WIDE-BAND SIGNAL PARAMETER ESTIMATION BASED ON HIGHER-ORDER STATISTICS

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ABSTRACT

In this paper we develop an algorithm to improve the accuracy of the wideband signal parameter estimation. It is well known that in the presence of an unknown noise, these estimates may be grossly inaccurate. The proposed algorithm uses both the fourth order cumulant for the suppression of the gaussian noise, the transformation matrices for estimating the coherent cumulant matrix and a noneigenvector algorithm for the characterization of the sources. We show that the performances of bearing estimation algorithm improve substantially when the proposed algorithm is used. This method is tested on simulated data and its performances are clearly pointed out.

1. INTRODUCTION

In many problems in multidimensional signal processing, the observation vector can be modeled as a superposition of a finite number of elementary signals and an additive noises. Generally, in sensor array processing applications, the estimation of multiple narrow-band or wideband source parameters is a classic problem[1-6]. Many bearing estimation procedures have been reported in literature, among them the various eigenanalysis based methods which have been the focus of many studies[3-6]. These methods are also dependent on the statistical properties of the noises. A fundamental assumption for most direction finding algorithms, developed in the last decade, is that the noise is spatially and temporally white or the spatial correlation structure of the background noise is known to within a multiplicative scalar. Then, the localization algorithm can be usually modified in a straightforward manner to include it in the treatment. In practice, this assumption is rarely fulfilled. This is due to the fact that noise has several origins, such as traffic noise, ambient sea noise, or flow noise, and sometimes the source signals with low power or undetected which are assimilated to noise and are often spatially correlated. In recent years, there has been a growing interest in the problem of improving high resolution eigenstructure techniques in the presence of Gaussian noise or the spatially colored noise[3,6]. The ambient noise is unknown in practice; therefore its modelisation or its estimation is necessary. The methods developed for this problem are very few and there is not a definitive solution to this problem. In this paper, the direction-of-arrivals of the sources are estimated in the presence of the noise assumed as a sum of a Gaussian noise and an unknown noise. By means of the property of the fourth-order cumulant, the gaussian noise contribution is suppressed. Thus we use the coherent signal subspace method with spatial fourth order cumulant matrices. Instead of spectral matrices which are used by the coherent signal subspace algorithm[1], the focusing matrices are used in order to transform the signal subspace of the fourth order cumulant matrices at each analysis frequency bin into a signal subspace at the selected frequency.

2. PROBLEM FORMULATION

Consider an uniform linear array composed of N identical sensors separated from each other by a distance d. Let P (P< N) sources impinge on the array from the directions $\{\theta_1, \theta_2, \theta_3, ..., \theta_p\}$. The signal received at the ith sensor can be expressed as:

$$r_i(t) = \sum_{p=1}^{P} s_p(t-\tau_{ip}) + n_i(t) + g_i(t), \quad i = 1, ..., N$$
$$-T/2 < t < T/2 \qquad (1)$$

where $n_i(t)$ and $g_i(t)$ are the additive Gaussian noise and spatially colored noise respectively at the ith sensor, $s_p(t)$ is the signal emitted by the pth source and τ_{ip} is the propagation delay associated with the pth source and the ith sensor. Rewriting (1) in matrix notation, in the frequency domain, we obtain:

$$\mathbf{r}(f_j) = \mathbf{A}(f_j)\,\mathbf{s}(f_j) + \mathbf{n}(f_j) + \mathbf{g}(f_j), \quad j = 1, ..., M \qquad (2)$$

where **r**(fj), **s**(fj), **n**(fj) is an additive and gaussian noise and **g**(fj) are the Fourier transforms of the observation, signal and noise vectors, and **A**(f_j) is the NxP transfer matrix of the sources-sensor array systems with respect to some chosen reference point.

Assume that the signals and the additive noises are uncorrelated. It follows from these assumptions that the spatial fourth order cumulant matrix [7] at j-th temporal frequency bin is defined as

$$\mathbf{H}(f_j) = \mathrm{cum}(r_1(f_j), r^*_1(f_j), \mathbf{r}(f_j), \mathbf{r}^+(f_j)),$$

substituting (1) into this expression and taking account that the noise $\mathbf{n}$(fj) is Gaussian, after some calculations, we obtain the (NxN) matrix:

$$\mathbf{H}(f_j) = \sum_{i=1}^{P} h_{si}(f_j) \left|\mathbf{A}_j(1,i)\right|^2 \mathbf{a}_i(f_j)\mathbf{a}_i^+(f_j) + \mathrm{Cum}(g_1(f_j), g^*_1(f_j), \mathbf{g}(f_j), \mathbf{g}^+(f_j))$$

or

$$\mathbf{H}(f_j) = \mathbf{A}(f_j)\mathbf{H}_s(f_j)\mathbf{A}^+(f_j) + \mathbf{G}(f_j) \qquad (3)$$

