

Distributed Causal Memory: Modular Specification and Verification in Higher-Order Distributed Separation Logic

Technical Appendix

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A Summary of the Specification of the Causally Consistent Distributed Database

Given client-provided *Addrlist* and *Keys*, we provide types, functions, predicates, and lemmas as in Section 3 (Mathematical Model), and given those, the specification of the causally-consistent distributed database is as follows. The specification existentially quantifies over abstract predicates; then follows all the laws governing the abstract predicates from Section 4 (Specification) and Section 7 (HOCAP Style Specification for Write) (here we only show a few) and finally the specification for the initialization operation.

$$\begin{aligned}
 & \exists \text{GlobalInv} : iProp. \\
 & \exists (\cdot) \rightarrow_s (\cdot) : \text{Keys} \rightarrow \mathcal{P}_{\text{fin}}(\text{WriteEvent}) \rightarrow iProp. \\
 & \exists (\cdot) \rightarrow_u (\cdot) : \text{Keys} \rightarrow \mathcal{P}_{\text{fin}}(\text{WriteEvent}) \rightarrow iProp. \\
 & \exists \text{Seen} : \mathbb{N} \rightarrow \text{LocalHistory} \rightarrow iProp. \\
 & \exists \text{Snap} : \text{Keys} \rightarrow \mathcal{P}_{\text{fin}}(\text{WriteEvent}) \rightarrow iProp. \\
 & \exists \text{initToken} : \mathbb{N} \rightarrow iProp. \\
 & \exists \Phi_{\text{DB}} : \text{Message} \rightarrow iProp. \\
 & \quad \text{Snap}(x, h) * \text{Snap}(x, h') \vdash \text{Snap}(x, h \cup h') \\
 & \wedge \quad x \rightarrow_u h \vdash x \rightarrow_u h * \text{Snap}(x, h) \\
 & \wedge \quad \dots \\
 & \wedge \quad \boxed{\text{GlobalInv}}^{N_{\text{Gl}}} * \text{Seen}(i, s) * x \rightarrow_u h \vdash \models_{\mathcal{E}} \forall a \in s, w \in h. w.t < a.t \Rightarrow \exists a' \in s_{|x}. [a'] = w \\
 & \wedge \quad \text{True} \vdash \models_{\mathcal{E}} \boxed{\text{GlobalInv}}^{N_{\text{Gl}}} * \left(\bigstar_{0 \leq i < \text{length}(\text{Addrlist})} \text{initToken}(i) \right) * \left(\bigstar_{k \in \text{Keys}} k \rightarrow_u \emptyset \right) * \text{initSpec}(\text{init})
 \end{aligned}$$

where

$$\begin{aligned}
 \text{initSpec}(\text{init}) & \triangleq \\
 & \left\{ \begin{array}{l} \text{initToken}(i) * \text{Fixed}(A) * \text{IsNode}(ip_i) * \text{Addrlist}[i] = (ip_i, p) * \\ (ip_i, p) \in A * \text{isList}(\text{Addrlist}, v) * \text{FreePorts}(ip_i, \{p\}) * \bigstar_{z \in \text{Addrlist}} z \models \Phi_{\text{DB}} \end{array} \right\} \\
 & \quad \langle ip_i; \text{init}(i, v) \rangle \\
 & \quad \{ (rd, wr). \text{Seen}(i, \emptyset) * \text{readSpec}(rd, i) * \text{writeSpec}(wr, i) \} \\
 \text{readSpec}(\text{read}, i) & \triangleq
 \end{aligned}$$

