Multiplicative Interpretation of Neutrosophic Cubic Set on B-Algebra

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Abstract

Purpose of this paper is to interpret the multiplication of neutrosophic cubic set. Here we define the notation of ɤ-multiplication of neutrosophic cubic set and study it with the help of neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal and neutrosophic cubic closed normal ideal. We also study ɤ-multiplication under homomorphism and cartesian product through significant characteristics.

Keywords: B-algebra, Neutrosophic cubic set, ɤ-Multiplication, Cartesian product, Homomorphism.

1.Introduction

Theory of existing and non-existing value was first introduced by Zadeh [1,2]. Cubic set was defined by Jun et al. [3] in 2012, which was the modern form of interval-valued fuzzy set. Cubic set with the help of subalgebras, ideals and closed ideals of *B*-algebra was studied by Senapati et al. [4]. After the defing of *BCK*-algebra and *BCI*-algebra by Imai et al. [5] and Iseki [6], cubic set through subalgebras and q-ideals in BCK/BCI-algebra was investigated by Jun et al. [7, 8]. Notion of M-subalgebra on G-algebra is introduced and analyzed by Khalid et al. [9]. Interval-valued fuzzy set on *B*-algebra was studied by Senapati et al. [10,11]. Intuitionistic fuzzy translation and multiplication of G-algebra were deeply studied by by Khalid et al. [19]. Neutrosophic cubic set is the extended form of interval valued intuitionistic fuzzy theory with indeterminacy was introduced by Smarandache [12]. Neutrosophic logics and neutrosophic probability gave the new idea of research were interpret by Smarandache [13]. Neutrosophic cubic was introduced by Jun et al. [14]. Neutrosophic cubic point, (α,β)-fuzzy ideals and neutrosophic cubic (α,β)-ideals were analyzed by Gulistan et al. [15]. A new idea of normal ideal and closed normal ideal under neutrosophic cubic set was given and investigated by Khalid et al. [16]. Neutrosophic cubic set was investigated by Jun et al. [17]. PS fuzzy ideals were studied by Priya et al. [18]. Rosenfeld's fuzzy subgroup was studied by Biswas [20]. B-homomorphism was deeply studied by Neggers et al. [21]. Neutrosophic soft cubic subalgebra was extensively studied by Khalid et al. [22]. A B-algebra is an important logical class of algebra was defined by Neggers et al. [23]. T-Neutrosophic Cubic Set was defined and deeply investigated by Khalid et al. [24].

In this paper, we define ɤ-multiplication of neutrosophic cubic set and investigate the neutrosophic cubic M-subalgebra, neutrosophic cubic normal ideal (NCNID) and neutrosophic cubic closed normal ideal (NCCNID) under ɤ-multiplication with the help of P-intersection, P-union etc. We also study the cartesian product and homomorphism of ɤ-multiplication of neutrosophic cubic normal ideal (ɤMNCNID) and ɤ-multiplication of neutrosophic cubic closed normal ideal (ɤMNCCNID) with important results.

2. Preliminaries

Definition 2.1 [19] A nonempty set X with a constant 0 and $*$ is said to be B-algebra if it fulfills these conditions:

1: $ţ * ţ = 0$,

2: $ţ * 0 = 0$, for all $ţ \in X$.

3: $(ţ * ŧ) * t = ţ * (t * (0 * ŧ))\ \forall\ ţ, ŧ, t \in X$.

Definition 2.2 [21] A nonempty subset K of B-algebra X is called a subalgebra of Y if $ţ * ŧ \in K\ \forall\ ţ, ŧ \in K$, a mapping $f : X \to Y$ of B-algebra is called B-homomorphism if $f(ţ * ŧ) = f(ţ) * f(ŧ)\ \forall\ ţ, ŧ \in X$.

Definition 2.3 [1] Let X be a collection of elements like ţ. Then a FS J in X is defined as $J = \{< ţ, \nu_J(ţ) > | ţ \in X\}$, where $\mu_J(ţ)$ is called the existenceship value of ţ in J and $\nu_J(ţ) \in [0,1]$.

