

STABLE NONLINEAR FILTERS WITH SPATIAL PREDICTION

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ABSTRACT

The paper deals with nonlinear two-dimensional filters for image restoration. The filter employs classic predictor structure with nonlinear predictor and prediction error processed by a static (memoryless) nonlinear element or by a nonlinear two-dimensional filter. The filters discussed are both recursive and nonrecursive. Even in the case of recursive structures stability is guaranteed. The filters are suitable for impulsive noise removal from color images. Preservation of textures and even one-pixel thin lines are advantages of the filters proposed. The experimental data prove that these filters outperforms classic nonlinear median-based filters like vector median, recursive median and weighted median.

1 INTRODUCTION

Many types of nonlinear filters [1] have been proposed and examined during last two decades. Most of them were nonrecursive, i.e. without feedback loops. Such filters are inherently stable and easy to design but recursive structures are more efficient from the point of view of numbers of arithmetic operations needed in their implementations. Therefore, nonlinear recursive filter design is recently also a matter of interest [2-4]. Main obstacles in development of these filters are the lack of general design techniques as well as the stability testing problem.

Later, application of the system structure commonly used in predictive coding of signals (DPCM) has been proposed [5-7]. Such a structure is well known because it is being investigated in the context of image compression since at least 30 years [8].

Concept of passive digital systems showed to be useful to solve the stability problems. In particular, the theory of l_1 -passivity is very promising [9,12].

This paper deals with two-dimensional nonlinear digital filters which include nonlinear spatial predictors and nonlinear a network for nonlinear processing of the prediction error. The filters are designed mostly for color image processing. The goal of the paper is to examine how

an additional prediction path improves the performance of median filters of different kind.

Some kinds of filters with a prediction path are considered in this paper. Among them are vector filters that calculate the color distances in the RGB and $L^*a^*b^*$ spaces.

The paper deals also with a new original structures which includes a filter that proceeds the prediction error signal. This structure has been introduced with the aim to reduce degradation of thin lines caused by classic median filters.

2 THE NONLINEAR FILTERS WITH PREDICTION

The structure considered in the paper (Fig.1) includes a predictor being a nonlinear sorting filter. In the experiments, various types of median filter were used:

- median filter (MF),
- recursive median filter (RMF),
- vector median filter (VMF),
- recursive vector median filter (RVMF),
- centered weighted median filter (RCWMF),
- recursive centered weighted median filter (RCWMF).

Median filters has been recognized as useful restoration tools because of their edge preservation characteristics and their simple implementations. However, they still tend to remove fine details, such as thin lines and corners. Therefore median filters of various types are used here only as predictors. Prediction error d is calculated as a difference between the median filter output p and the input signal u to the whole structure. The output is a sum of the prediction and the prediction error proceeded by a special nonlinear memoryless element. Large values of prediction errors are zeroed because they are classified as caused by impulsive noise samples. Small prediction error values pass the nonlinear element unchanged as they are treated as necessary correction which ought to be added to the prediction p in order to obtain the actual output sample. Soft switching is implemented by multiplication of the

prediction error d by a factor k defined as an even function of $f_k(d)$ (see Fig. 2).

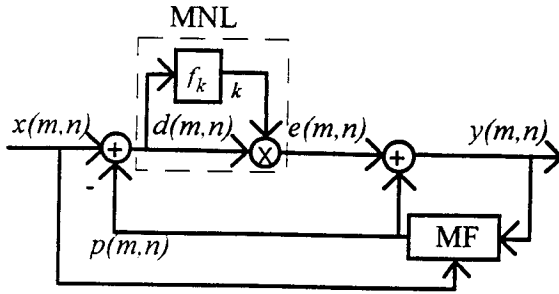


Figure 1: Filter structure: MNL - memoryless nonlinearity, MF - median-based nonlinear filter (recursive or nonrecursive).