$$\begin{aligned}
& \{ \text{Seen}(i, s) \} \\
& \langle ip_i; \text{read}(x) \rangle \\
& \left\{ \begin{array}{l} v. \exists s' \supseteq s. \text{Seen}(i, s') * \\ \left((v = \text{None} \wedge s'_{|x} = \emptyset) \vee \right. \\ \left. (\exists a \in s'_{|x}. v = \text{Some}(a.v) * \text{Snap}(x, \{ \lfloor a \rfloor \}) * a \in \text{Maximals}(s'_{|x}) * \text{Observe}(s'_{|x}) = a) \right) \end{array} \right\} \\
& \text{writeSpec}(\text{write}, i) \triangleq \\
& \forall \mathcal{E}, k, v, s, P, Q. \mathcal{N}_{\text{GI}} \subseteq \mathcal{E} \Rightarrow \\
& \square (\forall s', a. (s \subseteq s' * a \notin s' * a.k = k * a.v = v * P) \\
& \quad \begin{array}{l} \vdash^{\mathcal{E}} \forall h. \left(\begin{array}{l} \lfloor a \rfloor \notin h * \lfloor a \rfloor \in \text{Maximals}(h \uplus \{ \lfloor a \rfloor \}) * k \rightarrow_s h * \\ \text{Seen}(i, s' \uplus \{a\}) * \text{Maximum}(s' \uplus \{a\}) = a \end{array} \right) \\ \Rightarrow_{\mathcal{E} \setminus \mathcal{N}_{\text{GI}}} k \rightarrow_s h \uplus \{ \lfloor a \rfloor \} * \mathcal{E} \models^{\top} Q \ a \ h \ s' \rangle \\ \{ P * \text{Seen}(i, s) \} \langle ip_i; \text{write}(k, v) \rangle \{ v.v = () * \exists h, s', a. s \subseteq s' * Q \ a \ h \ s' \}
\end{array}
\end{aligned}$$

B Client Reasoning about Causality: Indirect Causal Dependency

The proof of the indirect causal dependency example [?] makes use of predicates

$$\begin{aligned}
& \text{pending} : iProp \\
& \text{shot} : \text{WriteEvent} \rightarrow iProp \\
& \text{hist} : \mathcal{P}_{\text{fin}}(\text{WriteEvent}) \xrightarrow{\text{fin}} iProp
\end{aligned}$$

satisfying the laws below. The predicates pending and shot are defined using the *oneshot* resource algebra [?] whereas the hist predicate is defined using a fractional agreement resource algebra. We refer to the Coq formalization for all the details and the full proof.

Laws for ghost resources

$$\begin{aligned}
& \text{pending} \Rightarrow * \text{shot}(w) \\
& \text{shot}(w) * \text{shot}w' \vdash w = w' \\
& \text{pending} * \text{shot}(w) \vdash \text{False} \\
& \lceil \diamond \rceil * \lceil \diamond \rceil \vdash \text{False} \\
& \text{hist}(h_1) * \text{hist}(h_2) \vdash h_1 = h_2 \\
& \text{hist}(h_1) * \text{hist}(h_1) \vdash \text{hist}(h_2) * \text{hist}(h_2)
\end{aligned}$$

Invariants

$$\begin{aligned}
& \text{Inv}_x \triangleq \exists h. x \rightarrow_u h * \text{hist}(h) * ((\text{pending} * \forall w \in h. w.v \neq 1) \vee \\
& \quad (\lceil \diamond \rceil * \exists w. \text{shot}(w) * w.v = 37 * \text{Maximum}(h) = \text{Some}(w) * \\
& \quad \forall w' \in h. w'.v = 37 \Rightarrow w = w')) \\
& \text{Inv}_y \triangleq \exists h. y \rightarrow_u h * \forall w \in h. w.v = 1 \Rightarrow \exists w'. w'.t < w.t * \text{shot}(w')
\end{aligned}$$

