

Locality and non-locality

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Abstract—Aim: Whether, under all the circumstances considered, a relativistic concept of locality and non-locality may fully reproduce the quantum probabilities for outcomes of experiments, is re-investigated.

Methods: The usual methods and rules of statistics, probability theory and quantum mechanics were used.

Results: The interior logic of the variance has been re-investigated. A relationship between the Pythagorean theorem and the variance has been established. A n -dimensional Pythagorean theorem has been derived. The problem of locality and non-locality and the relationship to the variance has been analysed.

Conclusion: It may no longer stay an open question how to deal with the notions of locality and non-locality.

Index Terms—Locality, Non-locality, Pythagorean theorem, Unified field theory, Causality.

I. INTRODUCTION

THE Pythagorean Theorem and Euler's identity [51] are arguably one of the most beautiful equations in mathematics[7]. However, in physics, the Pythagorean Theorem is of importance too. Meanwhile, there are more than 371 Pythagorean Theorem proofs known [37]. Pythagoras of Samos, as one of the well known ancient Greek philosophers, lived from ca. 570 to ca. 490 before current era (BCE). There is neither evidence that Pythagoras is an author of original scientific and other publications nor are there any detailed accounts of his thought written by contemporaries available. Euclid of Megara (ca. 435 – ca. 365 BC), the “founder of geometry” [22] and a pupil of Socrates lived about 200 years later than Pythagoras but does not give any credit to Pythagoras in connection with Proposition 47 in his book Elements. The glorification of Pythagoras is immortally linked with the Pythagorean Theorem which itself seems to stem from the authority of several Greek and Latin authors like Plutarch and Cicero. However, these authors wrote half a millennium after Pythagoras and in turn, rely on a single source – Apollodorus. In other words, it is unknown, what the historical Pythagoras actually did and thought and what were the practices and beliefs of Pythagoras. However, in order to determine the historical origin of the Pythagorean Theorem, it is important to consider the concrete evidence of the discovery and of the proof of the Pythagorean Theorem by Babylonian mathematicians [38] more than 1000 years before [40] Pythagoras was born. Moreover, the implications of the Pythagorean Theorem seemingly has no ending.

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II. MATERIAL AND METHODS

The assumptions, foundations, and implications of mathematics and physics and other sciences need to be logically consistent otherwise controversial arguments, disagreements, conflicts and fundamental misunderstandings are prone to come.

A. Definitions

A logically sound definition of a mathematical object is extremely valuable for any further scientific investigation. However, even this publication has no answer for the question what is **the mathematical definition of a mathematical definition?**

1) *Definition:*

2) *The number +0:*

Definition II.1 (The number +0). Let c denote the speed of light in vacuum [24], [45], [48], [49], let ϵ_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary number [20]. The number +0 is defined as the expression

$$\begin{aligned} +0 &\equiv +1 - 1 \\ &\equiv +1 + i^2 \equiv +1 + e^{i\pi} \\ &\equiv + (c^2 \times \epsilon_0 \times \mu_0) + e^{i\pi} \quad (1) \end{aligned}$$

while ‘ \equiv ’ or ‘ \equiv ’ denotes the equals sign [41] or equality sign [42] used to indicate equality and ‘ $-$ ’ [39], [50] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and ‘ $+$ ’ denotes the plus [41] signs used to represent the operations of addition and the notions of positive as well.

Remark II.1. Roger Cotes (1682 – 1716) [23] or Leonhard Euler's (1707 – 1783) identity [28] is regarded as one of the most beautiful equations [51]. In this context, it is provisionally presumed, that Euler's identity [28] is logically sound and correct. However, the definition of the basic numbers +1 and +0 in terms of Euler's identity and physical ‘constants’ offer us the possibility to test classical logic or mathematical theorems et cetera by reproduceable physical experiments. In particular, it is very remarkable that Leibniz [34] himself published in 1703 the first self-consistent binary number system representing all numeric values while using typically 0 (zero) and 1 (one).

3) *The number +1:*

Definition II.2 (The number +1). Again, let c denote the speed of light in vacuum [24], [45], [48], [49], let ϵ_0 denote the electric constant and let μ_0 the magnetic constant. Let i

denote the imaginary number [20]. The number $+1$ is defined as the expression

$$\begin{aligned} +1 &\equiv +1 + 0 \equiv +1 - 0 \\ &\equiv -i^2 \equiv -e^{i\pi} \\ &\equiv + (c^2 \times \epsilon_0 \times \mu_0) \end{aligned} \quad (2)$$

while again ' $=$ ' or ' \equiv ' may denote the equals sign [41] or equality sign [42] used to indicate equality and ' $-$ ' [39], [50] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and ' $+$ ' denotes the plus [41] signs used to represent the operations of addition and the notions of positive as well.

4) Wave function:

Definition II.3 (Wavefunction). Let $p({}_R X_t)$ represent the probability from the point of view of a stationary observer R for finding a certain particle X at a given point in space at a given time / Bernoulli trial t . Let $E({}_R X_t^2)$ denote the expectation value of ${}_R X_t^2$. Let $E({}_R X_t)$ denote the expectation value of ${}_R X_t$. Let $\sigma({}_R X_t)$ denote the standard deviation of ${}_R X_t$. Let $\sigma({}_R X_t)^2$ denote the variance of ${}_R X_t$. Let the wavefunction represent the probability amplitude [21] of an event or of finding an event (i. e. a particle) at a given point in space at a given (period of) time / Bernoulli trial [47] t . In general, it is

$$\begin{aligned} p({}_R X_t) &\equiv \frac{E({}_R X_t)}{{}_R X_t} \\ &\equiv \frac{E({}_R X_t)^2}{E({}_R X_t^2)} \\ &\equiv \frac{p({}_R X_t)^2 \times ({}_R X_t)^2}{p({}_R X_t) \times ({}_R X_t)^2} \\ &\equiv \Psi({}_R X_t) \times \Psi^*({}_R X_t) \end{aligned} \quad (3)$$

