FRACTIONALLY SPACED LINEAR MMSE TURBO EQUALIZATION

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ABSTRACT

We extend previous work on linear MMSE (minimum mean square error) SISO (soft-in/soft-out) equalizers for turbo equalization to the case of fractional sampling of the received signal. We present a time-recursive algorithm also for this case. Special attention is paid to the influence of the receiver filter preceeding the sampler.

1. INTRODUCTION

Turbo equalization, or iterative equalization and decoding, was first introduced in [1] to improve communication system performance over channels imposing intersymbol interference (ISI). Turbo equalization can be applied whenever the data bits are protected by an error-correcting code (ECC) and the code bits are interleaved before transmission over an ISI channel. The principle, shown in Fig. 1, is to exhange reliability (soft) information on the code bits, usually in the form of log-likelihood ratios (LLRs), between the equalizer for the ISI channel and the decoder for the ECC in an iterative fashion.

The equalizer and decoder in turbo equalization are SISO modules, accepting soft information on the code bits as input and providing soft information on the code bits as output. The optimal (w.r.t. bit error rate, BER) SISO equalizer utilizes the APP (*a posteriori* probability) algorithm of [2], but this is too complex for many applications, especially if the modulation order and/or the length of the channel impulse response (CIR) is large. For such applications, suboptimal SISO equalizers must be used, of which a popular choice is soft ISI cancellation combined with linear filtering [3, 4, 5], see Fig. 2: The input soft information is used to compute the *a priori* average value (mean) of the ISI, which is subtracted from the received signal. The remaining signal, which consists of the transmitted signal, noise, and residual ISI, is passed through a linear filter, and finally the output signal from the linear filter is converted to LLRs on the code bits in a soft-output demapper.

In most previous literature on this topic, a symbol-spaced discrete-time model has been assumed, as implicitly shown in Figs. 1-2. The equalizer filter $\mathbf{f}_n$ can be computed using different criteria, of which the MMSE-criterion taking the input soft information into account performs best [4, 5]. This approach, called linear MMSE SISO equalizer, has computational complexity proportional to N^2 per symbol interval and iteration, and yields a time-varying filter even when the channel impulse response is constant. In [6] this solution was extended to the case of time-varying channels, and in the present paper we extend it further to time-varying channels using a fractionally spaced discrete-time model (two samples/symbol). It is well known that fractional sampling performs better than symbol-spaced sampling in the case of inexact symbol synchronization or when the channel exhibits spectral nulls in the roll-off region of the transmitted spectrum (see e.g. [7])

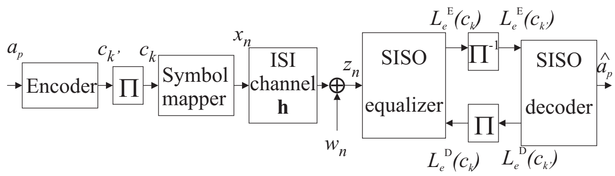

Figure 1: Turbo equalization system model.

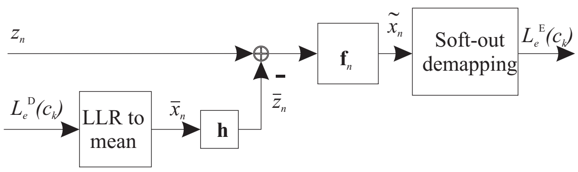

Figure 2: SISO equalizer using soft ISI cancellation and linear filtering, operating at one sample/symbol.

We note that independent from [4, 5] a fractionally spaced linear MMSE SISO equalizer was derived in [8], inspired by ideas from iterative multiuser detection [9]. The differences between our approach and that presented in [8] are that we allow the CIR to be time-varying, and that we do not assume a whitened matched filter preceeding the sampler. In this paper we assume the CIR to be known at every time instant; incorporating channel estimation is also part of our research but is left out of this paper for simplicity.

2. DISCRETE-TIME CHANNEL MODELS

In this section we describe well-known channel models in the notation used in this paper.

