

CHR 20 April 1993

Final calculations for Figure 2 in the hydrodynamics paper... I leave the Garcia de la Torre results at the values already calculated (in Figure 2 data-- "Garcia de la Torre Final" or Cricket Graph file) and use my Kirkwood subroutine on radii and intersphere distances calculated for this example, which is particularly simple since the DNA is treated in the rigid limit.

Notice that the Yamakawa and Fujii equation for the sedimentation behavior in the limit of an infinite persistence length (rigid rod) reduces to $3.620 (\log[\text{masstrue}]-3.552)$ For Kovacic and van Holde's fit to the DNA data.

CHR started 18 Feb 2020. Test of reproducibility for the ReScience 10-year reproducibility challenge (<https://rescience.github.io/ten-years/>). Chosen paper is Robert, C.H. (1995) Estimating Friction Coefficients of Mixed Globular/Chain Molecules, such as Protein/DNA Complexes. Biophys J. 69, 840-48.

All code in this file came from the original notebooks "Example 2 1-over-r.nb" and "Example 2 dinucleosomes.nb" and run using *Mathematica* 5.2 and 12.0.

Changes:

- lines were added to save results for lollipop and dumbbell assemblies and to plot them at the end of the notebook
- semicolons at end of commands producing graphics were suppressed to see the output
- removed the mysterious "t=" command starting this notebook. A misprint? Or a sign of higher intelligence?

Results to compare to originals are indicated by blue text boxes.

```
In[1]:= Off[General::spell];
Off[General::spell1];
```

```
In[3]:= NAvo=6.02 10^23;
ro=1.003; (* density, g/cm^3 *)
visc=0.01016; (* viscosity, g/(cm s) or "Poise" *)
persistencelength=34000 10^-8; (* persistence length, cm *)
el=3.4 10^-8; (* height of basepair, cm *)
lx={0,0,1}; (* chain vector for a basepair *)
ctorsion=2.4 10^-19; (* not used-- ergs/radian? *)
mhistone=108500; (* octamer mol. weight, g/mole *)
mbp=660; (* basepair mol. weight, g/mole *)
vdna=0.55; (* specific volume of DNA, cm^3/g *)
vhistone=0.75; (* same but of protein, cm^3/g *)
score=10.7 10^-13; (* core sedimentation coeff, sec *)
dDNA=27 10^-8; (* Yamagawa-Fujii (Y-F) diam, cm *)
switchbp=50; (* point at which we switch from *)
(* Y-F to ellipsoid model, bp *)
```

```

In[18]:= doKirkwoodG:=Block[{i,j,sum,sumradii},
  (* Calculates friction and sedimentation coefficients, f1 and
    s1, according to the standard Kirkwood-Riseman theory. *)
  sumradii=Sum[radii[[i]],{i,ntot}];
  sum=0;
  Do[ Do[ rirj=N[radii[[i]] radii[[j]]];
    If[ rirj>0,
      delsum=rirj/rij[[i,j]],
      delsum=0
    ];
    If[ test,
      Print[" i,j,rij(cm): ",i," ",j," ",N[rij[[i,j]]]
    ];
    sum=sum+delsum,
    {j,i+1,ntot}],
  {i,ntot}];
  (* Notice we only counted the distance of each pair once,
    but we multiplied by two to make up for it. Now calculate
    frictional coeff and convert from basepairs to cm: *)
  ffree=6 Pi visc sumradii g/s;
  f1=6 Pi visc sumradii/(1 + 2 sum/sumradii) g/s;
  Print[" Kirkwood results: {f free-draining,f1}"];
  Print[" ",N[{ffree,f1}]]
];

```

```

In[19]:= snaked[i_]:=Module[{j},
  s1=3.620 (Log[10,i 660]-3.552) 10^-13;
  rstokes=((i 660) (1-vdna ro)/(NAvo s1))/(6 Pi visc)
];

```

Define radii of rigid-model DNA spheres and of globular region in cm:

```

In[20]:= r1=57 10^-8;
r2=16.534 10^-8; (* Equal volume achieved
                  by r2=Sqrt[3/2] 13.5 A *)

```

Values of the ratio $L = \text{length}/r1$ to use in the comparison.

```

In[22]:= (* Values of length/r1 to use in comparison and corresponding
  numbers of basepairs: *)
lengthp={3,4,5,6,8,10,12,14,16};
nbasepairs=(lengthp r1)/(3.4 10^-8);

```

```

In[25]:= nbasepairs

```

```

Out[25]= {50.2941, 67.0588, 83.8235, 100.588, 134.118, 167.647, 201.176, 234.706, 268.235}

```

```

In[26]:= test = False;

