MULTIUSER DETECTION IN MULTIPATH NON-GAUSSIAN CHANNELS *

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ABSTRACT

This paper addresses the problem of multiuser detection in multipath fading code-division multiple-access (CDMA) channels with non-Gaussian noise. A robust multipath decorrelator is proposed to mitigate the multiple-access interference and the multipath interference, and to ameliorate non-Gaussian ambient noise. The performance of the proposed multiuser detector is assessed via computer simulations and compared with that of the linear multipath decorrelator and with that of the maximum-likelihood detector designed under the assumption of perfect knowledge of the channel fading coefficients.

1 INTRODUCTION

Several important multiple-access channels, including mobile radio channels, indoor wireless channels and underwater acoustic channels, exhibit severe multipath fading. In frequency-selective fading CDMA channels receiver performance in limited by additional interference resulting from multipath propagation and fading induced near-far problem. Recent work on multiuser detection has been focused on the development of advanced detection techniques to mitigate the channel impairments due to multipath fading. In [8] the optimum multiuser detector for CDMA frequency-selective Rayleigh fading channels has been synthesized and it has been shown that it rivals the optimum detector for isolated transmission. Motivated by high complexity of the optimum multiuser detector, in [9] a multiuser receiver for frequency-selective channels, which performs decorrelation before multipath combining, the linear multipath decorrelator, is developed. Specifically, a coherent multiuser detector, for binary phase-shift keying signals, which requires perfect channel estimation available at the receiver and a multipath decorrelator with equal-gain combining for differential coherent detection of differential phase-shift keying (DPSK) signals, are proposed and analyzed.

All the aforementioned work considers multiuser detection in Gaussian channels. However, in many physical channels in which multiple-acces communications is applied, the ambient noise in known through experimental results to be decidedly non-Gaussian due to the impulsive nature of man-made electromagnetic interference and a great deal of natural noise as well [2]. The optimum (in the maximum-likelihood sense) multiuser detector for data detection in additive impulsive noise has been derived in [3] and its performance has been analyzed in [4]. It has been shown that the performance gains afforded by maximum likelihood multiuser detection in impulsive noise can be substantial when compared to multiuser detection strategies based on a Gaussian noise assumption. Since the maximum-likelihood strategy is computationally intensive, in [6] a lower complexity (with respect to the optimum multiuser detector) M-estimator-based multiuser detector has been proposed and analyzed. Specifically, in [6] is shown that the proposed multiuser detector offers significant performance gain over the linear decorrelating detector when the ambient channel noise is non-Gaussian, with little attendant increase in the algorithmic complexity. Moreover, in [5] a low complexity robust multiuser receiver for data detection in flat-fading CDMA non-Gaussian channels has been proposed.

In the present paper, the problem of multiuser detection in multipath fading impulsive channels is addressed. To study a CDMA system where the modulation technique employed for each transmission allows noncoherent demodulation, attention is focused on CDMA systems where the modulation technique employed for each transmission is DPSK. A robust multipath decorrelator with differential detection is proposed to mitigate the multiple-access interference and the multipath interference, and to ameliorate non-Gaussian ambient noise. The performance of the proposed multiuser detector is analyzed via computer simulations and compared with that of the linear multipath decorrelator with differential detection [9] and with that of the maximum-likelihood detector designed under the assumption of perfect knowledge of the channel fading coefficients.

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2 SYSTEM MODEL

In a CDMA system, the waveform received by a given terminal can be modeled as consisting of a set of superimposed modulated data signals observed in additive noise

$$r(t) = S(t) + w(t) \tag{1}$$

where $S(t)$ and $w(t)$ represent the useful signal and the ambient channel noise, respectively. The CDMA system considered in this paper is a synchronous binary communication system with K users who simultaneously transmit through a multipath fading channel. It is assumed that the signaling interval T is smaller than the channel coherence time, such that the fading process can be assumed to be constant for the duration of at least one signaling interval. Under this slow fading condition, the useful signal can be modeled as

$$S(t) = \Re\left\{\sum_{k=1}^{K}\sum_{i=-M}^{M} A_k b_k(i) \sum_{l=0}^{L-1} \boldsymbol{c}_{l,k}(i) s_k(t - iT - \frac{l}{B})\right\} \tag{2}$$