For a family $J_i = \{< ţ, \nu_{J_i}(ţ) > | ţ \in X\}$ of FSs in X, where $i \in k$ and k is index set, Then join ($\vee$) and meet ($\wedge$) are as follows:

$$\bigvee_{i\in k} J_i = (\bigvee_{i\in k} \nu_{J_i})(ţ) = \sup\{\nu_{J_i} | i \in k\}$$

and

$$\bigwedge_{i\in k} J_i = (\bigwedge_{i\in k} \nu_{J_i})(ţ) = \inf\{\nu_{J_i} | i \in k\},$$

respectively, $\forall\ ţ \in X$.

Definition 2.4 [2] An IVFS B is of the form $B = \{< ţ, \tilde{\nu}_B(ţ) > | ţ \in X\}$, where $\tilde{\nu}_B | X \to D[0,1]$, here $D[0,1]$ is the collection of all subintervals of [0,1]. The intervals $\tilde{\nu}_B(ţ) = [\nu_B^-(ţ), \nu_B^+(ţ)]\ \forall\ ţ \in X$ denote the degree of existence of ţ to the set B, also $\tilde{\nu}_B^c = [1 - \nu_B^-(ţ), 1 - \nu_B^+(ţ)]$ shows the complement of $\tilde{\nu}_B$.

For a family $B_i = \{< ţ, \tilde{\nu}_B(ţ) > | ţ \in X\}$ of IVFSs in X where k is an index set and $i \in k$, the union $G = \bigcup_{i\in k} \tilde{\nu}_{B_i}(ţ)$ and the intersection $F = \bigcap_{i\in k} \tilde{\nu}_{B_i}(ţ)$ are defined below:

$$G(ţ) = \mathrm{rsup}\{\tilde{\nu}_{B_i}(ţ) i \in k\}$$

and

$$F(ţ) = \mathrm{rinf}\{\tilde{\nu}_{B_i}(ţ) | i \in k\},$$

respectively, $\forall\ ţ \in X$.

Definition 2.5 [20] Consider two elements $D_1, D_2 \in D[0,1]$. If $D_1 = [ţ_1^-, ţ_1^+]$ and $D_2 = [ţ_2^-, ţ_2^+]$, then $\mathrm{rmax}(D_1, D_2) = [\max(ţ_1^-, ţ_2^-), \max(ţ_1^+, ţ_2^+)]$ which is denoted by $D_1 \vee^r D_2$ and $\mathrm{rmin}(D_1, D_2) = [\min(ţ_1^-, ţ_2^-), \min(ţ_1^+, ţ_2^+)]$ which is denoted by $D_1 \wedge^r D_2$. Thus, if $D_i = [ţ_i^-, ţ_2^+] \in D[0,1]$ for $i = 1,2,3, \ldots,$ then we define $\mathrm{rsup}_i(D_i) = [\sup_i(ţ_i^-), \sup_i(ţ_i^+)]$, i.e., $\vee_i^r D_i = [\vee_i ţ_i^-, \vee_i ţ_i^+]$. Similarly we define $\mathrm{rinf}_i(D_i) = [\inf_i(ţ_i^-), \inf_i(ţ_i^+)]$, i.e., $\wedge_i^r D_i = [\wedge_i ţ_i^-, \wedge_i ţ_i^+]$. Now we call $D_1 \geq D_2 \Leftarrow ţ_1^- \geq ţ_2^-$ and $ţ_1^+ \geq ţ_2^+$. Similarly the relations $D_1 \leq D_2$ and $D_1 = D_2$ are defined.

Definition 2.6 [19] A fuzzy set $B = \{< ţ, \nu_B(ţ) > | ţ \in X\}$ is called a fuzzy subalgebra of X if $\nu_B(ţ * ƀ) \geq \min\{\nu_B(ţ), \nu_B(ƀ)\}\ \forall\ ţ, ƀ \in X$.