Consider the baseband-equivalent model shown in Fig. 3(a). The transmitted symbols $x(t) = \sum_{n=-\infty}^{\infty} x_n \delta(t - nT)$ are passed through a transmitter filter $g_T(\tau)$ and a time-varying possibly complex-valued CIR $h(t,\tau)$, added with white Gaussian noise $w(t)$ with two-sided spectral density $N_0/2$, and passed through a receiver filter $g_R(\tau)$ before being sampled either once or twice per symbol period. The received signal prior to sampling is

$$z(t) = \int_{-\infty}^{\infty} h_E(t,\tau)x(t-\tau)d\tau + \int_{-\infty}^{\infty} g_R(\tau)w(t-\tau)d\tau \quad (1)$$

where the equivalent channel impulse response $h_E(t,\tau)$ is the convolution

$$h_E(t,\tau) = \int_{-\infty}^{\infty} h(t,u)g_E(\tau-u)du \quad (2)$$

and $g_E(\tau)$ is the convolution of $g_T(\tau)$ and $g_R(\tau)$.

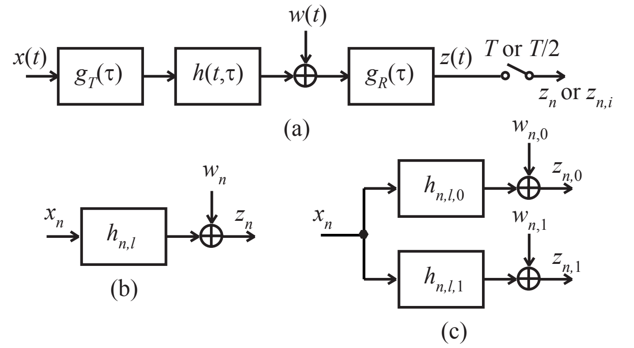

Figure 3: Channel models: (a) Continuos-time with sampled output. (b) Discrete-time symbol-spaced. (c) Discrete-time fractionally spaced.

In the equivalent discrete-time symbol-spaced model shown in Fig. 3(b) the received samples $z_n = z(nT)$ are given by

$$z_n = \sum_{l=-\infty}^{\infty} x_{n-l} h_{n,l} + w_n \tag{3}$$

where

$$h_{n,l} = h_E(nT, lT) = \int_{-\infty}^{\infty} h(nT,\tau) g_E(lT-\tau) d\tau \tag{4}$$

and $w_n = \int_{-\infty}^{\infty} g_R(\tau) w(nT-\tau) d\tau$. If the folded spectrum of $g_R(\tau)$ is equal to 1 for all frequencies in the Nyquist range $[-1/2T, 1/2T]$, the noise samples w_n will be independent complex Gaussian variables with variance N_0/T. We then obtain the equivalent channel model usually assumed in the derivation of symbol-spaced equalizers.

Sampling the received signal twice per symbol can equivalently be considered as two parallel symbol-spaced channels, as shown in Fig. 3(c). The received samples $z_{n,i} = z(nT + iT/2)$ are given by

$$z_{n,i} = \sum_{l=-\infty}^{\infty} x_{n-l} h_{n,l,i} + w_{n,i}, \quad i \in \{0,1\} \tag{5}$$

where

$$h_{n,l,i} = \int_{-\infty}^{\infty} h(nT+iT/2,\tau) g_E(lT + iT/2 - \tau) d\tau \tag{6}$$

and $w_{n,i} = \int_{-\infty}^{\infty} g_R(\tau) w(nT + iT/2 - \tau) d\tau$. Note that for the fractionally sampled noise sequence to be white, i.e., *all* $w_{n,i}$ being independent Gaussian variables, the folded spectrum of $g_R(\tau)$ must be equal to 1 for all frequencies in the Nyquist range which has now been extended to $[-1/T, 1/T]$. The variance of each noise sample is then $2N_0/T$ due to the doubled noise bandwidth, but this does not pose a performance degradation since we have twice as many noise samples to average over.

The received signal energy per symbol, E_s, for uncorrelated symbols x_n from a unit-energy constellation transmitted over a constant CIR and in the abscence of noise, is given by

$$\begin{aligned} E_s &= \int_{-\infty}^{\infty} h_E(t,\tau) h_E^*(t,\tau) d\tau \\ &\approx \begin{cases} T \sum_{l=-\infty}^{\infty} h_{n,l} h_{n,l} = TE\{z_n z_n^*\} \\ \frac{T}{2} \sum_{l=-\infty}^{\infty} \sum_{i=0}^{1} h_{n,l,i} h_{n,l,i} = TE\{z_{n,i} z_{n,i}^*\} \end{cases} \end{aligned} \tag{7}$$