where $\Re\{\}$ denotes real part; A_k is the amplitude of the $k-$th user; $2M+1$ is the number of symbols per user in the data frame of interest; L is the number of resolvable paths for each user; B is the bandwith of the spread-spectrum signal; $\{b_k(i) : i = -M, ..., M\}$ denote the symbol stream for the $k - th$ user; $\boldsymbol{c}_{l,k}(i)$ is the complex channel gain process for the $l-$th path of the $k-$th user's signal. The KL channel fading processes $\boldsymbol{c}_{l,k}(i), 1 \le k \le K, 1 \le l \le L$ are modeled as zero-mean complex Gaussian processes and are assumed to be statistically independent of each other.

For the data signaling interval much longer than the multipath delay spread T_m, i.e., $T \gg T_m \simeq \frac{L}{B}$, any intersimbol interference due to the channel dispersion can be neglected. Therefore, to detect the $i-$th symbol of the K users, the received signal in the $i-$th signaling interval can be utilized, i.e.,

$$r(t) = \Re\left\{\sum_{k=1}^{K} A_k b_k(i) \sum_{l=0}^{L-1} \boldsymbol{c}_{l,k}(i) s_k(t - iT - \frac{l}{B})\right\} + w(t). \tag{3}$$

The signaling constellation (i.e., $s_1, s_2, \dots, s_K$) consists of a set of non-orthogonal signals

$$s_k(t) = \begin{cases} \sqrt{\frac{2}{T}} a_k(t) e^{j[\omega_c t + \phi_k]} & , \quad t \in [0, T] \\ 0 & , \quad t \notin [0, T] \end{cases} \tag{4}$$

where ω_c is a common carrier frequency, ϕ_k is the phase of the $k-$th user relative to some reference, and the spreading waveforms $a_k(t)$ are of the form:

$$a_k(t) = \sum_{n=0}^{N-1} a_n^k \, p_{T_c}(t - nT_c). \tag{5}$$

Here, $\{a_0^k, a_1^k, \dots, a_{N-1}^k\}$ is a signature sequence of $+1$'s and -1's assigned to the $k-$th user, and p_{T_c} is a unit-amplitude pulse of duration T_c (where $NT_c = T$).

At the receiver, a sequence is derived from the received waveform $r(t)$ by (complex) basebanding, chip matched filtering, and chip rate sampling; that is

$$\boldsymbol{r}_n(i) = \sqrt{\frac{2}{T_c}} \int_{iT+nT_c}^{iT+(n+1)T_c} r(t) e^{-j\omega_c t} dt, \tag{6}$$

where j denotes the imaginary unit: $j = \sqrt{-1}$.

Accounting for (3)-(6), the resulting discrete-time signal sample corresponding to the $n-$th chip of the $i-$th symbol is given by

$$\boldsymbol{r}_n(i) = \boldsymbol{s}_n(i) + \boldsymbol{w}_n(i) \quad n \in [0, N-1] \tag{7}$$

where

$$\boldsymbol{s}_n(i) \triangleq \frac{1}{\sqrt{N}} \sum_{k=1}^{K} \sum_{l=0}^{L-1} \boldsymbol{\theta}_{l,k}(i) a_{n-l}^k \tag{8}$$

with $\boldsymbol{\theta}_{l,k}(i) \triangleq A_k b_k(i) \boldsymbol{c}_{l,k}(i)$ and $\boldsymbol{w}_n(i)$ is the channel ambient noise. It is assumed that the sequence of noise samples $\{\boldsymbol{w}_n(i)\}$ is a sequence of independent and identically distributed complex random variables whose inphase and quadrature components are statistically independent non-Gaussian random variables with a common probability density function f. Specifically, in this paper the commonly used and highly tractable $\epsilon-$mixture model has been adopted, that is

$$f = (1 - \epsilon)\mathcal{N}(0, \sigma_n^2) + \epsilon\mathcal{N}(0, \gamma^2\sigma_n^2) \tag{9}$$

where $\mathcal{N}(0, \sigma_n^2)$ represents the nominal background noise, (Gaussian noise with zero mean and variance σ_n^2), and the $\mathcal{N}(0, \gamma^2\sigma_n^2)$ term is the impulsive component, with ϵ representing the probability that impulses occur.