```

LOLLIPOPS

```

In[27]:= f1Kirk={};
f1robert={};
frictionratio={};
rij={};
Do[
  Print[nbasepairs[[k]]," bp spacer"];

  Print["My model:"];
  ntot=2;
  snaked[nbasepairs[[k]]];
  radii={r1,rstokes};
  rij={{0,r1 + lengthp[[k]] r1/2},{r1 + lengthp[[k]] r1/2,0}};
  doKirkwoodG;
  f1robert=Append[f1robert,f1];

  Print["Kirk's model:"];
  ntot=lengthp[[k]]+1;
  radii={r1};
  Do[radii=Append[radii,r2],{i,lengthp[[k]]}];
  rij=Table[0,{i,ntot},{j,ntot}];
  Do[ Do[
    rij[[i,j]]=radii[[i]] + (j-i-1) ^2 r2 + r2,
    {j,i+1,ntot}],
    {i,ntot}];
  doKirkwoodG;
  f1Kirk=Append[f1Kirk,N[f1]],

```

```
{k,Length[lengthp]]};
```

50.2941 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{1.79634 \times 10^{-7} \text{ g}}{s}, \frac{1.36724 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.04155 \times 10^{-7} \text{ g}}{s}, \frac{1.06294 \times 10^{-7} \text{ g}}{s} \right\}$$

67.0588 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{1.92394 \times 10^{-7} \text{ g}}{s}, \frac{1.49327 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.3582 \times 10^{-7} \text{ g}}{s}, \frac{1.10619 \times 10^{-7} \text{ g}}{s} \right\}$$

83.8235 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.04736 \times 10^{-7} \text{ g}}{s}, \frac{1.61623 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.67484 \times 10^{-7} \text{ g}}{s}, \frac{1.15792 \times 10^{-7} \text{ g}}{s} \right\}$$

100.588 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.16701 \times 10^{-7} \text{ g}}{s}, \frac{1.73621 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.99149 \times 10^{-7} \text{ g}}{s}, \frac{1.21429 \times 10^{-7} \text{ g}}{s} \right\}$$

134.118 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.39706 \times 10^{-7} \text{ g}}{s}, \frac{1.96829 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.62478 \times 10^{-7} \text{ g}}{s}, \frac{1.33378 \times 10^{-7} \text{ g}}{s} \right\}$$

167.647 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.61743 \times 10^{-7} \text{ g}}{s}, \frac{2.19157 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{4.25807 \times 10^{-7} \text{ g}}{s}, \frac{1.45695 \times 10^{-7} \text{ g}}{s} \right\}$$

201.176 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.83031 \times 10^{-7} \text{ g}}{s}, \frac{2.40771 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{4.89136 \times 10^{-7} \text{ g}}{s}, \frac{1.58096 \times 10^{-7} \text{ g}}{s} \right\}$$

234.706 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.03719 \times 10^{-7} \text{ g}}{s}, \frac{2.61794 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{5.52465 \times 10^{-7} \text{ g}}{s}, \frac{1.70467 \times 10^{-7} \text{ g}}{s} \right\}$$

268.235 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.2391 \times 10^{-7} \text{ g}}{s}, \frac{2.82316 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{6.15794 \times 10^{-7} \text{ g}}{s}, \frac{1.82757 \times 10^{-7} \text{ g}}{s} \right\}$$

CHR Feb 2020. Lollipop geometry friction coefficients are identical to the corresponding results in "Figure 2 calculations.nb" with the exception of the number of reported significant figures (6 here versus 4 in the original). The 1st and 3rd columns of figures were plotted as the solid lines in Figure 2. (The dotted lines and black circles in Figure 2 were obtained from previous work.)

```
In[32]:= MatrixForm[Transpose[{N[nbasepairs, 3], N[f1robert, 4], N[f1Kirk, 4]}]]
```

```
Out[32]//MatrixForm=
```

$$\begin{pmatrix} 50.2941 & \frac{1.36724 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.06294 \times 10^{-7} \text{ g}}{\text{s}} \\ 67.0588 & \frac{1.49327 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.10619 \times 10^{-7} \text{ g}}{\text{s}} \\ 83.8235 & \frac{1.61623 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.15792 \times 10^{-7} \text{ g}}{\text{s}} \\ 100.588 & \frac{1.73621 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.21429 \times 10^{-7} \text{ g}}{\text{s}} \\ 134.118 & \frac{1.96829 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.33378 \times 10^{-7} \text{ g}}{\text{s}} \\ 167.647 & \frac{2.19157 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.45695 \times 10^{-7} \text{ g}}{\text{s}} \\ 201.176 & \frac{2.40771 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.58096 \times 10^{-7} \text{ g}}{\text{s}} \\ 234.706 & \frac{2.61794 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.70467 \times 10^{-7} \text{ g}}{\text{s}} \\ 268.235 & \frac{2.82316 \times 10^{-7} \text{ g}}{\text{s}} & \frac{1.82757 \times 10^{-7} \text{ g}}{\text{s}} \end{pmatrix}$$

```
In[33]:=
```

```
xLollipop = nbasepairs;  
yLollipop = f1robert;  
yLollipop2 = f1Kirk;
```