Proof sketch*Node i, proof outline*

$$\begin{array}{l}
\{ \text{Seen}(i, s) * \text{hist}(h) * [\diamond] * h \subseteq [s] * [\text{Inv}_x] \} \\
\left| \text{open } \text{Inv}_x \right. \begin{array}{l}
\{ \text{hist}(h) * \text{hist}(h) * x \rightarrow_u h * \text{pending} * [\diamond] \} \\
\text{write}(x, 0) \\
\left\{ \begin{array}{l}
\exists a, s' \supseteq s. \text{Seen}(i, s' \uplus \{a\}) * \text{hist}(h \uplus \{[a]\}) \\
* \text{hist}(h \uplus \{[a]\}) * x \rightarrow_u h \uplus \{[a]\} * \text{pending} * [\diamond]
\end{array} \right\}
\end{array} \\
\{ \exists h', s'. \text{Seen}(i, s') * \text{hist}(h) * [\diamond] * h' \subseteq s' * [\text{Inv}_x] \} \\
\{ \text{Seen}(i, s') * \text{hist}(h') * [\diamond] * h' \subseteq [s'] * [\text{Inv}_x] \} \\
\left| \text{open } \text{Inv}_x \right. \begin{array}{l}
\{ \text{hist}(h) * \text{hist}(h) * x \rightarrow_u h * \text{pending} * [\diamond] \} \\
\text{write}(x, 37) \\
\left\{ \begin{array}{l}
\exists s'' \supseteq s', a'. \text{Seen}(i, s'' \uplus \{a'\}) * \text{hist}(h \uplus \{[a']\}) * \text{hist}(h \uplus \{[a']\}) * x \rightarrow_u h \uplus \{[a']\} \\
* [\diamond] * \text{Maximum}(h \uplus \{[a']\}) = \text{Some}([a']) * \text{shot}([a'])
\end{array} \right\}
\end{array} \\
\{ \exists h', s''. \text{Seen}(i, s'') * \text{hist}(h') * [\text{Inv}_x] \}
\end{array}$$

Node j, proof outline

$$\begin{array}{l}
\{ \text{Seen}(j, s) * [\text{Inv}_x] * [\text{Inv}_y] \} \\
\text{wait}(x = 37) \\
\{ \exists s' \supseteq s, a_x. \text{Seen}(j, s) * \text{shot}([a_x]) * w_x \in s' * \dots \} \\
\{ \text{Seen}(j, s') * \text{shot}(w_x) \} \\
\left| \text{open } \text{Inv}_y \right. \begin{array}{l}
\{ \exists h_y, s'. x \rightarrow_u h_y * \dots \} \\
\text{write}(y, 1) \\
\left\{ \begin{array}{l}
\exists a_y, s'' \supseteq s'. \text{Seen}(j, s'' \uplus \{a_y\}) * x \rightarrow_u h_y \uplus \{[a_y]\} * \text{shot}(w_x) * \\
a_y.v = 1 * a_y \in s'' * [a_x].t < [a_y].t
\end{array} \right\}
\end{array} \\
\{ \text{Seen}(j, s'') * [\text{Inv}_y] \}
\end{array}$$

Node k, proof outline

$$\begin{array}{l}
\{ \text{Seen}(k, s) * [\text{Inv}_x] * [\text{Inv}_y] * \} \\
\text{wait}(y = 1) \\
\{ \exists s' \subseteq s, a_y \in s', w_x. \text{Seen}(k, s') * \text{shot}(w_x) * w_x.t < [a_y].t * a_y.v = 1 \} \\
\{ \text{Seen}(k, s') * \text{shot}(w_x) \} \\
\left| \text{open } \text{Inv}_x \right. \begin{array}{l}
\{ \exists h_x. x \rightarrow_u h_x * \text{shot}(w_x) * \text{shot}(w_x) * \text{Maximum}(h_x) = \text{Some}(w_x) * w_x.v = 37 \} \\
\text{read}(x) \\
\{ v. \exists s''. \text{Seen}(k, s'') * v = \text{Some}(37) \}
\end{array} \\
\{ v. \exists s''. \text{Seen}(k, s'') * v = \text{Some}(37) \}
\end{array}$$