From this definition follows that

$$\begin{aligned} \Psi({}_R X_t) &\equiv \frac{1}{\Psi^*({}_R X_t)} \times p({}_R X_t) \\ &\equiv \frac{p({}_R X_t)}{\Psi^*({}_R X_t \times f({}_R X_t))} \times f({}_R X_t) \\ &\equiv \frac{p({}_R X_t)}{\underbrace{\Psi^*({}_R X_t) \times f({}_R X_t)}_{{}_R A_t}} \times f({}_R X_t) \\ &\equiv {}_R A_t \times f({}_R X_t) \\ &\equiv \frac{1}{\Psi^*({}_R X_t)} \times \frac{E({}_R X_t)}{{}_R X_t} \\ &\equiv \frac{1}{\Psi^*({}_R X_t) \times {}_R X_t} \times E({}_R X_t) \end{aligned} \quad (4)$$

Lemma II.1. It is

$${}_R A_t \equiv \frac{\Psi({}_R X_t)}{f({}_R X_t)} \quad (5)$$

Proof. Multiplying the equation

$$\Psi({}_R X_t) \equiv \Psi({}_R X_t) \quad (6)$$

by the term $f({}_R X_t)/f({}_R X_t)$ it is

$$\Psi({}_R X_t) \equiv \frac{\Psi({}_R X_t)}{f({}_R X_t)} \times f({}_R X_t) \quad (7)$$

At the same time it has to be that $\Psi({}_R X_t) \equiv {}_R A_t \times f({}_R X_t) \equiv \frac{\Psi({}_R X_t)}{f({}_R X_t)} \times f({}_R X_t)$ and it follows that

$${}_R A_t \equiv \frac{\Psi({}_R X_t)}{f({}_R X_t)} \quad (8)$$

Quod erat demonstrandum.

5) The variance:

Definition II.4 (The variance). Sir Ronald Aylmer Fisher (1890 – 1962), an English statistician, “the single most important figure in 20th century statistics” [26] coined the term variance as follows: “It is therefore desirable in analysing the causes of variability to deal with the square of the standard deviation as the measure of variability. We shall term this quantity the Variance ...” [see 30, p. 399] Again, let $p({}_R X_t)$ represent the probability from the point of view of a stationary observer R for finding a certain particle X at a given point in space at a given time / Bernoulli trial t . Let $E({}_R X_t)$ denote the expectation value of ${}_R X_t$. The expectation value of ${}_R X_t$ is defined as

$$E({}_R X_t) \equiv p({}_R X_t) \times ({}_R X_t) \equiv \Psi({}_R X_t) \times {}_R X_t \times \Psi^*({}_R X_t) \quad (9)$$

The expectation value of the other of ${}_R X_t$, of ‘the local hidden variable’ of ${}_R X_t$, of the complementary of ${}_R X_t$, of the opposite of ${}_R X_t$, of the **anti** ${}_R X_t$, denoted by $\underline{{}_R X_t}$, is defined as

$$E(\underline{{}_R X_t}) \equiv (1 - p({}_R X_t)) \times ({}_R X_t) \quad (10)$$

Let $E({}_R X_t^2)$ denote the expectation value of ${}_R X_t^2$. The expectation value of ${}_R X_t^2$ is defined as

$$E({}_R X_t^2) \equiv p({}_R X_t) \times ({}_R X_t^2) \equiv p({}_R X_t) \times ({}_R X_t \times {}_R X_t) \quad (11)$$

Let $\sigma({}_R X_t)$ denote the standard deviation of ${}_R X_t$. Let $\sigma({}_R X_t)^2$ denote the variance of ${}_R X_t$. In general, the variance [see 33, p. 42] is defined as

$$\begin{aligned} \sigma({}_R X_t)^2 &\equiv \sigma({}_R X_t) \times \sigma({}_R X_t) \\ &\equiv E({}_R X_t - E({}_R X_t))^2 \\ &\equiv E({}_R X_t^2) - (E({}_R X_t))^2 \\ &\equiv ({}_R X_t^2 \times p({}_R X_t)) - (p({}_R X_t) \times {}_R X_t)^2 \\ &\equiv ({}_R X_t^2) \times (p({}_R X_t) - p({}_R X_t)^2) \\ &\equiv ({}_R X_t^2) \times (p({}_R X_t) \times (1 - p({}_R X_t))) \\ &\equiv {}_R X_t \times (p({}_R X_t) \times {}_R X_t \times (1 - p({}_R X_t))) \end{aligned} \quad (12)$$

From equation 12 follows that

$$\begin{aligned} p({}_R X_t) \times (1 - p({}_R X_t)) &\equiv \frac{\sigma({}_R X_t)^2}{{}_R X_t^2} \\ &\equiv \frac{E({}_R X_t^2)}{{}_R X_t^2} - \frac{(E({}_R X_t))^2}{{}_R X_t^2} \\ &\equiv p({}_R X_t) - p({}_R X_t)^2 \end{aligned} \quad (13)$$

6) The complex conjugate:

Definition II.5 (The complex conjugate). The conjugate of a complex number denoted as conjugate $(a({}_R X_t) + (i \times b({}_R X_t)))$, where $i^2 \equiv -1$ is the imaginary [20], is defined as

$$\begin{aligned} \text{conjugate}(a({}_R X_t) + (i \times b({}_R X_t))) \\ \equiv (a({}_R X_t) - (i \times b({}_R X_t))) \end{aligned} \quad (14)$$

As proofed somewhere else, any complex number multiplied by its complex conjugate is a real number. It is

$$\begin{aligned} (a({}_R X_t) + (i \times b({}_R X_t))) \times (a({}_R X_t) - (i \times b({}_R X_t))) \\ \equiv (a({}_R X_t)^2) - (i^2 \times b({}_R X_t)^2) \\ \equiv (a({}_R X_t)^2) + (b({}_R X_t)^2) \end{aligned} \quad (15)$$

7) The right-angled triangle:

Definition II.6 (The right-angled triangle). A right-angled triangle is a triangle in which one angle is 90-degree angle. Let ${}_R C_t$ denote the hypotenuse, the side opposite the right angle (side ${}_R C_t$ in the figure 1). The sides a_t and b_t are called legs. In a right-angled triangle ABC, the side AC, which is abbreviated as b_t , is the side which is adjacent to the angle α , while the side CB, denoted as a_t , is the side opposite to angle α . The following figure 1 may illustrate an aright-angled triangle [see 18].