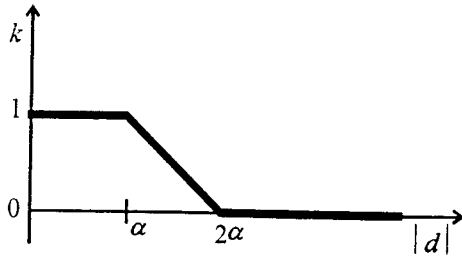


Figure 2: Function $f_k(d)$.

3 COLOR IMAGE PROCESSING

In the simplest case, the filter MF is not a vector filter and the image is processed in the RGB color space or in a space related to the RGB coordinates by a linear transformation. In such a case all three image components are processed independently.

Some modification of this simple scheme is related to application of a vector filter [10] as the filter MF.

In vector filter implemented in the $L^*a^*b^*$ color space, the prediction error is not fed directly into the memoryless nonlinear element but color difference between input and prediction is calculated in the $L^*a^*b^*$ space. Only this difference value is fed into the memoryless element.

4 MODIFIED FILTER STRUCTURE

The above described filters process textures and small details much better than median filters of different kind. Nevertheless filters described above degrade thin lines exhibiting high contrast to background.

In order to avoid degradation of thin lines a modification of the filter structure is suggested (Fig.3). It consists in processing of the prediction error by a nonlinear filter rather than by memoryless nonlinear element.

At first, parameter s is calculated.

Step 1:

- if $d(m,n)$ and $d(m-1,n-1) > 2\alpha$ then $r = 1$;
- if $d(m,n)$ and $d(m, n-1) > 2\alpha$ then $r = 1$;
- if $d(m,n)$ and $d(m+1, n-1) > 2\alpha$ then $r = 1$;

- if $d(m,n)$ and $d(m-1,n) > 2\alpha$ then $r = 1$;
- if $d(m,n)$ and $d(m-1,n-1) < -2\alpha$ then $r = -1$;
- if $d(m,n)$ and $d(m, n-1) < -2\alpha$ then $r = -1$;
- if $d(m,n)$ and $d(m+1, n-1) < -2\alpha$ then $r = -1$;
- if $d(m,n)$ and $d(m-1,n) < -2\alpha$ then $r = -1$;
- else $r = 0$

Step 2:

- if $r(m,n) = 1$ and $r(m-1, n-1) = 1$ then $s = 1$;
- if $r(m,n) = 1$ and $r(m, n-1) = 1$ then $s = 1$;
- if $r(m,n) = 1$ and $r(m+1, n-1) = 1$ then $s = 1$;
- if $r(m,n) = 1$ and $r(m-1, n) = 1$ then $s = 1$;
- if $r(m,n) = -1$ and $r(m-1, n-1) = -1$ then $s = 1$;
- if $r(m,n) = -1$ and $r(m, n-1) = -1$ then $s = 1$;
- if $r(m,n) = -1$ and $r(m+1, n-1) = -1$ then $s = 1$;
- if $r(m,n) = -1$ and $r(m-1, n) = -1$ then $s = 1$;
- else $s = 0$

The filter structure is augmented as shown in Fig. 3. Prediction error d is multiplied by the sum $(k+s)$.

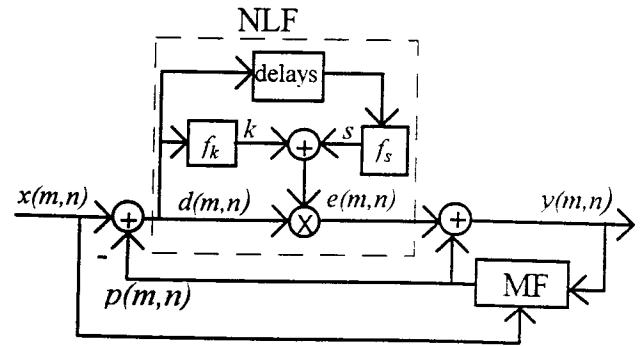


Figure 3: The modified filter structure. f_s denotes calculations of s .