C Case Study: Towards Session Guarantees for Client-Centric Consistency

C.1 Session Manager Library Implementation

```

(* Client stub *)
type db_key
type db_val

type sm_req =
  | InitReq
  | ReadReq of db_key
  | WriteReq of db_key * db_val

type sm_res =
  | InitRes
  | ReadRes of db_val
  | WriteRes

let rec listen_wait_for_seqid skt seq_id =
  let res_raw = listen_wait skt in
  let res = deser_res (fst res_raw) in
  let tag = fst res in
  let vl = snd res in
  if (tag = !seq_id) then
    (seq_id := !seq_id + 1;
     vl)
  else
    listen_wait_for_seqid skt seq_id

let session_exec skt seq_id lock server_addr
  req =
  acquire lock;
  let msg = ser_req req in
  sendTo skt msg server_addr;
  let res = listen_wait_for_seqid skt seq_id
  in
  release lock;
  res

(* Request handler *)
let rec request_handler skt rd_fn wr_fn =
  let req_raw = listen_wait skt in
  let sender = snd req_raw in
  let req = deser_req (fst req_raw) in
  let seq_id = fst req in
  let res =
    match (snd req) with
    | Some InitReq → InitRes
    | Some ReadReq k → ReadRes (rd_fn k)
    | Some WriteReq (k, v) → wr_fn k v;
    WriteRes
    | None → assert false
  in
  sendTo skt (ser_res (seq_id, res)) sender;
  request_handler skt rd_fn wr_fn

let server dbs db_id req_addr =
  let fns = init dbs db_id in
  let rd_fn = fst fns in
  let wr_fn = snd fns in
  let skt = socket () in
  socketbind skt req_addr;
  request_handler skt rd_fn wr_fn

let sm_setup client_addr =
  let skt = socket () in
  socketbind skt client_addr;
  let seq_id = ref 0 in
  let lock = newlock () in
  let connect_fn server_addr =
    session_exec skt seq_id lock server_addr
    InitReq;
    ()
  in
  let read_fn server_addr key =
    session_exec skt seq_id lock server_addr
    (ReadReq key)
  in
  let write_fn server_addr key vl =
    session_exec skt seq_id lock server_addr
    (WriteReq (key, vl));
    ()
  in
  (connect_fn, read_fn, write_fn)

```

C.2 Session Manager Specifications

SM-INIT

$$\{\top\} \langle ip_{client}; \text{sconnect}(ip_i) \rangle \left\{ \exists s. \text{Seen}(i, s) * \bigstar_{k \in \text{Keys}} \exists h_k. \text{Snap}(k, h_k) * \boxed{\text{GlobalInv}}^{\mathcal{N}_{\text{GI}}} \right\}$$

SM-READ

$$\{ip_i \Rightarrow \Phi_i * \text{Seen}(i, s) * \text{Snap}(k, h)\} \\ \langle ip_{client}; \text{sread}(ip_i, k) \rangle \\ \left\{ \begin{array}{l} v. \exists s' \supseteq s, h' \supseteq h. * \text{Seen}(i, s') * \text{Snap}(k, h') * \\ (v = \text{None} * s'|_k = \emptyset) \\ \vee (\exists a, w. v = \text{Some}(w) * a.v = w * a.k = k * a \in \text{Maximals}(s'|_k) * [a] \in h') \end{array} \right\}$$

SM-WRITE

$$\{ip_i \Rightarrow \Phi_i * \text{Seen}(i, s) * \text{Snap}(k, h)\} \\ \langle ip_{client}; \text{swrite}(ip_i, k, v) \rangle \\ \left\{ \begin{array}{l} \exists a, s' \subseteq s, h' \subseteq h. a.k = k * a.v = v * \text{Seen}(i, s') * \text{Snap}(k, h') \\ * a \notin s * a' \in s' * [a] \notin h * [a'] \in h' * [a] \in \text{Maximals}(h') * \text{Maximum}(s') = \text{Some}(a) \end{array} \right\}$$

C.3 Session Guarantees Specifications

SM-READ-YOUR-WRITES

$$\{ip_i \Rightarrow \Phi_i\} \\ \langle ip_{client}; \text{sconnect}(ip_i); \text{swrite}(ip_i, k, v_w); \text{sread}(ip_i, k) \rangle \\ \left\{ \begin{array}{l} \exists s, a_w, a_r, v_r. v_o = \text{Some}(v_r) * a_w.k = k * a_w.v = v_w * a_r.k = k * a_r.v = v_r \\ v_o. * \text{Seen}(a, s) * a_w, a_r \in s * \neg(a_r.t < a_w.t) \end{array} \right\}$$