Definition 2.7 [14] Let X be a nonempty set. A NCS is $P_k = (B, \Lambda)$, where $B = \{\langle ţ; B_T(ţ), B_I(ţ), B_F(ţ)\rangle | ţ \in X\}$ is an interval neutrosophic set in X and $\Lambda = \{\langle ţ; \lambda_T(ţ), \lambda_I(ţ), \lambda_F(ţ)\rangle | ţ \in X\}$ is a neutrosophic set in X.

Definition 2.8 [3] Let U be a universe and cubic set in U, we mean a structure $\{ţ, \bar{\nu}_A(ţ), \lambda_A(ţ) |\ ţ \in U\}$ in which $\bar{\nu}_A$ is an IVF set in U and λ_A is a fuzzy set in U. A cubic set $A = \{ţ, \bar{\nu}_A(ţ), \lambda_A(ţ) |\ ţ \in U\}$ is simply denoted by C(U), which is the set of all cubic sets in U.

Definition 2.9 [3] Let $C = \{\langle ţ, C(ţ), \lambda(ţ)\rangle\}$ be a cubic set, where $C(ţ)$ is an IVFS in Y, $\lambda(ţ)$ is a fuzzy set in Y. Then A is cubic subalgebra under $*$ if it fulfills these axioms:

C1: $C(ţ * ƀ) \geq \mathrm{rmin}\{C(ţ), C(ƀ)\}$,

C2: $\lambda(ţ * ƀ) \leq \max\{\lambda(ţ), \lambda(ƀ)\}\ \forall\ ţ, ƀ \in X$.

Definition 2.10 [18] A fuzzy set $B = \{< ţ, \nu_B(ţ) > | ţ \in X\}$ is called a fuzzy ideal of X if

(i) $\nu_B(0) \geq \nu_B(ţ)$,

(ii) $\nu_B(ţ) \geq \min\{\nu_B(ţ * ƀ), \nu_B(ƀ)\}\ \ \forall\ ţ, ƀ \in X$.

Definition 2.11 [14] For any $C_i = (A_I, F_I)$, where $A_i = \{\langle t_1; A_{iT}(ţ), A_{iI}(ţ), A_{iF}(ţ)\rangle | ţ \in Y\}$, $F_i = \{\langle t_1; F_{iT}(ţ), F_{iI}(ţ), F_{iF}(ţ)\rangle | ţ \in Y\}$ for i ∈ k, then

P-union: $\bigcup_{\substack{P\\ i\in k}} C_i = (\bigcup_{i\in k} A_I, \bigvee_{i\in k} F_i)$,

P-intersection: $\bigcap_{\substack{P\\ i\in k}} C_i = (\bigcap_{i\in k} A_i, \bigwedge_{i\in k} F_i)$,

R-union: $\bigcup_{\substack{R\\ i\in k}} C_i = (\bigcup_{i\in k} A_I, \bigwedge_{i\in k} F_i)$,

R-intersection: $\bigcap_{\substack{R\\ i\in k}} C_i = (\bigcap_{i\in k} A_i, \bigvee_{i\in k} F_i)$.

Definition 2.12 [16] A NCS $Ʀ = (R_{T,I,F}, \lambda_{T,I,F})$ of X is called a NCNID of X if it fulfills following axioms:

N3. $R_{T,I,F}(0) \geq R_{T,I,F}(ţ * \alpha)$ and $\lambda_{T,I,F}(0) \leq \lambda_{T,I,F}(ţ * \alpha)$,

N4. $R_{T,I,F}(ţ * \alpha) \geq \mathrm{rmin}\{R_{T,I,F}((ţ * \alpha) * (ƀ * \beta)), R_{T,I,F}(ƀ * \beta)\}$,

N5. $\lambda_{T,I,F}(ţ * \alpha) \leq \max\{\lambda_{T,I,F}(ţ * \alpha) * (ƀ * \beta)), \lambda_{T,I,F}(ƀ * \beta)\}, \forall\ ţ, ƀ$ X and $\alpha, \beta \in [0,1]$.