3 DETECTION STRUCTURES

To introduce the robust multipath decorrelator let us observe that the sequence derived at the receiver in the $i-$th symbol interval can be written in matrix notation as

$$\underline{r}(i) = \underline{H}\ \underline{\theta}(i) + \underline{w}(i), \tag{10}$$

where

$$\underline{r}(i) \triangleq [\Re\{\boldsymbol{r}_0(i)\}, .., \Re\{\boldsymbol{r}_{N-1}(i)\}, \Im\{\boldsymbol{r}_0(i)\}, .., \Im\{\boldsymbol{r}_{N-1}(i)\}]^T, \tag{11}$$

$$\underline{w}(i) \triangleq [\Re\{\boldsymbol{w}_0(i)\}, .., \Re\{\boldsymbol{w}_{N-1}(i)\}, \Im\{\boldsymbol{w}_0(i)\}, .., \Im\{\boldsymbol{w}_{N-1}(i)\}]^T, \tag{12}$$

and

$$\underline{\theta}(i) \triangleq [\Re\{\boldsymbol{\theta}_{0,1}(i)\}, .., \Re\{\boldsymbol{\theta}_{L-1,1}(i)\}, .., \Re\{\boldsymbol{\theta}_{0,K}(i)\}, .., \Re\{\boldsymbol{\theta}_{L-1,K}(i)\}, \Im\{\boldsymbol{\theta}_{0,1}(i)\}, .., \Im\{\boldsymbol{\theta}_{L-1,1}(i)\}, ..,$$

$$\Im\{\boldsymbol{\theta}_{0,K}(i)\},..,\Im\{\boldsymbol{\theta}_{L-1,K}(i)\}]^T, \quad (13)$$

where $\Im\{\}$ denotes imaginary part. Moreover, in equation (10)

$$\underline{H} \triangleq \begin{bmatrix} \underline{S} & \underline{0} \\ \underline{0} & \underline{S} \end{bmatrix}, \quad (14)$$

where

$$\underline{S} \triangleq [\underline{S}^1, \underline{S}^2, ..., \underline{S}^K] \quad (15)$$

with

$$\underline{S}^k \triangleq [\underline{s}_0^k, \underline{s}_1^k, ..., \underline{s}_{L-1}^k], \quad (16)$$

$$\underline{s}_l^k \triangleq \frac{1}{\sqrt{N}}[a_{0-l}^k, a_{1-l}^k, ..., a_{N-1-l}^k]^T \quad (17)$$

and $\underline{0}$ the $N \times KL$ matrix of all zeros.

The idea underlying robust multiuser detection is to detect the symbols in (10) by first estimating the vector $\underline{\theta}(i)$, and then extracting symbols estimates from these continuous estimates. Since the channel fading introduces, for each user and for each resolvable path, a π-radian phase ambiguity, differential encoding is considered. In this case by measuring each user's phase for each resolvable path in two successive symbol intervals, the transmitted bits can be recovered, provided that the fading rate is sufficiently slow.

The required estimates of $\underline{\theta}(i)$ are obtained by using an estimator of the class of the M-estimators proposed by Huber [1]. These estimators minimize a function $\rho(\cdot)$ (called the *penalty function*) of the residuals:

$$\hat{\underline{\theta}}(i) = \arg \min_{\underline{\theta}(i) \in R^{2KL}} \sum_{p=1}^{2N} \rho \left(r_p(i) - \sum_{l=1}^{2KL} h_{pl}\theta_l(i) \right), \quad (18)$$

where $r_p(i)$ and $\theta_l(i)$ are the $p-th$ and the $l-$th element of the vectors $\underline{r}(i)$ and $\underline{\theta}(i)$, respectively, and h_{pl} is the $pl-$th element of the matrix $\underline{H}$. Given such an estimator, the detected symbols are given by

$$\hat{b}_k(i) = \text{sgn} \left\{ \Re \left[\sum_{l=0}^{L-1} \hat{\boldsymbol{\theta}}_{l,k}(i) \hat{\boldsymbol{\theta}}_{l,k}^*(i-1) \right] \right\} \quad (19)$$

where

$$\hat{\boldsymbol{\theta}}_{l,k}(i) = \hat{\theta}_{l+(k-1)L}(i) + j\hat{\theta}_{l+(k-1)L+LK}(i). \quad (20)$$

and the superscript $*$ denotes complex conjugation.