SM-MONOTONIC-READS

$$\{ip_i \Rightarrow \Phi_i\} \\ \langle ip_{client}; \text{sconnect}(ip_i); \text{let } v_1 = \text{sread}(ip_i, k) \text{ in let } v_2 = \text{sread}(ip_i, k) \text{ in } (v_1, v_2) \rangle \\ \left\{ \begin{array}{l} \exists v_{o1}, v_{o2}. v_o = (v_{o1}, v_{o2}) \\ \left(\begin{array}{l} (\exists s. v_{o1} = \text{None} * v_{o2} = \text{None} * \text{Seen}(a_1, s) * s|_k = \emptyset) \\ \vee \\ (\exists s, v_2, a_2. v_{o1} = \text{None} * v_{o2} = \text{Some}(v_2) * \text{Seen}(a_1, s) \\ * a_2.k = k * a_2.v = v_2 * a_2 \in \text{Maximals}(s|_k)) \\ \vee \\ (\exists s, v_1, v_2, a_1, a_2. v_{o1} = \text{Some}(v_1) * v_{o2} = \text{Some}(v_2) * \text{Seen}(a_1, s) \\ * a_1.k = k * a_1.v = v_1 * a_2.k = k * a_2.v = v_2 * a_2 \in \text{Maximals}(s|_k) \\ * \neg(a_2.t < a_1.t)) \end{array} \right) \end{array} \right\}$$

SM-MONOTONIC-WRITES

 $\{ip_i \models \Phi_i\}$

$$\langle ip_{client}; sconnect(ip_i); swrite(ip_i, k_1, v_1); swrite(ip_i, k_2, v_2) \rangle$$

$$\left\{ \begin{array}{l} \exists s_1, a_1, a_2. a_1.k = k_1 * a_1.v = v_1 * a_2.k = k_2 * a_2.v = v_2 \\ * \text{Seen}(a_1, s_1) * a_1, a_2 \in s_1 * a_1.t < a_2.t \\ * (\forall a, s_2, a_2. \text{Seen}(a_2, s_2) * a \in s_2 * a_2.t \leq e.t \\ \quad \Rightarrow *_{\top} \exists a'_1, a'_2. \lfloor a'_1 \rfloor = \lfloor a_1 \rfloor * \lfloor a'_2 \rfloor = \lfloor a_2 \rfloor * a'_1, a'_2 \in s_2 * a'_1.t < a'_2.t) \end{array} \right\}$$

SM-WRITES-FOLLOW-READS

 $\{ip_i \models \Phi_i\}$

$$\langle ip_{client}; sconnect(ip_i); \text{let } v = \text{sread}(ip_i, k_r) \text{ in } swrite(ip_i, k_w, v_w); v \rangle$$

$$\left\{ \begin{array}{l} \exists s_1, a_w. a_w.k = k_w * a_w.v = v_w * \text{Seen}(a_1, s_1) * a_w \in s_1 \\ v_o. \left(\begin{array}{l} v_o = \text{None} \\ \vee \\ \exists a_r, v_r. v_o = \text{Some}(v_r) * a_r.k = k_r * a_r.v = v_r * a_r \in s_1 * a_r.t < a_w.t \\ * (\forall a, s_2, a_2. \text{Seen}(a_2, s_2) * a \in s_2 * a_w.t \leq e.t \\ \quad \Rightarrow *_{\top} \exists a'_r, a'_w. \lfloor a'_r \rfloor = \lfloor a_r \rfloor * \lfloor a'_w \rfloor = \lfloor a_w \rfloor * a'_r, a'_w \in s_2 * a'_r.t < a'_w.t) \end{array} \right) \end{array} \right\}$$