# MULTIPLE SNAPSHOT MATCHING PURSUIT FOR DIRECTION OF ARRIVAL (DOA) ESTIMATION

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## ABSTRACT

*The Matching Pursuit (MP) algorithm may be used to give a low complexity method of obtaining a solution to the directions of arrival (DOA) problem which arises in a large number of application areas. We present a novel estimation of the DOA in which we incorporate multiple snapshots of the received signal into each run of the MP algorithm. A tree based search is used in combination with MP to expand the search space. This new method, MSMPDOA, gives a substantial performance improvement over running the MP algorithm on each snapshot separately before the results are aggregated to give the DOA estimate. In a series of simulations, it is shown that the success rate in accurately determining arrival directions at low values of SNR is improved by 10-30% by utilizing the MSMPDOA algorithm.*

## 1. INTRODUCTION

The directions of arrival (DOA) problem has a long history which can be traced back to sonar and radar [1]. More recently, there has been renewed interest in the problem with the practical application of smart antennas in mobile communications [2]. In each of these applications, the goal is to isolate signals which are arriving from several different directions. An antenna array is used to measure the incoming signals and signal processing algorithms are used to process the data and provide the directions. This information can be used in a variety of ways such as the separation of a desired signal from interference or the processing of multipath components of a signal.

Many algorithms have been employed to obtain solutions to this problem. Initially, beamforming methods, which are essentially Fourier based algorithms, were intensively studied [3]. Subspace based methods such as MUSIC and ESPRIT were later developed and have very good performance which can approach the performance of a maximum likelihood algorithm [4]. However, this performance is achieved at the expense of a very high computational cost. There is a need for more computationally efficient algorithms which can be run on mobile devices, for instance, and still deliver very good performance.

In the DOA application, the received signal is due to a small number of directions. By forming a matrix where each column accounts for a possible direction, the problem is to use the received signal and extract the correct directions from this matrix. Since the number of possible arrival angles is much greater than the number of sensors, the system of equations which must be solved is underdetermined and the problem of extracting the correct directions has been shown to be NP-hard in [5]. Indeed, this problem is a linear inverse problem and similar problems arise in many different application areas. One of the most computationally efficient approaches to finding a suboptimal solution is the Matching Pursuit algorithm proposed in [6]. The EDAMP algorithm described in [7] used Matching Pursuit (MP) to find a solution to the DOA problem. In an extensive simulation study in [8], EDAMP was shown to provide improved DOA estimation over ESPRIT and MUSIC at low signal to noise ratio (SNR). It also has the advantage that it can be used with a small number of snapshots and has a lower computational complexity than other algorithms.

In this paper, we propose an enhanced MP based algorithm for DOA estimation which incorporates groups of snapshots into the MP algorithm. We term this new algorithm Multiple Snapshot Matching Pursuit Direction of Arrival (MSMPDOA) Estimation. In contrast, the EDAMP algorithm [8] considers each snapshot in isolation in running the MP algorithm and the results are then aggregated to produce the final DOA estimates. It is shown through simulation that at low values of SNR the MSMPDOA gives a success rate in identifying DOA which is 10-30% better than the EDAMP method proposed in [8].

The outline of the paper is as follows. In section 2, the DOA problem is formulated and the notation used in the paper is detailed in full. The Matching Pursuit (MP) algorithm is briefly outlined in section 3. The new method, MSMPDOA, is detailed in section 4. A number of simulation scenarios are considered in section 5 and the results of the new algorithm are detailed. The results obtained using MSMPDOA and the EDAMP algorithm are compared. Finally, some conclusions and directions for future work are given in section 6.

## 2. PROBLEM FORMULATION

We consider a uniform linear array (ULA) with $M$ elements spaced a distance $d$ apart. The sources are far-field and the incoming waves are plane waves. The first element in the array is taken as the phase reference. This scenario is depicted in figure 1.