Copy-paste the original output here for comparison. The recalculated results match to all significant figures.

$$\begin{array}{ccc} & \frac{1.367 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.063 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 50.3 & & \\ & \frac{1.493 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.106 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 67.1 & & \\ & \frac{1.616 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.158 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 83.8 & & \\ & \frac{1.736 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.214 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 101. & & \\ & \frac{1.968 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.334 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 134. & & \\ & \frac{2.192 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.457 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 168. & & \\ & \frac{2.408 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.581 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 201. & & \\ & \frac{2.618 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.705 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 235. & & \\ & \frac{2.823 \cdot 10^{-7} \text{ g}}{\text{s}} & \frac{1.828 \cdot 10^{-7} \text{ g}}{\text{s}} \\ 268. & & \end{array}$$

DUMBBELLS

Here are the dumbbell calculations. We use a linear array that is the same as before, but another subunit is added at the end.

```

In[36]:= f1Kirk={};
f1robert={};
frictionratio={};
rij={};
Do[
  Print[nbasepairs[[k]], " bp spacer"];

  Print["My model:"];
  ntot=3;
  snaked[nbasepairs[[k]]];
  radii={r1,rstokes,r1};
  rij={{0,r1 + lengthp[[k]] r1/2,2 r1 + lengthp[[k]] r1},
        {0,0,r1 + lengthp[[k]] r1/2},
        {0,0,0}};
  doKirkwoodG;
  f1robert=Append[f1robert,f1];

  Print["Kirk's model:"];
  ntot=lengthp[[k]]+2;
  radii={r1};
  Do[radii=Append[radii,r2],{i,lengthp[[k]]}];
  radii=Append[radii,r1];
  rij=Table[0,{i,ntot},{j,ntot}];
  Do[ Do[ If[ j<ntot,
            rij[[i,j]]=radii[[i]] + (j-i-1) 2 r2 + r2,
            rij[[i,j]]=radii[[i]] + (j-i-1) 2 r2 + radii[[j]]
          ],
        {j,i+1,ntot}],
    {i,ntot}];
  doKirkwoodG;
  f1Kirk=Append[f1Kirk,N[f1]],

{k,Length[lengthp]}};

50.2941 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{2.88796 \times 10^{-7} \text{ g}}{\text{s}}, \frac{1.87331 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$


Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.13317 \times 10^{-7} \text{ g}}{\text{s}}, \frac{1.4686 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$


67.0588 bp spacer

```

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.01556 \times 10^{-7} \text{ g}}{s}, \frac{2.02566 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.44981 \times 10^{-7} \text{ g}}{s}, \frac{1.50246 \times 10^{-7} \text{ g}}{s} \right\}$$

83.8235 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.13898 \times 10^{-7} \text{ g}}{s}, \frac{2.1688 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.76646 \times 10^{-7} \text{ g}}{s}, \frac{1.54392 \times 10^{-7} \text{ g}}{s} \right\}$$

100.588 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.25863 \times 10^{-7} \text{ g}}{s}, \frac{2.30493 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{4.0831 \times 10^{-7} \text{ g}}{s}, \frac{1.59033 \times 10^{-7} \text{ g}}{s} \right\}$$

134.118 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.48868 \times 10^{-7} \text{ g}}{s}, \frac{2.56156 \times 10^{-7} \text{ g}}{s} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{4.71639 \times 10^{-7} \text{ g}}{s}, \frac{1.69212 \times 10^{-7} \text{ g}}{s} \right\}$$

167.647 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.70904 \times 10^{-7} \text{ g}}{\text{s}}, \frac{2.80287 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{5.34968 \times 10^{-7} \text{ g}}{\text{s}}, \frac{1.80055 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$

201.176 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{3.92193 \times 10^{-7} \text{ g}}{\text{s}}, \frac{3.03298 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{5.98297 \times 10^{-7} \text{ g}}{\text{s}}, \frac{1.91227 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$

234.706 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{4.12881 \times 10^{-7} \text{ g}}{\text{s}}, \frac{3.25447 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{6.61626 \times 10^{-7} \text{ g}}{\text{s}}, \frac{2.02559 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$

