\alpha) \quad &= \quad \sum_{i=1}^{\ell} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{\ell} \alpha_i \alpha_j y_i y_j k(\mathbf{x}_i, \mathbf{x}_j) \\ \text{subject to} \quad & \quad 0 \leq \alpha_i \leq C \text{ and } \textstyle\sum_{i=1}^{\ell} \alpha_i y_i = 0 \end{aligned}$$

Kernel trick

- Kernel function k: semi-definite positive $k(\mathbf{x}_i, \mathbf{x}_j) = \langle \Phi(\mathbf{x}_i), \Phi(\mathbf{x}_j) \rangle_{\mathcal{H}}$

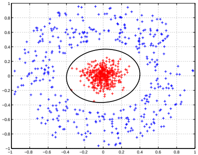


$\xrightarrow{\Phi}$



$\longrightarrow \langle \Phi(\mathbf{x}_i), \Phi(\mathbf{x}_j) \rangle_{\mathcal{H}}$

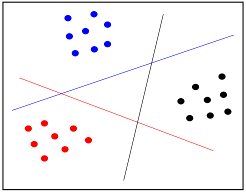
$k(\mathbf{x}_i, \mathbf{x}_j) \longrightarrow$



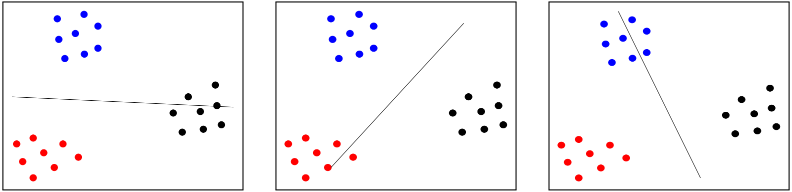
- Some kernels:
  - ⋆ Linear kernel: $k(\mathbf{x}_i, \mathbf{x}_j) = \langle \mathbf{x}_i, \mathbf{x}_j \rangle_{\mathbb{R}^n}$
  - ⋆ Polynomial kernel: $k(\mathbf{x}_i, \mathbf{x}_j) = \Big(\langle \mathbf{x}_i, \mathbf{x}_j \rangle_{\mathbb{R}^n} + q \Big)^p$
  - ⋆ Gaussian kernel: $k(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(- \frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2_{\mathbb{R}^n}}{2\sigma^2} \right)$

Multiclass problem

- One versus All: m binary classifiers

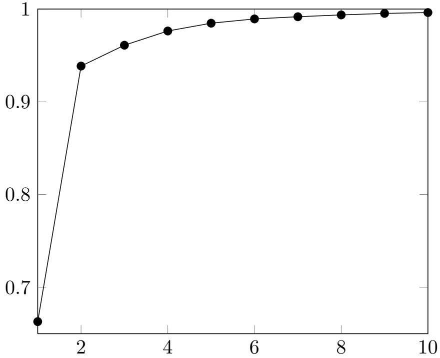


- One versus One: $m(m\text{-}1)/2$ classifiers



EMP

- Inputs: $\text{EMP}(\mathbf{x})$, $\dim\big(\text{EMP}(\mathbf{x})\big) = (2n+1)p$
- Parameters:
  - ⋆ Numbers of PCs $\rightarrow p$
  - ⋆ Size of MP $\rightarrow n$
  - ⋆ Hyperparameters



Cumulative variance

ROSIS-03

University Area, Pavia - Italy

- Airbone
- [H W]=[610 340]
- 103 channels
- 1.3 m/pixel
- 43 923 referenced samples
- 9 classes : Asphalts, Meadow, Gravel, Tree, Metal Sheet, Bare Soil, Bitumen, Brick and Shadow.

Quantitative analysis

Let's generate some results!

Discussion

- Mathematical Morphology:
  - Morphological operators extract relevant spatial features for classification
  - Connectivity is a key property
  - Extended profiles should be constructed with advance FE algorithms
  - The use of attributes profile allows the extraction of relevant spatial features
- SVM:
  - Ability to include additional spatial information
  - Robust to the dimensionality
- Classification:
  - Analyst intervention is minimized
  - Classification accuracy
  - Works with other geographical area

Perspectives

- Supervised or un-supervised FE?
- Best attribute?
- Time processing?

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Image analysis of hyperspectral data using mathematical morphology

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