The response of the antenna array at time $t$ to a signal arriving at angle $\theta$ is given by

$$\mathbf{b}(t) = s(t)\mathbf{a}(\theta) \tag{1}$$

where $s(t)$ is the signal amplitude, $\mathbf{a}(\theta) = [1 \; e^{-jhd\cos(\theta)} \dots e^{-j(M-1)hd\cos(\theta)}]^T \in \mathbb{C}^M$, and $h = 2\pi/\lambda$ for signal wavelength $\lambda$ [4]. Assuming that there are $R$ signals which make up the received signal and that these signals

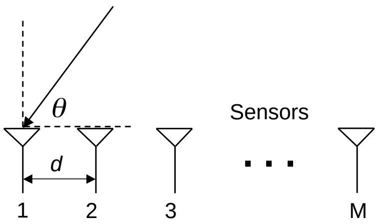


Figure 1: Uniform Linear Array (ULA) and the Direction of Arrival (DOA) problem.

arrive from directions $\theta_i, i = 1,\cdots,R$, the antenna response is given as

$$\mathbf{b}(t) = \sum_{i=1}^{R} s_i(t)\mathbf{a}(\theta_i) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{n}(t)$ is an additive noise term. The sensor outputs are sampled at time instances $t_l, l = 1,\cdots,N_S$ giving multiple snapshots of the received signal $\mathbf{b}(t_1),\mathbf{b}(t_2),\cdots,\mathbf{b}(t_{N_S})$.

We let the $N$ possible directions of arrival form an array $\mathbf{A} = [\mathbf{a}(\alpha_1),\cdots,\mathbf{a}(\alpha_N)]$. The resulting array has the following Vandermonde form

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\phi_1} & e^{j\phi_2} & \cdots & e^{j\phi_N} \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ e^{j(M-1)\phi_1} & e^{j(M-1)\phi_2} & \cdots & e^{j(M-1)\phi_N} \end{bmatrix} \tag{3}$$

where for convenience we have $\phi_i = (\frac{2\pi d}{\lambda})\cos(\alpha_i)$. The problem is to determine the small number of columns from $\mathbf{A}$ which correspond to the directions of arrival. When these columns are appropriately scaled and added together, the resulting signal should be a good approximation to the received signal. If the algorithm used to determine the DOA works perfectly, the columns selected should correspond to $\alpha_i = \theta_i$, $i = 1,\cdots,R$.

If we consider the vector $\mathbf{s}(t) \in \mathbb{C}^N$, the problem can be stated as finding the sparse solution vector $\mathbf{s}(t)$ with $R$ nonzero entries which gives an approximate solution to the problem $\mathbf{As}(t) = \mathbf{b}(t)$. Since the number of sensors $M$ is much smaller than the number of possible DOA, this is an underdetermined system of equations. The squared error $\|\mathbf{b}(t) - \mathbf{As}(t)\|$ can be used to determine the goodness of fit of a solution and when $\|\mathbf{b}(t) - \mathbf{As}(t)\| < \varepsilon$, where $\varepsilon$ is some small value, the solution is said to be found. The problem stated here fits the description of a sparse approximation problem. Matching Pursuit is one method for finding approximate solutions to this type of problem and this is outlined in the following section.

## 3. MATCHING PURSUIT

The Matching Pursuit (MP) algorithm was first proposed in [6] with application to time-frequency decompositions. The algorithm has since been applied in many different areas including audio representation [9] and image coding [10]. Many variations on the original algorithm have been proposed which offer improved performance at the cost of some additional computation. In this paper, we will use the Orthogonal Matching Pursuit algorithm [11, 12].