5 STABILITY

The MF filter calculates its output p as one of the input values, i.e. samples of a processed image x or, in the case of recursive filters, also samples of previously estimated output samples y .

The prediction error d is multiplied by a factor $(k+s)$, where $0 \leq (k+s) \leq 1$. Therefore even in a recursive structure, the absolute value of signal does not grow over maximum input value. Therefore the filter is always stable [9,12].

6 EXPERIMENTAL RESULTS

The test images selected are *lena*, *boats*, *clown*, and *penguin*. The images corrupted with impulsive noise were processed by filters with and without the prediction loop. Two types of noise statistics were examined:

- Type A - random disturbance of R, G, B components independently with impulsive noise of amplitude (0-255). The distribution of impulse values as well as of

pixel choice are uniform. The probability that a given pixel in a given component will be corrupted is p .

- Type B - random simultaneous change of all the R, G, B components of a pixel by applying additive impulsive noise with its amplitude being a half of the amplitude of a respective component. The probability that a given pixel will be corrupted is p .

For a comparison, PSNR is calculated for all the test images in two color spaces, RGB and $L^*a^*b^*$.

The PSNR calculated in the $L^*a^*b^*$ color space is more reliable as the color difference calculated in this space is a better measure of the color distance as perceived by a human being.

The performance of the proposed filters with prediction (PF) is superior to that of the median filters also in terms of subjective quality, as shown in Fig. 4, where the fine texture is preserved only in the output of the filter with a feedback loop. In order to prove good efficiency of the proposed filters, the objective quality calculated as the peak signal to noise ratio (PSNR) has been used. The increase of the PSNR [dB] when the prediction is applied is shown in Table 2. The improvement is highest for recursive vector median filters (RVMF).

Figure 5 illustrates preservation of thin lines implied by application of the modified structure. This advantageous property is obtained with small decrease of efficiency of rejection of impulsive noise (Table 3).

Table 1. The results for noise with $p=5\%$ (PSNR calculated in $L^*a^*b^*$ space)..

PSNR in[dB]	Type A noise				Type B noise			
Test images	Boats	Lena	Clown	Penguin	Boats	Lena	Clown	Penguin
Corrupted image	17.70	16.20	16.40	15.69	18.51	32.45	24.05	21.73
Output image								
MF	25.81	35.64	28.21	27.35	26.07	36.28	28.68	29.11
M-PF	29.10	31.38	30.26	31.54	29.24	36.69	32.91	31.61
RMF	25.76	35.34	27.57	27.13	26.06	35.76	27.84	28.13
RM-PF	28.19	37.83	32.06	28.35	29.15	36.68	32.66	31.06
CWMF	27.31	36.65	29.60	28.24	27.86	38.56	30.96	31.33
CWM-PF	28.42	35.98	30.45	28.15	30.03	36.37	32.87	32.95
RCWMF	27.07	36.77	29.33	28.26	27.63	38.00	30.51	30.84
RCWM-PF	28.36	36.45	30.55	28.31	30.11	36.35	32.86	33.03
VMF	25.61	35.45	27.89	29.06	25.78	35.97	28.37	29.37
VM-PF	28.55	37.69	32.07	29.92	29.33	36.68	33.07	32.01
RVMF	25.45	34.76	26.83	28.00	25.65	35.15	27.26	28.22
RVM-PF	28.43	37.53	32.05	29.60	29.26	36.67	32.79	31.38

Table 2. An improvement caused by application of the prediction (for noise $p=5\%$). PSNR is calculated both in RGB and $L^*a^*b^*$ color spaces. Testimage used is *lena*.