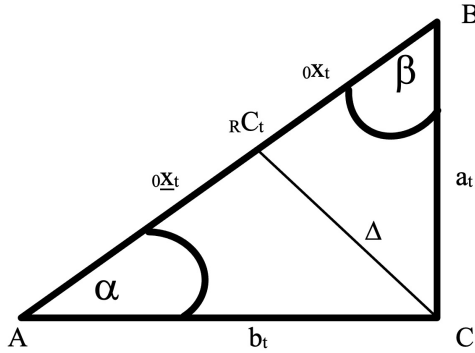


Figure 1. A right-angled triangle

8) The n-dimensional Pythagorean theorem:

Definition II.7 (The n-dimensional Pythagorean theorem). The famous Pythagorean theorem of Euclidean geometry is attributed to the Greek thinker Pythagoras of Samos (6th century, B.C.). The same is defined as

$${}_o a_t^2 + {}_o b_t^2 \equiv {}_R C_t^2 \quad (16)$$

where ${}_o$ may denote the point of view of a co-moving observer while ${}_R$ may denote the point of view of a stationary observer at a certain point in space-time t . In general, it is

$${}_o a_t^2 \equiv {}_o x_t \times {}_R C_t \quad (17)$$

or

$${}_o a_t^{2n} \equiv {}_o x_t^n \times {}_R C_t^n \quad (18)$$

Equally, it is

$${}_o b_t^2 \equiv {}_o x_t \times {}_R C_t \quad (19)$$

or

$${}_o b_t^{2n} \equiv {}_o x_t^n \times {}_R C_t^n \quad (20)$$

where n denotes the number of dimensions. The Pythagorean theorem can be extended to higher dimensions [see 52] too. In general, the Pythagorean theorem is based on the fundamental relationship [see 14]

$${}_o x_t + {}_o x_t \equiv {}_R C_t \quad (21)$$

Under conditions where ${}_o x_t$ and ${}_o x_t$ are described by a wave-function, **the superposition principle**, first stated by Daniel Bernoulli (1700 – 1782) in 1753 (“**Later (1753), Daniel Bernoulli formulated the principle of superposition ...**” [see 36, p. 2]) demands that $\Psi({}_o x_t) + \Psi({}_o x_t) \equiv \Psi({}_o x_t + {}_o x_t) \equiv \Psi({}_R C_t)$. In the n -dimensional case, the relationship before becomes

$$({}_o x_t + {}_o x_t)^n \equiv {}_R C_t^n \quad (22)$$

The Pythagorean theorem becomes something like

$$({}_o x_t + {}_o x_t)^n \times {}_R C_t^n \equiv {}_R C_t^n \times {}_R C_t^n \quad (23)$$

or as

$$\underbrace{{}_o x_t^n \times {}_R C_t^n}_{{}_o a_t^{2n}} + \underbrace{\dots}_{{}_o x_t^n \times {}_R C_t^n} \equiv {}_R C_t^n \times {}_R C_t^n \equiv {}_R C_t^{2n} \quad (24)$$

and at the end the n -dimensional Pythagorean theorem follows as

$${}_o a_t^{2n} + {}_o b_t^{2n} \equiv {}_R C_t^{2n} \quad (25)$$

Remark II.2. In general, it is [see 14]

$$\Delta^2 \equiv ({}_o x_t) \times ({}_o x_t) \quad (26)$$

(see figure 1). Under conditions where

$$E({}_o x_t) + E({}_o x_t) \equiv {}_R C_t \quad (27)$$

we obtain the identity of

$$\Delta^2 \equiv \sigma({}_R X_t)^2 \quad (28)$$

Especially general relativity is related to the Pythagorean theorem. General relativity is a theory of the geometrical properties of space-time too while the metric tensor $g_{\mu\nu}$ itself is of fundamental importance for general relativity. The metric tensor $g_{\mu\nu}$ is something like the generalisation of the Pythagorean theorem. Thus far, it does not appear to be necessary to restrict the validity of the Pythagorean theorem only to certain situations. The question is justified why the Riemannian geometry should be oppressed by the quadratic restriction. In this context, **Finsler geometry**, named after Paul Finsler (1894 - 1970) who studied it in his doctoral thesis [see 29] in 1918, appears to be a kind of a metric generalization of Riemannian geometry without the quadratic restriction and justifies the attempt to systematize and to extend the possibilities of general relativity.

		Curvature		
		YES	NO	
Momentum	YES	0	0	0
	NO	$(-\Lambda \times g_{\mu\nu})$	$(\frac{R}{2} \times g_{\mu\nu})$	$(\frac{R}{2} \times g_{\mu\nu} - \Lambda \times g_{\mu\nu})$

$$G_{\mu\nu} \quad \frac{R}{2} \times g_{\mu\nu} \quad R_{\mu\nu}$$

Table I

EINSTEIN FIELD EQUATIONS AND THE LAMBDAVACUUM SOLUTION

B. Axioms

1) *Axioms in general*: Axioms [32] and rules which are chosen carefully can be of use to avoid logical inconsistency and equally preventing science from supporting particular ideologies. Rightly or wrongly, long lasting advances in our knowledge of nature are enabled by suitable axioms [25] too. Einstein himself brings it again to the point. [see 27, p. 17]

“Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden.”[see 27, p. 17]

Einstein’s previous position now been translated into English: *The truly great advances in our understanding of nature originated in a manner almost diametrically opposed to induction*. It is worth mentioning in this matter, Einstein himself advocated especially basic laws (axioms) and conclusions derived from the same as a main logical foundation of any ‘theory’.