In section 2 we have described a data matrix $\mathbf{A}$ and a measurement vector $\mathbf{b}$ and the goal is to find a solution $\mathbf{s}$ which approximately satisfies $\mathbf{As} = \mathbf{b}$ (the time index has been omitted for convenience). We denote the columns in the matrix $\mathbf{A}$ by $\mathbf{a_l} = \mathbf{a}(\alpha_l)$, $l = 1,\cdots,N$. For the description of the OMP algorithm, we introduce $I_p = \{k_1,k_2,\cdots,k_p\}$, $I_0 = \emptyset$ which is the set of $p$ vectors selected. We let $\mathbf{S_p} = \left[\mathbf{a_{k_1}},\mathbf{a_{k_2}},\cdots,\mathbf{a_{k_p}}\right]$ (with $\mathbf{S_0} = \emptyset$) denote the matrix which has the selected vectors as columns. $\mathbf{P_{S_p}}$ represents the orthogonal projection matrix onto the range space of $\mathbf{S_p}$ and its orthogonal complement $\mathbf{P}^{\perp}_{\mathbf{S_p}} = \left(\mathbf{I} - \mathbf{P_{S_p}}\right)$, $\mathbf{P_{S_0}} = \mathbf{0}$, $\mathbf{P}^{\perp}_{\mathbf{S_0}} = \mathbf{I}$. With this notation at hand, the OMP algorithm is described next. A more detailed description is given in [11, 12], and some theoretical results on the performance of the algorithm have been presented in [13].

The first step in the algorithm is to find the column in the matrix $\mathbf{A}$ which is best aligned with the signal vector $\mathbf{b_0} = \mathbf{b}$ and this is denoted $\mathbf{a_{k_1}}$. Then the projection of $\mathbf{b_0}$ along this direction $\mathbf{a_{k_1}}$ is removed from $\mathbf{b_0}$ and the residual $\mathbf{b_1}$ is obtained. Now the column in $\mathbf{A}$, $\mathbf{a_{k_2}}$, which is best aligned with $\mathbf{b_1}$ is found and a new residual, $\mathbf{b_2}$, is formed by projecting $\mathbf{b_0}$ onto the space orthogonal to *both* $\mathbf{a_{k_1}}$ and $\mathbf{a_{k_2}}$, i.e., all the chosen vectors. The algorithm proceeds by sequentially choosing the column which best matches the residual until some termination criterion is met. The $p$th iteration of the OMP is described in the following paragraph.

In the $p$th iteration of the algorithm, the vector from $\mathbf{A}$ most closely aligned with the residual $\mathbf{b_{p-1}}$ is chosen, where the alignment is measured as the 2-norm (denoted by $\|\cdot\|$) of the projection of the residual onto the vector, i.e.

$$\begin{aligned} k_p &= \arg\max_l \|\mathbf{P_{a_l}}\mathbf{b_{p-1}}\| \qquad (4) \\ &= \arg\max_l \frac{|\mathbf{a_l}^H\mathbf{b_{p-1}}|^2}{\|\mathbf{a_l}\|^2},\ l = 1,\cdots,N,\ l \notin I_{p-1}. \end{aligned}$$

The new residual vector is obtained by projecting $\mathbf{b}$ onto the orthogonal complement of the range space of $\mathbf{S_p}$ where $\mathbf{S_p} = [\mathbf{S_{p-1}},\ \mathbf{a_{k_p}}]$

$$\mathbf{b_p} = \mathbf{P}^{\perp}_{\mathbf{S_p}}\mathbf{b}. \tag{5}$$

The new residual is the smallest residual given the chosen set $\mathbf{S_p}$.

The iteration may be terminated in one of two ways. If the number of directions of arrival is known or can be estimated, the algorithm is terminated once this number of columns has been selected from the data matrix $\mathbf{A}$. Alternatively, the iterations are terminated once the residual becomes sufficiently small, i.e., $\|\mathbf{b_p}\| < \varepsilon$.