268.235 bp spacer

My model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{4.33072 \times 10^{-7} \text{ g}}{\text{s}}, \frac{3.46903 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$

Kirk's model:

Kirkwood results: {f free-draining,f1}

$$\left\{ \frac{7.24955 \times 10^{-7} \text{ g}}{\text{s}}, \frac{2.13958 \times 10^{-7} \text{ g}}{\text{s}} \right\}$$

CHR Feb 2020. Dumbbell geometry friction coefficients are identical to the corresponding results in "Figure 2 calculations.nb" with the exception of the number of reported significant figures (6 here versus 4 in the original). The 1st and 3rd columns of figures were plotted as the solid lines in Figure

2. (The dotted lines and black circles in Figure 2 were obtained from previous work.)

```
In[41]:= MatrixForm[Transpose[{N[nbasepairs,3],N[f1robert,4],N[f1Kirk,4]}]]
```

```
Out[41]//MatrixForm=
```

50.2941	$\frac{1.87331 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.4686 \times 10^{-7} \text{ g}}{\text{s}}$
67.0588	$\frac{2.02566 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.50246 \times 10^{-7} \text{ g}}{\text{s}}$
83.8235	$\frac{2.1688 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.54392 \times 10^{-7} \text{ g}}{\text{s}}$
100.588	$\frac{2.30493 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.59033 \times 10^{-7} \text{ g}}{\text{s}}$
134.118	$\frac{2.56156 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.69212 \times 10^{-7} \text{ g}}{\text{s}}$
167.647	$\frac{2.80287 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.80055 \times 10^{-7} \text{ g}}{\text{s}}$
201.176	$\frac{3.03298 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.91227 \times 10^{-7} \text{ g}}{\text{s}}$
234.706	$\frac{3.25447 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{2.02559 \times 10^{-7} \text{ g}}{\text{s}}$
268.235	$\frac{3.46903 \times 10^{-7} \text{ g}}{\text{s}}$	$\frac{2.13958 \times 10^{-7} \text{ g}}{\text{s}}$

```
In[42]:=
```

```
xDumbbell = nbasepairs;  
yDumbbell = f1robert;  
yDumbbell2 = f1Kirk;
```

Copy-paste the original output here for comparison. The recalculated results match to all provided significant figures.

50.3	$\frac{1.873 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.469 \cdot 10^{-7} \text{ g}}{\text{s}}$
67.1	$\frac{2.026 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.502 \cdot 10^{-7} \text{ g}}{\text{s}}$
83.8	$\frac{2.169 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.544 \cdot 10^{-7} \text{ g}}{\text{s}}$
101.	$\frac{2.305 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.59 \cdot 10^{-7} \text{ g}}{\text{s}}$
134.	$\frac{2.562 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.692 \cdot 10^{-7} \text{ g}}{\text{s}}$
168.	$\frac{2.803 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.801 \cdot 10^{-7} \text{ g}}{\text{s}}$
201.	$\frac{3.033 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{1.912 \cdot 10^{-7} \text{ g}}{\text{s}}$
235.	$\frac{3.254 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{2.026 \cdot 10^{-7} \text{ g}}{\text{s}}$
268.	$\frac{3.469 \cdot 10^{-7} \text{ g}}{\text{s}}$	$\frac{2.14 \cdot 10^{-7} \text{ g}}{\text{s}}$

For convenience, the relevant values are plotted below in the format close to that used in the article (the article's approach as solid lines versus the standard approach as dashes). The data points indicated as filled circles in Figure 2 were taken from previous results and are not shown.

```
In[95]:= xlabel = "Spacer length, bp";
ylabel = Row[{"frictional coefficient x", Superscript[10, 7], "g/s"}];

ClearAll[plotdark];
plotdark[xs_, ys_, plotstyle_:Automatic, plotrange_:{{0, 290}, {0, 3.5}}] :=
  ListLinePlot[Transpose[{xs, ys}], PlotStyle -> plotstyle, PlotRange -> plotrange,
    AspectRatio -> 1, DisplayFunction -> Identity]

Show[
  plotdark[xLollipop, 10^7 yLollipop /. {g -> 1, s -> 1}],
  plotdark[xLollipop, 10^7 yLollipop2 /. {g -> 1, s -> 1}, Dashing[{0.03, 0.015}]],
  plotdark[xDumbbell, 10^7 yDumbbell /. {g -> 1, s -> 1}],
  plotdark[xDumbbell, 10^7 yDumbbell2 /. {g -> 1, s -> 1}, Dashing[{0.025, 0.015}]],
  AspectRatio -> 1,
  Frame -> True, FrameLabel -> {xlabel, ylabel},
  DisplayFunction -> $DisplayFunction
]
```