Improvement in PSNR [dB]				
	calculated in the RGB color space		calculated in the $L^*a^*b^*$ color space	
	Type A noise	Type B noise	Type A noise	Type B noise
MF	3.63	2.38	2.58	2.36
RMF	3.60	3.14	2.94	2.51
CWMF	1.70	0.75	0.99	0.30
RCWMF	2.28	1.18	1.34	0.56
VMF	3.86	2.50	2.63	2.02
RVMF	4.74	3.44	3.46	2.53



Fig.4. Rejection of noise (type A) with $p=5\%$. From left: A corrupted image, an output of a recursive median filter, an output of the proposed filter without modification.

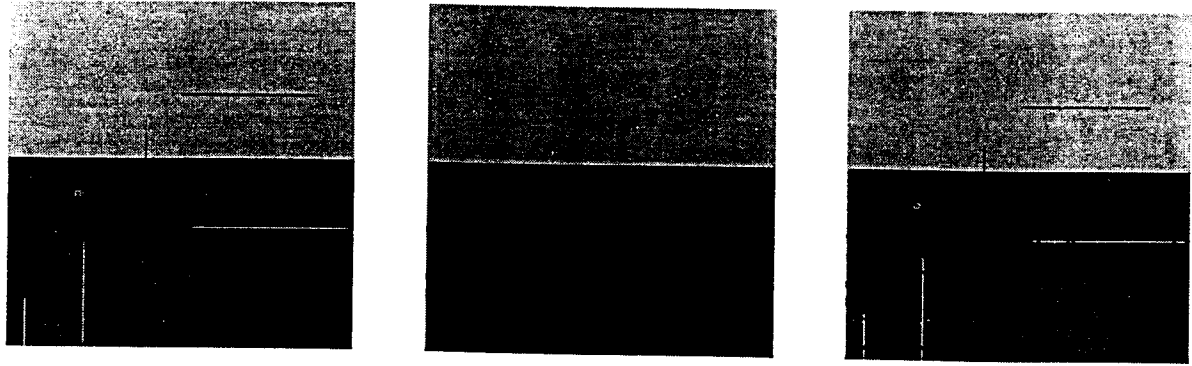


Fig.5. Rejection of noise (type A) with $p=5\%$. Upper images from left to right: the original. output of the filter from Fig.1 with a recursive median filter and output of the filter from Fig. 3, also with a recursive median filter.

Table 3. The results for type A ($p=5\%$) noise rejection in filters from Fig.3 and 1, i.e. with and without calculation of the factor s . PSNR is calculated in the RGB color space.

Test Images	PSNR in [dB]	
	Boats	Clown
Corrupted Image	21.00	21.61
Filtered Image		
MF with k+s factor	29.65	32.75
MF with k factor	31.75	33.69
RM with k+s factor	29.46	32.63
RM with k factor	30.20	35.04
VMF with k+s factor	29.35	32.57
VMF with k factor	30.36	35.15
RVMF with k+s factor	29.21	32.40
RVMF with k factor	29.90	34.70

The results depend on parameter α . The above results (Tables 1-3, Figs. 4 and 5) have been obtained for $\alpha = 25$.

The optimum value of α depends on a test image. Nevertheless $\alpha = 25$ is usually not far from the optimum (Fig. 6).

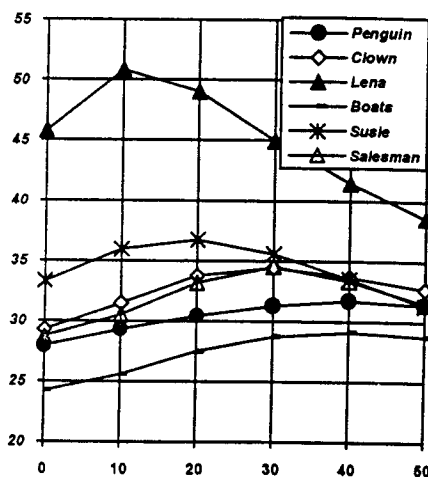


Fig.6. PSNR at the output a filter from Fig.1 with a recursive median filter. PSNR calculated in the RGB color space. in PSNR with variable threshold value.

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