### 3.1 Tree Based OMP and EDAMP

The extension of MP to a tree based search was introduced in [14]. The MP algorithm is a greedy search through the columns of $\mathbf{A}$ to find a subset of columns which can be used to represent $\mathbf{b}$. The search can be broadened to include not just the best matching column but also close matches. Two different methods were proposed. In the first method, the tree

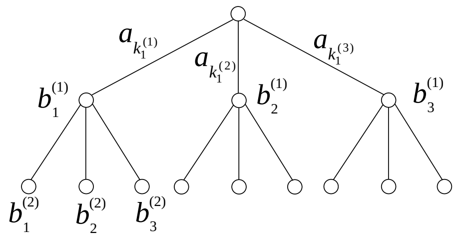


Figure 2: Tree Search Matching Pursuit

of possibilities was expanded by keeping the $Q$ best matching vectors from each MP iteration, i.e., $\{k_p^{(1)}, k_p^{(2)}, \cdots, k_p^{(Q)}\}$ at step $p$. This is shown in figure 2 above for $Q = 3$ where each branch corresponds to the data vector and the node represents the residual vector which results. Each of the residuals generated was then used as the input to another stage of the MP algorithm. This quickly led to an exponential growth in the number of nodes considered. For a practical implementation, the number of residuals was limited. In a similar manner to the M-L algorithm [15], a subset of residuals with the smallest magnitudes were selected at each level.

This algorithm was adapted to the DOA problem in the EDAMP algorithm proposed in [7, 8] and several heuristics were introduced to prune the tree of search possibilities. In particular, the number of branches considered was made variable in size and level dependent. At level $p$, the number of branches from each node was limited to $Q = \lceil N/\delta^p \rceil$ where $\delta$ is a decay parameter. Furthermore, the alignment between vector was used to reduce the set of nodes considered. With the definition

$$\rho_{ij} = \frac{|a_i^H a_j|}{\|a_i\|\|a_j\|}. \tag{6}$$

the alignment between the best matching vector $k_p^{(1)}$ and each of the other chosen vectors was obtained. A user-defined threshold was used and vectors which satisfied $\rho_{k_p^{(1)} k_p^{(i)}} \leq \xi$ were discarded. These enhancements kept the search wide at the start of the algorithm and pruned the search space as the depth increased leading to an efficient tree-based algorithm.

In EDAMP, each of the snapshots was processed one at a time resulting in a different solution for each of the snapshots. A final solution was obtained by aggregating the results obtained from each of the individual solution vectors. In the next section, we consider multiple snapshots at each node of the search tree.

## 4. MULTIPLE SNAPSHOT MATCHING PURSUIT FOR DIRECTION OF ARRIVAL ESTIMATION (MSMPDOA)

In the DOA problem, there are a number of snapshots $N_S$ available as described in section 2. The availability of multiple measurements in matching pursuit problems was first considered in [16]. In the MSMPDOA algorithm, we propose incorporating multiple snapshots into each iteration of the MP algorithm to estimate the DOA. Furthermore, we expand the search by considering a number of possible solutions at each level of the tree. The combination of these two innovations results in a novel algorithm which is shown to improve greatly on the DOA estimation accuracy obtained by processing one snapshot at a time.

A subset of the available snapshots is denoted by $\mathbf{B} = [\mathbf{b}(t_{j_1}), \mathbf{b}(t_{j_2}), \cdots, \mathbf{b}(t_{j_L})]$ where $L = 1$ gives just one snapshot and $L = N_S$ groups all snapshots together in $\mathbf{B}$. The algorithm uses exactly the same methodology outlined for OMP as described in section 3 but the alignment with matrix $\mathbf{B}$, instead of just with a single snapshot $\mathbf{b}$, is considered. The $Q$ columns in the matrix $\mathbf{A}$ which are best aligned with the measurement vectors $\mathbf{B_0} = \mathbf{B}$ are selected and these column indices are denoted as $k_1^{(1)}, \cdots, k_1^{(Q)}$. The alignment is measured using the Frobenius norm instead of the 2-norm because of the multiple snapshots. Residual matrices $\mathbf{B_1}^{(1)}, \cdots, \mathbf{B_1}^{(Q)}$ are formed corresponding to each chosen column. The selection of a column vector and formation of a new residual is repeated for each of the residual matrices at a given level. The procedure is iterated until a satisfactory solution is obtained to the inverse problem. At the $p$th step, there are residual matrices $\mathbf{B_{p-1}}^{(j)}, j = 1, \cdots, J$. We denote one of these residuals as $\mathbf{B_{p-1}}$ to save on notation.