“Grundgesetz (Axiome) und Folgerungen zusammen bilden das was man eine ‘Theorie’ nennt.”[see 27, p. 17]

Albert Einstein’s (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a ‘theory’* has still to get round. However, it is currently difficult to ignore completely these historical and far reaching words of wisdom. The same taken more seriously and put into practice, will yield an approach to fundamental scientific problems which is more creative and sustainably logically consistent. Historically, Aristotle himself already cited **the law of excluded middle** and **the law of contradiction** as examples of axioms. However, **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and

equally as the most simple axiom of science. Something which is really just itself is equally different from everything else. In point of fact, is such an equivalence which everything has to itself inherent or must the same be constructed by human mind and consciousness. Following Gottfried Wilhelm Leibniz (1646-1716):

“Chaque chose est ce qu’elle est.
Et dans autant d’exemples qu’on voudra
A est A, B est B.”[see 35, p. 327]

or **A = A, B = B** or **+1 = +1**. In this context, **lex contradictionis**, the negative of **lex identitatis**, or **+0 = +1** is of no minor importance too.

2) *Axiom I. Lex identitatis*: To say that +1 is identical to +1 is to say that both are the same.

$$+1 \equiv +1 \quad (29)$$

However, even such a numerical identity which seems in itself wholly unproblematic, for it indicates just to a relation which something has to itself and nothing else, is still subject to controversy. Another increasingly popular view is that the same numerical identity implies the controversial view that we are talking about two different numbers +1. The one +1 is on the left side on the equation, the other +1 is on the right side of an equation. The basicness of the relation of identity implies the contradiction too while circularity is avoided. In other words, how can the same +1 be identical with itself and be equally different from itself? We may usefully state that identity is an utterly problematic notion and might be the most troublesome of all.

3) *Axiom II. Lex contradictionis*:

$$+0 \equiv +1 \quad (30)$$

A considerable obstacle to understanding contemporary usage of the term contradiction, however, is that contradiction does not seem to be a unitary one. How can something be both, itself (**a path is a straight line** from the standpoint of a co-moving observer at a certain point in space-time) **and** the other of itself, its own opposition (**the same path is not a straight line**, the same path is curved, from the standpoint of a stationary observer **at a certain point in space-time**) [16]. We may simply deny the existence of objective or of any other contradictions. Furthermore, even if it remains especially according to Einstein’s special theory of relativity that it is not guaranteed that the notion of an absolute contradiction is justified, Einstein’s special theory of relativity insist that contradictions are objective and real. That this is so highlights the fact that from the standpoint of a co-moving observer, under certain circumstances, **a path is a straight line** and nothing else. However, under the same circumstances of special theory of relativity where the relative velocity $v > 0$, from the standpoint of a stationary observer **the same path is a not a straight line, the path is curved**. The justified question is, why should and how can an identical be a contradictory too?

4) *Axiom III. Lex negationis:*

$$\neg (0) \times (+0) \equiv (+1) \quad (31)$$

where \neg denotes the (natural/logical) process of negation [see 17].

III. RESULTS

A. The local hidden variable

Theorem III.1 (The local hidden variable). *Let ${}_RX_t$ denote something, a random variable as seen from the point of view of a stationary observer R , a quantum mechanical entity et cetera existing independently and outside of human mind and consciousness. Let the probability of ${}_RX_t$ be $p({}_RX_t)$, let the expectation value of ${}_RX_t$ be*

$$E({}_RX_t) \equiv p({}_RX_t) \times ({}_RX_t) \quad (32)$$

. Let the expectation value of the other of ${}_RX_t$, of 'the local hidden variable' of ${}_RX_t$, of the complementary of ${}_RX_t$, of the opposite of ${}_RX_t$ be

$$E({}_R\underline{X}_t) \equiv (1 - p({}_RX_t)) \times ({}_R\underline{X}_t) \quad (33)$$

In general, it is

$${}_RX_t \equiv E({}_RX_t) + E({}_R\underline{X}_t) \quad (34)$$

Proof by modus ponens.. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \quad (35)$$

is true, **then** the conclusion

$${}_RX_t \equiv E({}_RX_t) + E({}_R\underline{X}_t) \quad (36)$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \quad (37)$$

is true. Multiplying this premise (i. e. axiom) by ${}_RX_t$ it is

$${}_RX_t \equiv {}_RX_t \quad (38)$$

Equation 38 can be rearranged as

$${}_RX_t \equiv (+1) \times {}_RX_t \quad (39)$$

too or as

$${}_RX_t \equiv (1 + 0) \times {}_RX_t \quad (40)$$

and equally as

$${}_RX_t \equiv (1 + p({}_RX_t) - p({}_RX_t)) \times {}_RX_t \quad (41)$$

or as

$${}_RX_t \equiv (p({}_RX_t) + (1 - p({}_RX_t))) \times {}_RX_t \quad (42)$$

too. Equation 42 simplifies as

$${}_RX_t \equiv (p({}_RX_t) \times {}_RX_t) + ((1 - p({}_RX_t)) \times {}_RX_t) \quad (43)$$

Equation 43 simplifies further according to the definition given (see equation 32) as

$${}_RX_t \equiv E({}_RX_t) + ((1 - p({}_RX_t)) \times {}_RX_t) \quad (44)$$

However, equation 44 itself simplifies again according to the definition given (see equation 33) as

$${}_RX_t \equiv E({}_RX_t) + E({}_R\underline{X}_t) \quad (45)$$

In other words, our conclusion is true.

Quod erat demonstrandum.

B. The inner contradiction

Theorem III.2 (The inner contradiction). *In general, it is*

$$\sigma({}_RX_t)^2 \equiv E({}_RX_t) \times E({}_R\underline{X}_t) \quad (46)$$

Proof by modus ponens.. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \quad (47)$$

is true, **then** the conclusion

$$\sigma({}_RX_t)^2 \equiv E({}_RX_t) \times E({}_R\underline{X}_t) \quad (48)$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \quad (49)$$

is true. Multiplying this premise (i. e. axiom) by ${}_RX_t$

$${}_RX_t \equiv {}_RX_t \quad (50)$$

According to theorem III.1, equation 45 it is ${}_RX_t \equiv E({}_RX_t) + E({}_R\underline{X}_t)$. Equation 50 changes to

$${}_RX_t \equiv E({}_RX_t) + E({}_R\underline{X}_t) \quad (51)$$

Rearranging equation 51, it is

$${}_RX_t - E({}_RX_t) \equiv E({}_R\underline{X}_t) \quad (52)$$

Taking the expectation value, it is [see 33, p. 42]

$$E({}_RX_t - E({}_RX_t)) \equiv \sigma({}_RX_t) \equiv E(E({}_R\underline{X}_t)) \quad (53)$$