If $\rho(\cdot)$ has a derivative $\psi(\cdot) = \rho'(\cdot)$, then the solution to (18) satisfies the implicit equation system

$$\sum_{p=1}^{2N} \psi \left(r_p(i) - \sum_{l=1}^{2KL} h_{pl}\theta_l(i) \right) h_{pm} = 0, \quad m = 1, 2, ..., 2KL. \quad (21)$$

Different choices of the penalty function $\rho(\cdot)$ and of the corresponding derivative $\psi(\cdot)$ lead to different M-estimators and then to different multiuser detectors. If $\rho(x) = x^2/2\beta$, where β is any positive constant, one has the well known least-squares estimator and the corresponding detector is the linear multipath decorrelator with equal-gain combining proposed for differential coherent multiuser detection in Gaussian channels [9].

The robust multiuser detector proposed in this paper is based on the minimax M-estimator proposed by Huber [1]. For the noise pdf given in (9) the penalty function and its derivative of the Huber minimax M-estimator are given, respectively, by

$$\rho_H(x) = \begin{cases} \dfrac{x^2}{2\sigma_n^2} & , \quad \textit{for} \mid x \mid \leq \mathrm{k}\sigma_n^2 \\ -\dfrac{\mathrm{k}^2\sigma_n^2}{2} + \mathrm{k} \mid x \mid & , \quad \textit{for} \mid x \mid > \mathrm{k}\sigma_n^2 \end{cases} \quad (22)$$

$$\psi_H(x) = \begin{cases} \dfrac{x}{\sigma_n^2}, & , \quad \textit{for} \mid x \mid \leq \mathrm{k}\sigma_n^2 \\ \mathrm{k}\,\mathrm{sgn}(x) & , \quad \textit{for} \mid x \mid > \mathrm{k}\sigma_n^2 \end{cases} \quad (23)$$

with the parameter k connected to ϵ and σ_n^2 through

$$\frac{\phi(\mathrm{k}\sigma_n^2)}{\mathrm{k}\sigma_n^2} - Q(\mathrm{k}\sigma_n^2) = \frac{\epsilon}{2(1-\epsilon)} \quad (24)$$

where $\phi(x) \triangleq (\frac{1}{\sqrt{2\pi}})e^{-(x^2/2)}$, and $Q(x) \triangleq (\frac{1}{\sqrt{2\pi}}) \int_x^\infty e^{-(t^2/2)}dt$. This penalty function has the property that it incorporates small residuals similarly to least-squares, while reducing the influence of larger residuals (which could be the result of impulses in the ambient noise).

For performance comparison purposes in the next section will be also considered the single-shot maximum-likelihood multiuser detector having a perfect knowledge of the channel coefficients. Such a detector, in the considered noise environment (see (9)), maximizes over the bit vector the log-likelihood function

$$\ell^{ml-ng} = \sum_{n=0}^{N-1} \Re\{\boldsymbol{r}_n(i)\boldsymbol{s}_n^*(i)\} - \frac{1}{2}\sum_{n=0}^{N-1} \mid \boldsymbol{s}_n(i) \mid^2$$

$$+\gamma^2\sigma_n^2 \sum_{n=0}^{N-1} \log \frac{1 + \dfrac{1-\epsilon}{\epsilon}\,\gamma\,\exp\left[-\dfrac{(\Re\{\boldsymbol{r}_n(i) - \boldsymbol{s}_n(i)\})^2}{2\sigma_n^2}\Gamma\right]}{1 + \dfrac{1-\epsilon}{\epsilon}\,\gamma\,\exp\left[-\dfrac{(\Re\{\boldsymbol{r}_n(i)\})^2}{2\sigma_n^2}\Gamma\right]}$$

$$+\gamma^2\sigma_n^2 \sum_{n=0}^{N-1} \log \frac{1 + \dfrac{1-\epsilon}{\epsilon}\,\gamma\,\exp\left[-\dfrac{(\Im\{\boldsymbol{r}_n(i) - \boldsymbol{s}_n(i)\})^2}{2\sigma_n^2}\Gamma\right]}{1 + \dfrac{1-\epsilon}{\epsilon}\,\gamma\,\exp\left[-\dfrac{(\Im\{\boldsymbol{r}_n(i)\})^2}{2\sigma_n^2}\Gamma\right]}, \quad (25)$$

where

$$\Gamma = \left(1 - \frac{1}{\gamma^2}\right). \quad (26)$$

Maximum-likelihood detection generally requires exhaustation over 2^K choices.