1. For residual $\mathbf{B_{p-1}}$ form $\mathbf{E_{p,k}} = P_{\mathbf{a_k}}^{\perp} \mathbf{B_{p-1}}$
2. Select the $Q$ columns which give the lowest values of $\|\mathbf{E_{p,k}}\|_F^2 = \mathrm{tr}(\mathbf{E_{p,k}}^H \mathbf{E_{p,k}})$. Denote the indices of these columns by $k_p^{(1)}, \cdots, k_p^{(Q)}$.
3. Calculate each new residual $\mathbf{B_p^{(i)}} = \mathbf{P}_{\mathbf{S_p^{(i)}}}^{\perp} \mathbf{B_{p-1}}, i = 1, \cdots, Q$ where $\mathbf{S_p^{(i)}} = [\mathbf{a_{k_1}}, \cdots, \mathbf{a_{k_{p-1}}}, \mathbf{a_{k_p^{(i)}}}]$.
4. Return to step 1 for each residual at the current level. Once the current level is complete, advance to the next level and continue from step 1 with the new residual list. Continue to loop unless the termination criterion has been met. As was already noted in section 3, this criterion can be in the form of a known number of directions or a threshold which the residual norm must fall below, i.e., $\|\mathbf{B_p}\|_F \leq \varepsilon$.

Denote the number of columns selected at the end of the iterations by $\rho$ and the selected column indices in one solution set by $[k_1, \cdots, k_\rho]$. These are easily related to the DOA from the manner in which the matrix $\mathbf{A}$ was formed. In a similar manner to [8] we employ some heuristics to limit the exponential growth in the size of the tree. In particular, the number of child nodes from a parent node is limited to $\lceil N/d^p \rceil$ at level $p$ where $d$ is an algorithm parameter.

## 5. SIMULATIONS

In [8], the performance of EDAMP was compared to ESPRIT and MUSIC and was shown to outperform both at low SNR. Through a series of simulations, we demonstrate that the MSMPDOA algorithm yields even better performance than EDAMP at low values of SNR.

For each of the simulations below, we considered an Uniform Linear Array (ULA) with 10 elements in which the elements were spaced a distance $\lambda/2$ apart. There were 100 possible angles of arrival and these were distributed uniformly over the range $(-90°, 90°)$. These angles were used to form the columns in the matrix $\mathbf{A}$. There were 2 incoming signals of equal amplitude and Gaussian noise was added to give the received waveform. The decay parameter $\delta$, which was described in section 3, was set to 10 to limit the number of nodes at each level. In each trial 100 snapshots were considered. The DOA were estimated and if these matched the true incident DOA, the algorithm was deemed to have

| Parameter | Symbol | Value |
|---|---|---|
| Number of angles | $N$ | 100 |
| Number of snapshots | $N_S$ | 100 |
| Snapshot grouping | $L$ | 1,5,10 |
| Number of antennas | $M$ | 10 |
| Number of DOAs | $R$ | 2 |
| Decay parameter | $\delta$ | 10 |

Table 1: Simulation parameters.

successfully resolved the incident DOA. The simulation was repeated over 1000 different trials and the percentage success in accurately determining the DOA was obtained from the trial results. For convenience, the important parameters of the simulation are summarized in table 1.

We detail a number of different simulation scenarios in sections 5.2–5.4 to fully compare the performance of the EDAMP algorithm to the MSMPDOA method introduced in this paper. However, we first examine the task of obtaining a final estimate of the DOA given the solutions obtained from each run of the algorithm with each run incorporating a fixed number of the available snapshots.