3. PROPOSED METHOD

Note that the matrix $\mathbf{A}(f_j)\mathrm{H}_s(f_j)\mathbf{A}^+(f_j)$ has full rank P(uncorrelated sources), and let its P nonzero eigenvalues: $\lambda_1 \geq \ldots \geq \lambda p$. Then the eigenvalues and eigenvectors of $\mathbf{H}$(fj) are: $\lambda_1 \geq .. \geq \lambda p > \lambda p{+}1 > \ldots > \lambda_N$ and $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, .., \mathbf{v}_N]$. Let $\mathbf{V}$s = $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_p]$ and $\mathbf{V}$g = $[\mathbf{v}_{p+1}, ..., \mathbf{v}_N]$ are the signal and the noise subspaces respectively. Consider the smallest eigenvalue λ_N, by using some algebraic properties[9], one can show that :

$$\lim_{m \to \infty} \left(\frac{\mathbf{H}(f_j)}{\lambda_N(f_j)}\right)^{-m} = \mathbf{V}_g(f_j)\mathbf{V}^+_g(f_j)$$

The signal matrix projection is

$$\mathbf{V}_s(f_j)\mathbf{V}^+_s(f_j) = \mathbf{I} - \mathbf{V}_g(f_j)\mathbf{V}^+_g(f_j)$$

Note that the estimation of the noise matrix projection is obtained without eigendecomposition and without prior knowledge of the number of the sources. The fast algorithm developed in [9] is used to compute the inversion matrix.
To eliminate the spatial colored noise the projection of the received signals on the signal subspace is used, let

$$\mathbf{r}_s(f_j) = \mathbf{V}_s(f_j)\mathbf{V}^+_s(f_j)\mathbf{r}(f_j) \qquad (4)$$

We then compute a new slice cumulant matrix from the vector $\mathbf{r}_s(f_j)$.

Using the assumption, that the matrix $\mathbf{A}(f_j)$ is of rank P, there exists a (N-P) x P matrix such that :

$$\mathbf{A}_2(f_j) = \Pi^+(f_j)\mathbf{A}_1(f_j), \text{ for j=1,...,M} \qquad (5)$$

where $\mathbf{A}_1(f_j)$ and $\mathbf{A}_2(f_j)$ are two block matrices of the transfert matrix $\mathbf{A}(f_j)$, of dimensions (PxP) and (N-P)xP respectively. The matrix $\Pi(f_j)$ is the propagator operator. For estimating $\Pi(f_j)$, the fourth-order cumulant matrix(3) is used. Using (5), the cumulant matrix of the vector $\mathbf{r}_s(f_j)$ can be written

$$\mathbf{H}_{11}(f_j) = \begin{bmatrix} \mathbf{H}^{11}_{11}(f_j) & \mathbf{H}^{12}_{11}(f_j) \\ \mathbf{H}^{21}_{11}(f_j) & \mathbf{H}^{22}_{11}(f_j) \end{bmatrix}, \text{ where}$$

$$\mathbf{H}^{11}_{11}(f_j) = \mathbf{A}_1(f_j)\mathbf{H}_s(f_j)\mathbf{A}_1^+(f_j) \qquad (6)$$

$$\mathbf{H}^{12}_{11}(f_j) = \mathbf{H}^{11}_{11}(f_j)\Pi(f_j) \qquad (7)$$

$$\mathbf{H}^{21}_{11}(f_j) = \Pi^+(f_j)\mathbf{H}^{11}_{11}(f_j) \qquad (8)$$

$$\mathbf{H}^{22}_{11}(f_j) = \Pi^+(f_j)\mathbf{H}^{11}_{11}(f_j)\Pi(f_j). \qquad (9)$$

For example, one can estimate the propagator by :

$$\Pi(f_j) = \left(\mathbf{H}^{11}_{11}(f_j)\right)^{-1}\mathbf{H}^{12}_{11}(f_j). \qquad (10)$$

The obtained propagator is used to calculate the localization function at the frequency fj, given by :

$$L(\theta) = \frac{1}{\left|\mathbf{Q}^+(f_j)a(f_j,\theta)\right|^2}, \quad \text{for} \quad \theta \in [-90°, 90°],$$

where $\mathbf{Q}^+(f_j) = \left[\Pi^+(f_j) \;\middle|\; -\mathbf{I}\right]$ (11).

In order to exploit the advantage provided by the fourth-order cumulant domain and the coherent treatment, the transformation matrices are applied to the propagator at each frequency bin $\mathbf{T}(f_j)\Pi(f_j) = \hat{\Pi}(f_c)$ for j =1,..,M, where $\hat{\Pi}(f_c) = \left[\hat{\mathbf{A}}_1(f_c)\hat{\mathbf{H}}_s(f_c)\hat{\mathbf{A}}_1^+(f_c)\right]^{-1}\mathbf{H}^{12}_{11}(f_c)$ and $\hat{\mathbf{H}}_s(f_c) = \frac{1}{M}\sum_{j=1}^{M}\hat{\mathbf{A}}_1^{-1}(f_j)\mathbf{H}^{11}_{11}(f_j)\left(\hat{\mathbf{A}}_1^+(f_j)\right)^{-1}$, $\hat{\mathbf{A}}_1(f_j)$ is obtained by using an initial estimates of the direction of arrival of the sources. The transformation matrix is given by :

$$\mathbf{T}(f_j) = \hat{\Pi}(f_c)\left(\mathbf{H}_{11}^{12}(f_j)\right)^{+}\left[\mathbf{H}_{11}^{12}(f_j)\mathbf{H}_{11}^{12+}(f_j)\right]^{-1}\left(\mathbf{H}_{11}^{11}(f_j)\right)^{+}$$