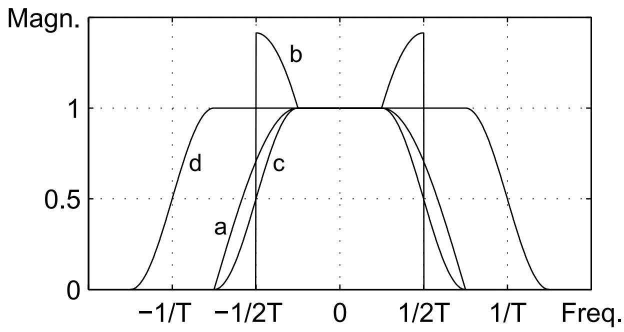

Figure 4: Filter frequency magnitude responses: (a) SRRC, $\alpha = 0.5$. (b) SRRC folded at the Nyquist frequency $1/2T$. (c) Raised cosine (RC) Nyquist filter symmetric around $1/2T$, $\alpha = 0.5$. (d) RC Nyquist filter symmetric around $1/T$, $\alpha = 0.25$.

for one and two samples/symbol, respectively. For a time-varying CIR, further averaging over t or n should be performed in these expressions.

For the following discussion, we refer to the illustration in Fig. 4. A commonly used filter for $g_T(\tau)$ and $g_R(\tau)$ has the SRRC (square root raised cosine) spectrum, such that their cascade is a Nyquist filter and the symbol-spaced sampled output is ISI-free if the channel is perfect, $h(t,\tau) = \delta(\tau)$. However, the symbol-spaced noise sequence will in this case not be white as the folded spectrum of $g_R(\tau)$ is 3 dB higher at the Nyquist frequency $1/2T$ than at zero frequency. Another choice of $g_R(\tau)$ is the "whitened matched filter" [10], matched either to $g_T(\tau)$ alone or to the convolution of $g_T(\tau)$ and $h(t,\tau)$. Choosing $g_R(\tau)$ as a whitened matched filter *or* as a Nyquist filter (symmetric around $1/2T$) will give a white noise sequence.

When sampling twice per symbol and $g_T(\tau)$ is an SRRC filter with roll-off factor α smaller than 1, the whitened matched filter *does not exist*. This subtle but seldom mentioned fact is caused by the matched filter (SRRC in the case of an ideal channel) having a frequency response equal to zero in the upper part $\pm[(1+\alpha)/2T, 1/T]$ of the extended Nyquist range, such that it can not be inverted to give a white noise sequence. Different choices for $g_R(\tau)$ for two samples per symbol can be SRRC or a matched filter, giving a colored noise sequence, or a Nyquist filter symmetric around $1/T$ giving a white noise sequence.

3. FRACTIONALLY SPACED MMSE SISO EQUALIZER

In this section we describe a fractionally spaced linear MMSE SISO equalizer. The derivation is similar to the symbol-spaced linear MMSE SISO equalizer of [4, 5, 6], and has been presented in more detail in a thesis [11].

Consider a turbo equalization-based receiver as shown in Fig. 1, with a filter-based SISO equalizer as in Fig. 2 but operating at two samples/symbol. The input LLRs $L_e^D(c_k)$ (fed back from the decoder from the previous iteration) are converted to *a priori* means $\bar{x}_n = E_L\{x_n\}$ and variances $v_n = E_L\{x_n x_n^*\} - \bar{x}_n \bar{x}_n^*$ of the transmitted symbols x_n [4], where E_L denotes expectation given the input LLRs. From the $\bar{x}_n$ we compute the *a priori* mean of the ISI, which is subtracted from the received signal.

The residual signal after soft ISI cancellation is fed through a linear filter $\mathbf{f}_n$ operating at two samples/symbol. The filter spans N_1 precursor symbols and N_2 postcursor symbols, such that the total number of filter coefficients is $2N$ with $N = N_1 + N_2 + 1$. We approximate the impulse response $h_{n,l,i}$ in (5)-(6) to be nonzero only for $0 \leq l < M$. The received signal vector processed by the equalizer at symbol interval n is then

$$\begin{aligned} \mathbf{z}_n &= [z_{n+N_1,1}\ z_{n+N_1,0}\ z_{n+N_1-1,1}\ \ldots\ z_{n-N_2,1}\ z_{n-N_2,0}]^T \\ &= \mathbf{H}_n \mathbf{x}_n + \mathbf{w}_n \end{aligned} \tag{8}$$