Squaring equation 53, it is

$$\sigma({}_RX_t)^2 \equiv E({}_RX_t - E({}_RX_t))^2 \equiv E(E({}_R\underline{X}_t))^2 \quad (54)$$

According to Kolmogoroff [see 33, p. 42], it is easy to calculate that $\sigma({}_RX_t)^2 \equiv \sigma({}_RX_t) \times \sigma({}_RX_t) \equiv E({}_RX_t^2) - (E({}_RX_t))^2$. In general, we obtain

$$\sigma({}_RX_t)^2 \equiv E({}_RX_t^2) - (E({}_RX_t))^2 \equiv E(E({}_R\underline{X}_t))^2 \quad (55)$$

However, equation 55 can be simplified further. The expectation value of ${}_RX_t$ is defined (see equation 9) as $E({}_RX_t) \equiv p({}_RX_t) \times ({}_RX_t)$. The expectation value of ${}_RX_t^2$ is defined (see equation 11) as $E({}_RX_t^2) \equiv p({}_RX_t) \times ({}_RX_t^2) \equiv p({}_RX_t) \times ({}_RX_t \times {}_RX_t)$. Equation 55 changes to

$$\begin{aligned} \sigma({}_RX_t)^2 &\equiv (p({}_RX_t) \times ({}_RX_t \times {}_RX_t)) - (p({}_RX_t) \times ({}_RX_t))^2 \\ &\equiv E(E({}_R\underline{X}_t))^2 \end{aligned} \quad (56)$$

and equally to

$$\begin{aligned}\sigma({}_R X_t)^2 &\equiv (p({}_R X_t) \times ({}_R X_t)^2) - (p({}_R X_t)^2 \times ({}_R X_t)^2) \\ &\equiv E(E({}_R \underline{X}_t))^2\end{aligned}\quad (57)$$

Equation 57 can be simplified as

$$\begin{aligned}\sigma({}_R X_t)^2 &\equiv ({}_R X_t)^2 \times (p({}_R X_t) - p({}_R X_t)^2) \\ &\equiv E(E({}_R \underline{X}_t))^2\end{aligned}\quad (58)$$

or as

$$\begin{aligned}\sigma({}_R X_t)^2 &\equiv ({}_R X_t)^2 \times (p({}_R X_t) \times (1 - p({}_R X_t))) \\ &\equiv E(E({}_R \underline{X}_t))^2\end{aligned}\quad (59)$$

We rearrange equation 59 further. It is

$$\begin{aligned}\sigma({}_R X_t)^2 &\equiv {}_R X_t \times (p({}_R X_t) \times {}_R X_t \times (1 - p({}_R X_t))) \\ &\equiv E(E({}_R \underline{X}_t))^2\end{aligned}\quad (60)$$

Equation 60 simplifies (see definition II.4, equation 10) as

$$\begin{aligned}\sigma({}_R X_t)^2 &\equiv ({}_R X_t \times p({}_R X_t)) \times E({}_R \underline{X}_t) \\ &\equiv E(E({}_R \underline{X}_t))^2\end{aligned}\quad (61)$$

Furthermore, equation 61 simplifies (see definition II.4, equation 9) as

$$\begin{aligned}\sigma({}_R X_t)^2 &\equiv E({}_R X_t) \times E({}_R \underline{X}_t) \\ &\equiv E(E({}_R \underline{X}_t))^2\end{aligned}\quad (62)$$

At the end, it is

$$\sigma({}_R X_t)^2 \equiv E({}_R X_t) \times E({}_R \underline{X}_t) \quad (63)$$

In other words, our conclusion is true.

Quod erat demonstrandum.

Remark III.1. The variance is determined by the relationship between something and its own other as $\sigma({}_R X_t)^2 \equiv E({}_R X_t) \times E({}_R \underline{X}_t)$ and appears to be equally a measure of the inner contradiction between something and its own other too. **The greater the variance, the greater the inner contradiction between something and its own other, between the rich and the non-rich, between the healthy and the non-healthy, between the local and the non-local et cetera.**

C. The probability of a single event

Theorem III.3 (The probability of a single event). The complex conjugate is of use to find the probability of a single event. A wave-function which defines the probability amplitude may be determined as a complex function like $\Psi({}_R X_t) \equiv (a({}_R X_t) + (i \times b({}_R X_t)))$ while the complex conjugate of the wave-function is determined as $\Psi^*({}_R X_t) \equiv (a({}_R X_t) - (i \times b({}_R X_t)))$. The probability of a single event is defined in terms of the complex conjugate as

$$\begin{aligned}\Psi({}_R X_t) \times \Psi^*({}_R X_t) &\equiv p({}_R X_t) \\ &\equiv (a({}_R X_t) + (i \times b({}_R X_t))) \times (a({}_R X_t) - (i \times b({}_R X_t))) \\ &\equiv (a({}_R X_t)^2) - (i^2 \times b({}_R X_t)^2) \\ &\equiv (a({}_R X_t)^2) + (b({}_R X_t)^2)\end{aligned}\quad (64)$$

Proof by modus ponens.. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \quad (65)$$

is true, **then** the conclusion

$$\Psi({}_R X_t) \times \Psi^*({}_R X_t) \equiv (a({}_R X_t)^2) + (b({}_R X_t)^2) \quad (66)$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \quad (67)$$

is true. Multiplying this premise (i. e. axiom) by $\Psi({}_R X_t) \times \Psi^*({}_R X_t)$ it is

$$\Psi({}_R X_t) \times \Psi^*({}_R X_t) \equiv \Psi({}_R X_t) \times \Psi^*({}_R X_t) \quad (68)$$

The wave-function is determined as a complex function like $\Psi({}_R X_t) \equiv (a({}_R X_t) + (i \times b({}_R X_t)))$ while the complex conjugate of the wave-function is determined as $\Psi^*({}_R X_t) \equiv (a({}_R X_t) - (i \times b({}_R X_t)))$. Substituting these relationships into equation 68 we obtain

$$\begin{aligned}\Psi({}_R X_t) \times \Psi^*({}_R X_t) &\equiv (a({}_R X_t) + (i \times b({}_R X_t))) \\ &\quad \times (a({}_R X_t) - (i \times b({}_R X_t)))\end{aligned}\quad (69)$$

or

$$\begin{aligned}\Psi({}_R X_t) \times \Psi^*({}_R X_t) &\equiv p({}_R X_t) \\ &\equiv (a({}_R X_t)^2) - (i^2 \times b({}_R X_t)^2) \\ &\equiv (a({}_R X_t)^2) + (b({}_R X_t)^2)\end{aligned}\quad (70)$$

Quod erat demonstrandum.