The coherent propagator is then :

$$\tilde{\Pi}(f_c) = \frac{1}{M}\sum_{j=1}^{M}\mathbf{T}(f_j)\,\Pi(f_j) \qquad (12)$$

Finally, $\tilde{\Pi}(f_c)$ may be used in (11) for the estimation of the direction-of-arrivals of the wide-band sources.

$$Q(\theta) = \frac{1}{\mathbf{a}^{+}(\theta, f_c)\tilde{\Pi}(f_c)\mathbf{a}(\theta, f_c)} \quad \theta \in [-90°, 90°] \qquad (13)$$

Therefore, the present algorithm for estimating the direction of arrival of wideband sources can be formulated as the following sequence of steps:

1) Compute $\mathbf{H}(f_j)$ (3).
2) Obtain signal matrix projection.
3) Transform the received data (4).
4) Compute $\mathbf{H}_{11}$(fj) and perform initial estimation of DOA.
5) Compute the propagator.
6) Form $\mathbf{T}(f_j)$, using initial estimates step 4), step 5) for j=1,...,M.
7) Form coherent propagator using (12).
8) Determine the peak positions in a spatial spectrum (13).

It should be pointed out that steps 4)- 8) above can be iterated to improve the estimates.

4. NUMERICAL EXAMPLE

We consider a linear array of N=7 sensors uniformly spaced at half wavelength of a central temporal frequency. The source signals are temporally stationary zero-mean, non-Gaussian wide-band. Five sources impinge on the array θ_1 = 8°, θ_2 = 10°, θ_3 = 15° , θ_4 = 20° and θ_5 = 22° respectively, in the presence of the colored noise and Gaussian noise. The array noise is stationary zero-mean, independent of the signals. The signal to noise ratio is 10 dB. From the array outputs, 256 snapshots of 64 samples each were selected and the frequency components were obtained via FFT. The directions 13° and 21° given by the beamformer method are used to estimate the focusing matrices. The MUSIC spectra based on the coherent propagator method before and after whitening the received data are plotted in Figures 1 and 2 respectively. From Figure 2 it is obvious that the two sources are localized by using the cumulant matrix of the transformed data while they are not resolved when the classic matrix cumulant is used.
One can remark the Music method can not separate the five sources even the number of the sources is taken equal to 5, but the proposed method gives the exact azimuth of the sources.

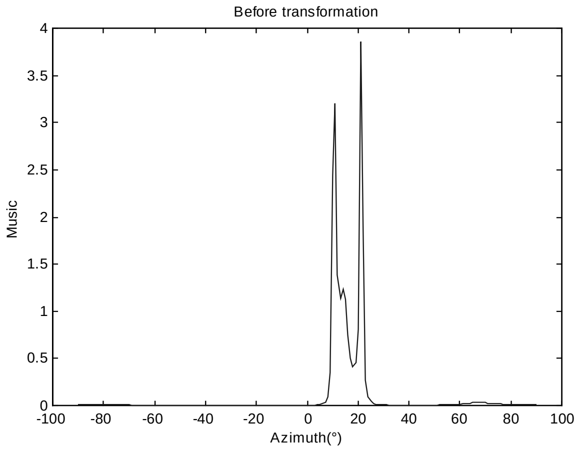

Figure1 : Cumulant before transformation

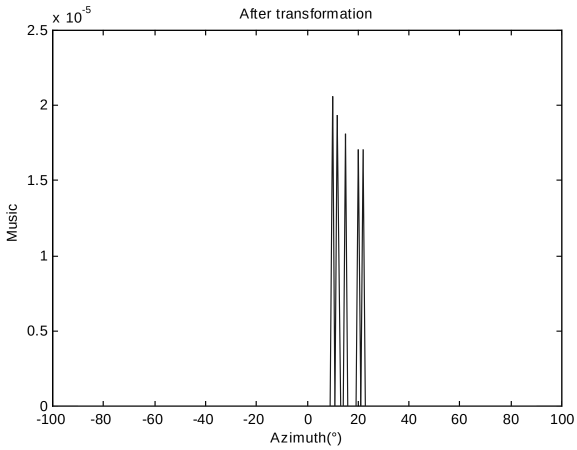

Figure 2 : Cumulant of transformed data

5. CONCLUSION

In this study, we have developed an algorithm for locating the wide-band sources in the presence of noises. The situation when unknown noise fields added to the Gaussian noise corrupt the received signals is considered. This method exploits the fourth-order cumulant properties (to eliminate the gaussian noise) and an approximation noise matrix projection (to eliminate the unknown noise). The obtained results show that the proposed algorithm is effective. It is insensitive to the noise contribution compared to the existing algorithms. This is due to the fact that our algorithm does not require the number of sources and the focusing operators are obtained from the cleaned cumulant matrices. These performances are confirmed by several studied examples (statistical study).

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