where

$$\mathbf{H}_n = \begin{bmatrix} \mathbf{h}^T_{n+N_1,1} & 0 & \cdots & 0 \\ \mathbf{h}^T_{n+N_1,0} & 0 & \cdots & 0 \\ 0 & \mathbf{h}^T_{n+N_1-1,1} & \cdots & 0 \\ 0 & \mathbf{h}^T_{n+N_1-1,0} & \cdots & 0 \\ & & \ddots & \\ 0 & & \cdots & \mathbf{h}^T_{n-N_2,1} \\ 0 & & \cdots & \mathbf{h}^T_{n-N_2,0} \end{bmatrix} \quad 2N \times (N+M-1) \tag{9}$$

$$\mathbf{h}_{n,i} = [h_{n,0,i}\ h_{n,1,i}\ \ldots\ h_{n,M-1,i}]^T \tag{10}$$

$$\mathbf{x}_n = [x_{n+N_1}\ x_{n+N_1-1}\ \ldots\ x_{n-N_2-M+1}]^T \tag{11}$$

$$\mathbf{w}_n = [w_{n+N_1,1}\ w_{n+N_1,0}\ w_{n+N_1-1,1}\ \ldots\ w_{n-N_2,0}]^T \tag{12}$$

We further define $\mathbf{s}_n$ as the $(N_1 + 1)$th column of $\mathbf{H}_n$, $\bar{\mathbf{x}}_n = E_L\{\mathbf{x}_n\}$, $\mathbf{V}_n = Cov_L\{\mathbf{x}_n, \mathbf{x}_n\} = E_L\{\mathbf{x}_n\mathbf{x}_n^H\} - \bar{\mathbf{x}}_n\bar{\mathbf{x}}_n^H$, and $\mathbf{\Sigma}_n = Cov\{\mathbf{w}_n, \mathbf{w}_n\}$. The transmitted symbols x_n can be considered independent (at least locally) due to the interleaver, such that $\mathbf{V}_n = \mathrm{Diag}[v_{n+N_1}\ v_{n+N_1-1}\ \ldots\ v_{n-N_2-M+1}]$.

Following [4, 6, 11], we find that the symbol estimates $\tilde{x}_n$ (which are converted to output LLRs in the soft-output demapper) from the MMSE-optimal linear SISO equalizer are given by

$$\tilde{x}_n = \mathbf{f}_n^H(\mathbf{z}_n - \mathbf{H}_n\bar{\mathbf{x}}_n + \bar{x}_n\mathbf{s}_n) \tag{13}$$

$$\mathbf{f}_n = (\mathbf{H}_n\mathbf{V}_n\mathbf{H}_n^H + (1 - v_n)\mathbf{s}_n\mathbf{s}_n^H + \mathbf{\Sigma}_n)^{-1}\mathbf{s}_n \tag{14}$$

and, using the matrix inversion lemma,

$$\mathbf{f}_n = \frac{\mathbf{g}_n}{1 + (1 - v_n)\mathbf{g}_n^H\mathbf{s}_n} \tag{15}$$

$$\mathbf{g}_n = (\mathbf{H}_n\mathbf{V}_n\mathbf{H}_n^H + \mathbf{\Sigma}_n)^{-1}\mathbf{s}_n = \mathbf{\Pi}_n^{-1}\mathbf{s}_n \tag{16}$$

The filter $\mathbf{f}_n$ is time-varying even when the channel is constant, due to the influence of input LLRs through $\mathbf{V}_n$. Direct computation of $\mathbf{f}_n$ has complexity proportional to $(2N)^3$ due to the inversion of the $2N \times 2N$ matrix $\mathbf{\Pi}_n = \mathbf{H}_n\mathbf{V}_n\mathbf{H}_n^H + \mathbf{\Sigma}_n$. For the symbol-spaced case a time-recursive update algorithm was derived [4, 6] to reduce the complexity from cubic to quadratic in N, for constant as well as time-varying channel conditions. Below, we present a similar time-recursive algorithm for two samples/symbol, with complexity proportional to $(2N)^2$.

We define $\mathbf{U}_n = \mathbf{\Pi}_n^{-1}$, and use the partitions

$$\mathbf{\Pi}_{n-1} = \left[\begin{array}{c|c} \mathbf{A}_n & \mathbf{G}_n \\ \hline \mathbf{G}_n^H & \mathbf{B}_n \end{array}\right], \mathbf{\Pi}_n = \left[\begin{array}{c|c} \mathbf{E}_n & \mathbf{F}_n^H \\ \hline \mathbf{F}_n & \mathbf{A}_n \end{array}\right] \tag{17}$$