D. Anti Chebyshev - The Chebyshev equality

Theorem III.4 (Anti Chebyshev - The Chebyshev equality). The Pafnuty Lvovich Chebyshev's (1821 – 1894) inequality (also called the Irénée-Jules Bienaymé [19] (1796 – 1878) – Chebyshev inequality) was proved by Chebyshev [46] in 1867 and late by his student Andrey Markov (1856–1922) in his 1884 Ph.D. thesis. Chebyshev's inequality [see 33, p. 42] is defined as $p(|{}_R X_t - E({}_R X_t)| \geq \sqrt{E({}_R X_t^2)}) \leq \frac{\sigma({}_R X_t)^2}{E({}_R X_t^2)}$ and provides in this form only an approximate value of the exact probability of a single event. The exact value of the probability of a single event (**Chebyshev's equality**) is given by

$$p({}_R X_t) \equiv 1 - \frac{\sigma({}_R X_t)^2}{E({}_R X_t^2)} \quad (71)$$

Proof by modus ponens.. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \quad (72)$$

is true, **then** the conclusion

$$p({}_R X_t) \equiv 1 - \frac{\sigma({}_R X_t)^2}{E({}_R X_t^2)} \quad (73)$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \quad (74)$$

is true. Multiplying this premise (i. e. axiom) by the variance $\sigma({}_R X_t)^2$

$$\sigma({}_R X_t)^2 \equiv \sigma({}_R X_t)^2 \quad (75)$$

Equation 75 can be rearranged (see definition II.4, equation 12) as

$$E({}_R X_t^2) - (E({}_R X_t))^2 \equiv \sigma({}_R X_t)^2 \quad (76)$$

or as

$$E({}_R X_t^2) \equiv (E({}_R X_t))^2 + \sigma({}_R X_t)^2 \quad (77)$$

The normalised form of the variance follows as

$$\frac{(E({}_R X_t))^2}{E({}_R X_t^2)} + \frac{\sigma({}_R X_t)^2}{E({}_R X_t^2)} \equiv +1 \quad (78)$$

Rearranging equation 78, it is

$$\frac{(E({}_R X_t))^2}{E({}_R X_t^2)} \equiv 1 - \frac{\sigma({}_R X_t)^2}{E({}_R X_t^2)} \quad (79)$$

Equation 79 simplifies (see definition II.3, equation 3) as

$$p({}_R X_t) \equiv 1 - \frac{\sigma({}_R X_t)^2}{E({}_R X_t^2)} \quad (80)$$

Quod erat demonstrandum.

E. Anti Fermat - Refutation of Fermat's Last Theorem

Theorem III.5 (Refutation of Fermat's Last Theorem). *Fermat's last theorem known as $({}_o a_t^n) + ({}_o b_t^n) \equiv {}_R C_t^n$ while no three positive integers a , b , and c satisfy the equation for any integer value of n greater than 2, is refuted.*

Proof by modus ponens.. If the premise

$$\underbrace{+1 = +1}_{(Premise)} \quad (81)$$

is true, **then** the conclusion

$$({}_o b_t^n) \equiv {}_R C_t^n \quad (82)$$

is also true, the absence of any technical errors presupposed, and Fermat's last theorem is refuted. The premise

$$+1 \equiv +1 \quad (83)$$

is true. Multiplying this premise (i. e. axiom or equation 83) by ${}_R C_t^n$, it is

$$+1 \times {}_R C_t^n \equiv +1 \times {}_R C_t^n \quad (84)$$

or

$${}_R C_t^n \equiv {}_R C_t^n \quad (85)$$

Pierre de Fermat's (1607 - 1665) Last Theorem (i. e. Observatio Domini Petri de Fermat) published 1670 in the book *Diophantus's Arithmetica* by Fermat's son, often considered simply as one of the most notable unsolved problems of mathematics, states that $({}_o a_t^n) + ({}_o b_t^n) \equiv {}_R C_t^n$ while no three positive integers a , b , and c satisfy the equation for any integer value of n greater than 2. Equation 85 changes to

$$({}_o a_t^n) + ({}_o b_t^n) \equiv {}_R C_t^n \quad (86)$$

Investigating the behaviour of **Fermat's Last Theorem** under conditions where $\mathbf{a} = +\mathbf{0}$, we obtain

$$((a_t \equiv +0)^n) + ({}_o b_t^n) \equiv {}_R C_t^n \quad (87)$$

or

$$({}_o b_t^n) \equiv {}_R C_t^n \quad (88)$$

In other words, at least one integer, the positive zero, is in compliance with Fermat's Last Theorem. Consequently, Fermat's Last Theorem is refuted. **Quod erat demonstrandum.**

Remark III.2. *Andrew Wiles's 1995 corrected proof of Fermat's Last Theorem appears to be only of very limited value. Three distinct positive integers ($a = +0$), b , and c can satisfy Fermat's equation*

$$({}_o a_t^n) + ({}_o b_t^n) \equiv {}_R C_t^n \quad (89)$$

In this context, there is reason to believe that it will be disputed that the positive zero (+0) is an integer. Well, in this case, a clear and convincing answer should be given to the question why a positive zero is not an integer. What than is a positive zero, a non-integer, an anti-integer, ...?