### 5.1 Generation of Solution

The results obtained from each of the snapshots or groupings of snapshots must be aggregated to obtain the final solution. We considered a couple of different methods of accomplishing this task. In the first method, a histogram of the different DOA solutions was compiled and the most frequent solution was chosen and used to provide the final DOA estimate. In the second method, the solutions obtained using each group of snapshots were averaged, and the largest entries (in magnitude) in the resulting vector were used to provide the final DOA estimate. The latter of these two methods gave superior results and was therefore used in the determination of the DOA.

### 5.2 Widely Spaced DOA

We first considered two widely spaced signals where the spacing was chosen greater than the Rayleigh spacing which is the minimum separation that can be resolved by a beamforming method [1, 4]. For an antenna array with $N$ elements, the Rayleigh spacing is $2\pi/N$ and the arrival angles in this simulation were chosen as $\pm 7.2°$ so that the separation exceeds the Rayleigh spacing.

The results of the simulation are shown in figure 3. $L = 1$ corresponds to the EDAMP algorithm, i.e., only 1 snapshot is processed at a time. The other values of $L$ incorporate $L = 5$ and $L = 10$ snapshots into the MSMPDOA algorithm. The figure gives the percentage success in resolving the DOA. All methods performed well in the higher SNR region. However, with SNR in the range $-5$dB to 5dB, the MSMPDOA algorithm gave a success rate which was 10-30% better than that of the EDAMP algorithm.

### 5.3 Closely Spaced DOA

Matching Pursuit allows the resolution of closely spaced signals as was first observed in [17] in the context of resolving two sines with frequency separation less than the Raleigh

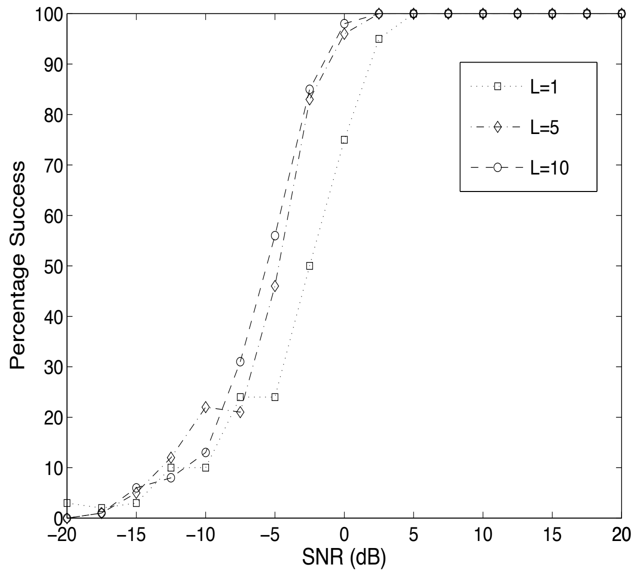


Figure 3: Resolution of widely spaced DOA $\pm 7.2°$ using MSMPDOA ($L = 5, 10$) and EDAMP ($L = 1$)

distance. We show that this resolution ability of the MP algorithm can similarly be exploited in the spatial domain.

We considered DOA of $\pm 3.6°$ and the results are given in figure 4. The format of the results is identical to that in figure 3. It is noted that for values of SNR above 5dB, the performance of the different algorithms was perfect in resolving the DOA. We found that there was a clear improvement in the success rate obtained using MSMPDOA with $L = 5$ and $L = 10$ over using EDAMP ($L = 1$) in the SNR region from -7.5dB to 5dB. The MSMPDOA($L = 10$) was also seen to offer a performance improvement over the MSMPDOA($L = 5$) method in the range from -5dB to 5dB.