$$\mathbf{U}_{n-1} = \left[\begin{array}{c|c} \mathbf{D}_n & \mathbf{N}_n \\ \hline \mathbf{N}_n^H & \mathbf{M}_n \end{array}\right], \mathbf{U}_n = \left[\begin{array}{c|c} \mathbf{K}_n & \mathbf{L}_n^H \\ \hline \mathbf{L}_n & \mathbf{C}_n \end{array}\right] \tag{18}$$

where $\mathbf{B}_n$, $\mathbf{E}_n$, $\mathbf{K}_n$, and $\mathbf{M}_n$ are 2×2 matrices, $\mathbf{F}_n$, $\mathbf{G}_n$, $\mathbf{L}_n$, and $\mathbf{N}_n$ are $(2N-2) \times 2$ matrices, and $\mathbf{A}_n$, $\mathbf{C}_n$, and $\mathbf{D}_n$ are $(2N-2) \times (2N-2)$ matrices. The time-recursive algorithm exploits the fact that we have a structured time-dependence between $\mathbf{\Pi}_{n-1}$ and $\mathbf{\Pi}_n$: They share a common submatrix $\mathbf{A}_n$, as can be verified by writing out the matrices elementwise.

Inserting (17)-(18) into the equations $\mathbf{\Pi}_n\mathbf{U}_n = \mathbf{I}_{2N}$ and $\mathbf{\Pi}_{n-1}\mathbf{U}_{n-1} = \mathbf{I}_{2N}$, it is straightforward to derive the time-recursive update:

$$\mathbf{A}_n^{-1} = \mathbf{D}_n - \mathbf{N}_n\mathbf{M}_n^{-1}\mathbf{N}_n^H \tag{19}$$

$$\mathbf{\Psi}_n = \mathbf{A}_n^{-1}\mathbf{F}_n \tag{20}$$

$$\mathbf{K}_n = (\mathbf{E}_n - \mathbf{F}_n^H\mathbf{\Psi}_n)^{-1} \tag{21}$$

$$\mathbf{L}_n = -\mathbf{\Psi}_n\mathbf{K}_n \tag{22}$$

$$\mathbf{C}_n = \mathbf{A}_n^{-1} + \mathbf{\Psi}_n\mathbf{K}_n\mathbf{\Psi}_n^H \tag{23}$$

where no matrix inversions are larger than 2×2.

At each symbol interval, the "new" elements $\mathbf{E}_n$ and $\mathbf{F}_n$ are computed as given by the definition of $\mathbf{\Pi}_n$. Then, $\mathbf{U}_n$ is computed from $\mathbf{U}_{n-1}$ using (19)-(23), and finally the filter coefficients $\mathbf{f}_n$ are computed using (15) and $\mathbf{g}_n = \mathbf{U}_n\mathbf{s}_n$. The algorithm can be initialized e.g. during an initial training sequence, where all transmitted symbols are *a priori* known such that $\mathbf{V}_n = \mathbf{0}$ and $\mathbf{U}_n = \mathbf{\Sigma}_n^{-1}$. This time-recursive algorithm has been implemented in C code as a "mex file" for Matlab, and used in the simulations presented below.

A point to note is that the inversion of a correlation matrix is equivalent to inverting the corresponding frequency spectrum. Therefore, if the receiver filter $g_R(\tau)$ has frequency response equal to zero somewhere in the extended Nyquist range $[-1/T, 1/T]$, neither the spectrum corresponding to $\mathbf{H}_n\mathbf{V}_n\mathbf{H}_n^H$ nor to $\mathbf{\Sigma}_n$ will have any content at these frequencies, and the matrix $\mathbf{\Pi}_n$ will be ill-conditioned such that the recursive algorithm trying to produce the inverse of $\mathbf{\Pi}_n$ will be numerically unstable. In order for the MMSE-optimal solution above to be applied directly, $g_R(\tau)$ must thus have a nonzero frequency response in the entire extended Nyquist range.

If $g_R(\tau)$ does not fulfill this criterion, e.g. being the SRRC filter in Fig. 4, there is strong correlation between adjacent noise samples and there is no unique MMSE solution. If one should still wish to use such a receiver filter, some untrue assumption on the noise correlation matrix $\mathbf{\Sigma}_n$ must be applied, such that a solution for $\mathbf{f}_n$ (which is no longer strictly MMSE-optimal) can be found. This can e.g. be assuming the filtered noise sequence is white, such that $\mathbf{\Sigma}_n$ is diagonal.