F. The n-dimensional Pythagorean theorem I

Theorem III.6 (The n-dimensional Pythagorean theorem). *The Pythagorean theorem as attributed to the Greek thinker Pythagoras of Samos (6th century, B.C.) is defined as*

$${}_o a_t^2 + {}_o b_t^2 \equiv {}_R C_t^2 \quad (90)$$

where ${}_o$ may denote the point of view of a co-moving observer while ${}_R$ may denote the point of view of a stationary observer at a certain point in space-time.

In general, the n-dimensional Pythagorean theorem is given by the equation

$$(({}_o a_t^2) + ({}_o b_t^2))^n \equiv {}_R C_t^{2n} \quad (91)$$

Proof by modus ponens. If the premise

$$\underbrace{+1 = +1}_{(Premise)} \quad (92)$$

is true, **then** the conclusion

$$(({}_o a_t^2) + ({}_o b_t^2))^n \equiv {}_R C_t^{2n} \quad (93)$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \quad (94)$$

is true. Multiplying this premise by x_t it is

$$x_t \equiv x_t \quad (95)$$

Adding ${}_o x_t$ to equation 95 it is

$$x_t + {}_o x_t \equiv x_t + {}_o x_t \quad (96)$$

Equation 96 changes (see definition II.7, equation 21) to

$$x_t + {}_o x_t \equiv {}_R C_t \quad (97)$$

which is equally the general foundation of the Pythagorean theorem [see 14]. However, the Pythagorean theorem can be extended to higher dimensions [see 52] too. In the n -dimensional case, equation 97 becomes

$$\begin{aligned} & \left({}_o x_t + {}_o \underline{x}_t \right)^n \\ & \equiv \left(\underbrace{\left({}_o x_t + {}_o \underline{x}_t \right) \times \left({}_o x_t + {}_o \underline{x}_t \right) \times \left({}_o x_t + {}_o \underline{x}_t \right) \times \dots}_{n \text{ times}} \right) \\ & \equiv {}_R C_t^n \end{aligned} \quad (98)$$

Multiplying equation 98 by the term ${}_R C_t^n$ we obtain

$$\begin{aligned} & \left({}_o x_t + {}_o \underline{x}_t \right)^n \times {}_R C_t^n \\ & \equiv \left(\underbrace{\left({}_o x_t + {}_o \underline{x}_t \right) \times \left({}_o x_t + {}_o \underline{x}_t \right) \times \left({}_o x_t + {}_o \underline{x}_t \right) \times \dots}_{n \text{ times}} \right) \times {}_R C_t^n \\ & \equiv {}_R C_t^n \times {}_R C_t^n \end{aligned} \quad (99)$$

Equation 99 simplifies as

$$\begin{aligned} & \left({}_o x_t + {}_o \underline{x}_t \right)^n \times {}_R C_t^n \\ & \equiv \left(\left({}_o x_t + {}_o \underline{x}_t \right) \times {}_R C_t \right)^n \\ & \equiv \left(\left({}_o x_t \times {}_R C_t \right) + \left({}_o \underline{x}_t \times {}_R C_t \right) \right)^n \\ & \equiv {}_R C_t^{2n} \end{aligned} \quad (100)$$

In general, it is ${}_o a_t^2 \equiv {}_o x_t \times {}_R C_t$ (see definition II.7, equation 17). Equation 100 simplifies as

$$\left(({}_o a_t^2) + ({}_o \underline{x}_t \times {}_R C_t) \right)^n \equiv {}_R C_t^{2n} \quad (101)$$

Furthermore, it is ${}_o b_t^2 \equiv {}_o \underline{x}_t \times {}_R C_t$ (see definition II.7, equation 19). The n -dimensional Pythagorean theorem follows as

$$\left(({}_o a_t^2) + ({}_o b_t^2) \right)^n \equiv {}_R C_t^{2n} \quad (102)$$

In other words, our conclusion is true.

Quod erat demonstrandum.

G. The n -dimensional Pythagorean theorem II

Theorem III.7 (The n -dimensional Pythagorean theorem II).

$$\left(({}_o a_t^2) + ({}_o b_t^2) \right)^n \equiv {}_R C_t^{2n} \quad (103)$$

Proof by modus ponens.. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \quad (104)$$

is true, **then** the conclusion

$$\left(({}_o a_t^2) + ({}_o b_t^2) \right)^n \equiv {}_R C_t^{2n} \quad (105)$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \quad (106)$$

is true. Multiplying this premise (i. e. axiom) by the term ${}_R C_t^2$, it is

$${}_R C_t^2 \equiv {}_R C_t^2 \quad (107)$$

The make a long story short, the Pythagorean theorem is defined as

$${}_o a_t^2 + {}_o b_t^2 \equiv {}_R C_t^2 \quad (108)$$

Raising to the power n , the n dimensional Pythagorean theorem is given as

$$\left(({}_o a_t^2) + ({}_o b_t^2) \right)^n \equiv {}_R C_t^{2n} \quad (109)$$

Quod erat demonstrandum.

Remark III.3. It follows from equation 109 the normalised n dimensional Pythagorean theorem as

$$\left(\frac{{}_o a_t^{2n}}{{}_R C_t^{2n}} \right) + \left(\frac{{}_o b_t^{2n}}{{}_R C_t^{2n}} \right) \equiv \left(\frac{{}_R C_t^{2n}}{{}_R C_t^{2n}} \right) \equiv +1 \quad (110)$$

Fermat's Last Theorem states that $({}_o a_t^n) + ({}_o b_t^n) \equiv {}_R C_t^n$ while no three positive integers a , b , and c satisfy the equation for any integer value of n greater than 2. Rearranging Fermat's Last Theorem, we obtain

$$\left(({}_o a_t^n) + ({}_o b_t^n) \right)^2 \equiv ({}_R C_t^n)^2 \quad (111)$$

while a lot of positive integers ${}_o a_t$, ${}_o b_t$, and ${}_R C_t$ satisfy equation 111 for any integer value of n (and even greater than 2). Simplifying equation 111 leads to a more general form of the equation before as

$$\left({}_o a_t^{2n} \right) + \left(2 \times {}_o a_t^n \times {}_o b_t^n \right) + \left({}_o b_t^{2n} \right) \equiv ({}_R C_t^{2n}) \quad (112)$$

and Fermat's Last Theorem appears to pass over into the n dimensional Pythagorean theorem. The metric tensor $g_{\mu\nu}$ of general relativity on a space is more or less a generalisation of Pythagoras' theorem for the distance for a certain distance between two points separated by different distances and reproduces the usual form of the Pythagorean Theorem. In general it is

$$g_{\mu\nu} dx^\mu dx^\nu \equiv ds^2 \quad (113)$$

while reproducing the usual form of the Pythagorean Theorem. The n dimensional form follows as

$$(g_{\mu\nu} dx^\mu dx^\nu)^n \equiv (ds^2)^n \equiv ({}_R C_t^{2n}) \quad (114)$$