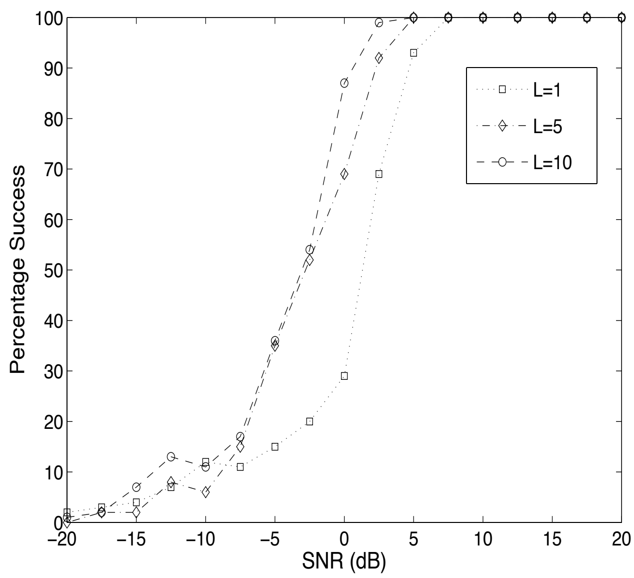


Figure 4: Resolution of closely spaced DOA $\pm 3.6°$ using MSMPDOA ($L = 5, 10$) and EDAMP ($L = 1$)

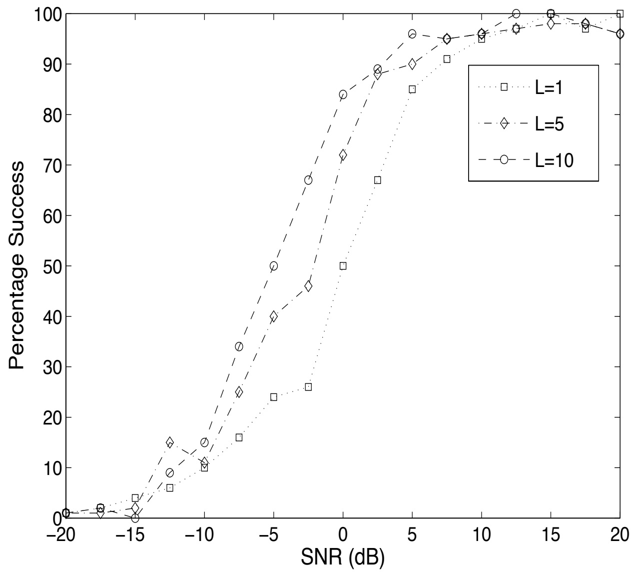


Figure 5: Performance with two DOA which are randomly selected from the range $(-63°, 63°)$ using MSMPDOA ($L = 5, 10$) and EDAMP ($L = 1$)

### 5.4 Randomly Spaced DOA

In many cases, the DOA will be unknown but will be confined to a certain range. In this simulation, the incident signal was composed of two signals with DOA which were chosen at random. From a practical point of view, the range can be limited and we chose a wide range of $(-63°, 63°)$. The results are plotted in figure 5 in a similar manner to figures 3 and 4. MSMPDOA with $L = 10$ is shown to give much better performance than $L = 5$ in this scenario. Both MSMPDOA($L = 5$) and MSMPDOA($L = 10$) performed much better than EDAMP ($L = 1$). The performance improvement is once again more apparent in the low SNR region. For instance at 0dB, MSMPDOA($L = 10$) gave a success rate of 84%, MSMPDOA($L = 5$) had a success rate of 72% while EDAMP had a success rate of 51%.

## 6. CONCLUSION

We have introduced a new algorithm for DOA estimation which incorporates multiple snapshots of the received signal into a Matching Pursuit (MP) based algorithm. A tree based search was used in combination with MP to expand the search space. This new method, MSMPDOA, was shown to give substantial performance improvement over running the MP algorithm on each snapshot separately before aggregating the results to give an estimation of the DOA. In a series of simulations, the successful detection of arrival directions at low values of SNR was improved by 10-30% with this new implementation. Further work on the algorithm will focus on the advantages of the algorithm in an environment in which the DOA are rapidly varying and the DOA must be calculated over a small number of snapshots.

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