4. SIMULATION RESULTS

We performed simulations using a standardized waveform for military HF (High Frequency, 3-30 MHz) ionospheric communications: MIL-STD-188-110B [12] operating at

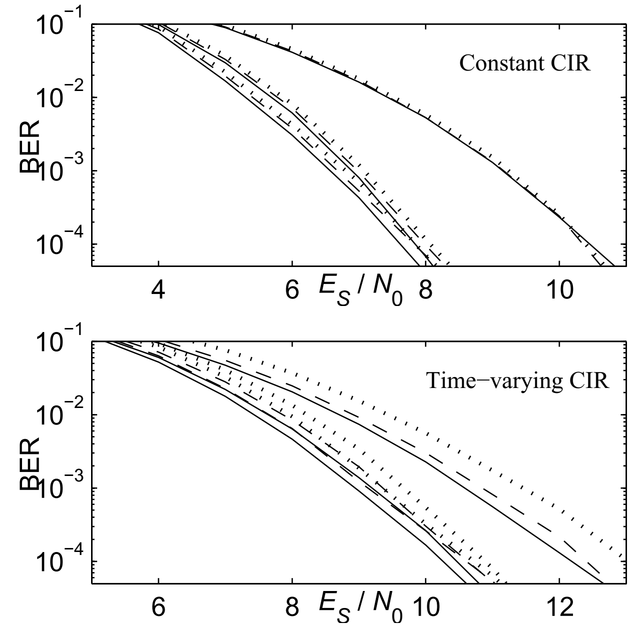

Figure 5: Simulated bit error rate vs E_s/N_0 after (from right to left) 0, 1, and 2 iterations. 150 interleaver blocks (≈ 1.7 million data bits) were simulated. Solid: Two samples/symbol, $g_R(\tau)$ Nyquist filter at symmetric around $1/T$. Dashed: Two samples/symbol, $g_R(\tau)$ SRRC filter matched to $g_T(\tau)$. Dotted: One sample/symbol, $g_R(\tau)$ as for dashed.

2400 bps in a 3 kHz bandwidth, with the long interleaver setting (4.8 s).

Blocks of 11514 independent data bits a_p are passed through a terminated convolutional rate-1/2 memory-6 ECC. The code bits $c_{k'}$ are passed through an interleaver permutation Π and mapped onto an 8-PSK signal constellation giving data symbols x_n. These are multiplexed with training symbols, which we also denote x_n for simplicity. The main pattern is 20 data symbols followed by 20 training symbols. The transmitted data and training symbols x_n, at a symbol rate of $1/T = 2400$ baud, are passed through an SRRC filter $g_T(\tau)$ with $\alpha = 0.35$ before upconversion and transmission over the radio channel.

The simulation setup used here is similar to that described in [13], except that the SISO equalizer now operates at two samples/symbol and that $g_T(\tau)$ and $g_R(\tau)$ are included. We have investigated two different channels with a delay spread of 1 ms (2.4 symbol intervals), namely the constant two-tap CIR $h(t,\tau) = \sqrt{\frac{2}{3}}\delta(\tau) + \sqrt{\frac{1}{3}}\delta(\tau - 1\text{ ms})$ and the time-varying two-tap CIR $h(t,\tau) = h_1(t)\delta(\tau) + h_2(t)\delta(\tau - 1\text{ ms})$, where $h_1(\tau)$ and $h_2(t)$ are equal-energy complex Gaussian processes (Rayleigh fading) with a Gaussian Doppler spectrum and a 2σ Doppler spread of 1 Hz, generated as described in [14]. The length of the equalizer filter $\mathbf{f}_n$ is given by $N_1 = N_2 = 20$, i.e. 41 taps for symbol-spaced sampling and 82 taps for fractional sampling.

Fig. 5 shows simulation results for symbol-spaced and fractionally-spaced turbo equalization, and for different choices of $g_R(\tau)$. For the dashed curves, the noise was assumed to be white with variance $2N_0/T$ when computing the equalizer coefficients. This performed better than assuming the noise to be white with variance N_0/T (not shown in the plots), which is the actual variance of the colored noise samples. This effect is probably caused by a variance of $2N_0/T$ corresponding to the correct noise power spectral density at low frequencies, below $\pm(1-\alpha)/2T$.

For both channels considered here, fractionally spaced equalization with $g_R(\tau)$ a Nyquist filter symmetric around $1/T$ performs best, as expected from theory, even though the differences are marginal. Note that due to limited resources, the simulation setup was by no means "optimized" to demonstrate large performance differences or large turbo processing gain. These results should therefore merely be considered as a demonstration that the proposed algorithm for fractionally spaced turbo equalization is functional.

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