H. Locality and non locality

Even if different columns of the Copenhagen interpretation of quantum mechanics like Heisenberg's uncertainty principle [6], [10], [12], Bell's theorem/inequality[8], [11], [16] and the CHSH [8], [11] inequality and other aspects are already refuted, this need not to mean that a logically and mathematically sound concept of non-locality is without any sense. In this context, let $p({}_R X_t)$ denote the probability from the point of view of a stationary observer R for finding a certain particle X somewhere **local** at a given point in space at a given time / Bernoulli trial t . Let $E({}_R X_t)$ denote the expectation value of ${}_R X_t$ for being local. The expectation value of locality of ${}_R X_t$ is defined as

$$E({}_R X_t) \equiv p({}_R X_t) \times ({}_R X_t) \equiv \Psi({}_R X_t) \times {}_R X_t \times \Psi^*({}_R X_t) \quad (115)$$

The expectation value of being **non-local**, denoted by $E(\underline{R}X_t)$, is defined as

$$E(\underline{R}X_t) \equiv (1 - p(\underline{R}X_t)) \times (\underline{R}X_t) \equiv p(\underline{R}X_t) \times (\underline{R}X_t) \quad (116)$$

Theorem III.8 (Locality and non locality).

$$\sigma(\underline{R}X_t)^2 \equiv E(\underline{R}X_t) \times E(\underline{R}X_t) \quad (117)$$

$$+1 \equiv +1 \quad (118)$$

Proof by modus ponens.. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \quad (119)$$

is true, **then** the conclusion

$$\sigma(\underline{R}X_t)^2 \equiv E(\underline{R}X_t) \times E(\underline{R}X_t) \quad (120)$$

is also true, the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \quad (121)$$

is true. Multiplying this premise (i. e. axiom) by the probability of being local, $p(\underline{R}X_t)$, it is

$$p(\underline{R}X_t) \equiv p(\underline{R}X_t) \quad (122)$$

which is equivalent with

$$p(\underline{R}X_t) \equiv 0 + p(\underline{R}X_t) \quad (123)$$

or with

$$p(\underline{R}X_t) \equiv +1 - 1 + p(\underline{R}X_t) \quad (124)$$

and with

$$p(\underline{R}X_t) \equiv 1 - (1 - p(\underline{R}X_t)) \equiv 1 - p(\underline{R}X_t) \quad (125)$$

In general, it is

$$p(\underline{R}X_t) \equiv 1 - p(\underline{R}X_t) \quad (126)$$

Locality is determined by its own non-locality and vice versa. The one passes over into its own other and vice versa. The more something is local, the less it is non-local and vice versa. Multiplying equation 126 by the term $(\underline{R}X_t)^2 \times (1 - p(\underline{R}X_t))$ it is

$$\begin{aligned} \sigma(\underline{R}X_t)^2 &\equiv p(\underline{R}X_t) \times \underline{R}X_t^2 \times (1 - p(\underline{R}X_t)) \\ &\equiv \underline{R}X_t \times p(\underline{R}X_t) \times \underline{R}X_t \times (1 - p(\underline{R}X_t)) \\ &\equiv \underline{R}X_t \times p(\underline{R}X_t) \times \underline{R}X_t \times p(\underline{R}X_t) \\ &\equiv E(\underline{R}X_t) \times E(\underline{R}X_t) \end{aligned} \quad (127)$$

IV. DISCUSSION

Investigating or developing specific concepts for non-locality in the light of empirical facts require logically sound mathematical or statistical methods too, in order to relate empirical facts with hypotheses of particular kind. Statistical methods and especially the variance is relied upon in almost all empirical scientific research. The correct understanding of the variance is of key importance to extrapolate from data to predictions and general facts and to communicate scientific findings the right way. Further debates that focuses on **the relation between locality and non-locality** may consider that the notions of locality and non-locality are completely described (see equation 127) by variance as used in statistics and in other science. Locality and non-locality can be evaluated in the light of sample data by the variance too. Only under conditions where

$$\begin{aligned} \sigma(\underline{R}X_t)^2 &\equiv p(\underline{R}X_t) \times \underline{R}X_t^2 \times (1 - p(\underline{R}X_t)) \\ &\equiv \underline{R}X_t \times p(\underline{R}X_t) \times \underline{R}X_t \times (1 - p(\underline{R}X_t)) \\ &\equiv \underline{R}X_t \times p(\underline{R}X_t) \times \underline{R}X_t \times p(\underline{R}X_t) \\ &\equiv E(\underline{R}X_t) \times E(\underline{R}X_t) \\ &\equiv 0 \end{aligned} \quad (128)$$

either the state of pure locality **or** the the state of pure locality is given, otherwise not.

V. CONCLUSION

In combination with other already published [6], [8], [10]–[12], [16] papers, the problem of a logically consistent description of locality and non-locality is solved.

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Ilija Barukčić studied human medicine at the University Hamburg (1982-1989), Germany. His main field of research is the relationship between a cause and an effect and the associated issues. Ilija Barukčić published several papers among them papers like the equivalence of time and gravitational field [7], the relativistic [9] wave equation and the unified field theory [14] and articles on indeterminate forms. Ilija Barukčić succeeded [31], [43], [44], [53] in mathematising the relationship between cause and effect [1]–[5], [13], [15]. Barukčić has been able to refute

Heisenberg's uncertainty principle [6], [10], [12], Bell's theorem/inequality [8], [11], [16] and the CHSH [8], [11] inequality.

Quod erat demonstrandum.

Jane Doe Biography text here.

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