4 NUMERICAL RESULTS

In this section the performance of the previously considered detectors is assessed via Monte Carlo computer simulations. In the simulations, the channel fading process for each user is modeled as lightly damped second order Auto-Regressive (AR) processes:

$$\boldsymbol{c}_{lk}(i) = -a_1\boldsymbol{c}_{lk}(i-1) - a_2\boldsymbol{c}_{lk}(i-2) + \boldsymbol{n}_{lk}(i) \qquad (27)$$

where $\boldsymbol{c}_{lk}(i)$ is the complex channel fading coefficient for the $l-$th path of the $k-$th user in the $i-th$ symbol interval, each driving noise process $\boldsymbol{n}_{lk}(i)$ is a complex zero-mean white Gaussian process and the AR parameters a_1 and a_2 are related to the physical parameters of the fading channel [7]. The mean-square value of the paths for each user are assumed to decay with the exponential law considered in [9]. Moreover, in the simulations the variance $\sigma^2 = (1-\epsilon)\sigma_n^2 + \epsilon\gamma^2\sigma_n^2$ of each of the in-phase and quadrature noise components has been held constant and equal to 1, whereas the power of each user has been varied to achieve the desired values of the signal-to-noise ratio defined as

$$SNR_k \triangleq \frac{A_k^2}{\sigma^2}\sum_{l=0}^{L-1} E\left\{\mid \boldsymbol{c}_{lk}(i)\mid^2\right\}. \qquad (28)$$

The figure shows the performance of the considered detectors as a function of the signal-to-noise ratio of the first user, in noise with a relatively high fraction of impulses ($\epsilon = 0.1, \gamma^2 = 100$) and for the case where the total received power from each user is one and the same. A synchronous six-user CDMA system with $B = 1.25$ MHz bandwith and 9600 bps data rate is considered. Three values of the multipath spread are considered: $T_m = 0.5\mu s$ (suburban environment), $T_m = 1.2\mu s$ and $T_m = 3\mu s$ (urban environment). Thus, accounting for the system bandwith, the number of resolvable fading paths is $L = 1$, $L = 2$ and $L = 4$, respectively. The spreading sequence of each user is a shifted version of an m-sequence of length $N = 127$ and the normalized fading rate is fixed at $f_dT = 0.008$. Note that for the given system parameters intersymbol interference is negligible. The results show that the proposed robust multipath decorrelator with equal-gain combining (labeled as RMD-EGC) clearly outperforms the linear multipath decorrelator with equal-gain combining (labeled as LMD-EGC). Specifically, in a wide range of values of SNR, the performance gain of the proposed RMD-EGC with respect to the LMD-EGC is almost independent on the order of diversity. Moreover, the performance loss of the proposed RMD-EGC with respect to the much more complex maximum-likelihood multiuser detector (labeled as ML) is contained and decreases as the number of paths increases.

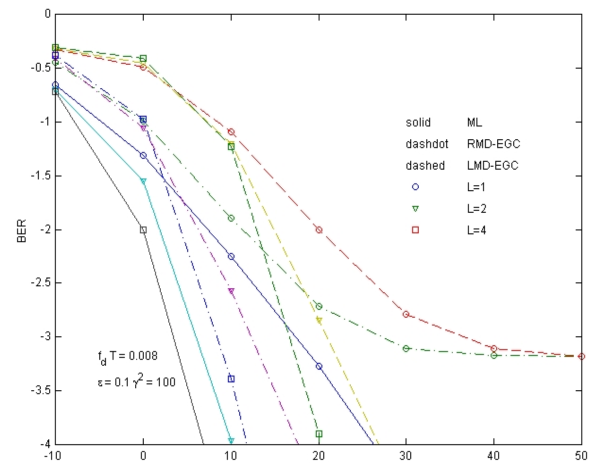

Fig.1

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