Let $Ʀ = \{R_{T,I,F}, \lambda_{T,I,F}\}$ be a NCS X then it is called NCCNID of X if it fulfills N4, N5 and N6: $R_{T,I,F}(0 * (ţ * \alpha)) \geq R_{T,I,F}(ţ * \alpha)$ and $\lambda_{T,I,F}(0 * (ţ * \alpha)) \leq \lambda_{T,I,F}(ţ * \alpha)$, $\forall\ ţ \in X$ and $\alpha \in [0,1]$.

Definition 2.13 [16] Let $Ʀ = (R_{T,I,F}, \lambda_{T,I,F})$ and $\mathcal{B} = (B_{T,I,F}, \upsilon_{T,I,F})$ are two NCSs of X and Y respectively. The Cartesian product $Ʀ \times \mathcal{B} = (X \times Y, R_{T,I,F} \times B_{T,I,F}, \lambda_{T,I,F} \times \upsilon_{T,I,F})$ is defined by $(R_{T,I,F} \times B_{T,I,F})(ţ * \alpha, ƀ * \beta) = \mathrm{rmin}\{R_{T,I,F}(ţ * \alpha), B_{T,I,F}(ƀ * \beta)\}$ and $(\lambda_{T,I,F} \times \upsilon_{T,I,F})(ţ * \alpha, ƀ * \beta) = \max\{\lambda_{T,I,F}(ţ * \alpha), \upsilon_{T,I,F}(ƀ * \beta))\}$, where $R_{T,I,F} \times B_{T,I,F} \mid X \times Y \rightarrow D[0,1]$ and $\lambda_{T,I,F} \times \upsilon_{T,I,F} \mid X \times Y \rightarrow [0,1]\ \forall\ (ţ, ƀ) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

Definition 2.14 [16] A neutrosophic cubic subset $Ʀ \times Ғ = (X \times Y, R_{T,I,F} \times F_{T,I,F}, \lambda_{T,I,F} \times \mu_{T,I,F})$ is called a NCNID if satisfies these conditions:

1. $(R_{T,I,F} \times F_{T,I,F})(0,0) \geq (R_{T,I,F} \times F_{T,I,F})((ţ * \alpha), (ƀ * \beta))$ and $(\lambda_{T,I,F} \times \mu_{T,I,F})(0,0) \leq (\lambda_{T,I,F} \times \mu_{T,I,F})((ţ * \alpha), (ƀ * \beta))\ \forall\ (ţ, ƀ) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

2. $(R_{T,I,F} \times F_{T,I,F})(ţ_1 * \alpha, ƀ_1 * \beta) \geq \mathrm{rmin}\{(R_{T,I,F} \times F_{T,I,F})((ţ_1 * \alpha, ƀ_1 * \beta) * (ţ_2 * \alpha, ƀ_2 * \beta)), (R_{T,I,F} \times F_{T,I,F})(ţ_2 * \alpha, ƀ_2 * \beta)\}$.

3.$(\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_1 * \alpha, ƀ_1 * \beta) \leq \max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_1 * \alpha, ƀ_1 * \beta)(ţ_2 * \alpha, ƀ_2 * \beta)), (\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_2 * \alpha, ƀ_2 * \beta)\}$ and $R \times Ғ$ is closed normal ideal if it satisfies 2, 3, and 4. $(R_{T,I,F} \times F_{T,I,F})((0,0) * (ţ_1 * \alpha, ƀ_1 * \beta)) \geq (R_{T,I,F} \times F_{T,I,F})(ţ * \alpha, ƀ * \beta)$ and $(\lambda_{T,I,F} \times \mu_{T,I,F})((0,0) * (ţ * \alpha, ƀ * \beta)) \leq (\lambda_{T,I,F} \times \mu_{T,I,F})(ţ * \alpha, ƀ * \beta)\ \forall\ (ţ_1, ƀ_1)$ and $(ţ_2, ƀ_2) \in X \times Y$ and $\alpha, \beta \in [0,1]$.

Definition 2.15 [9] Let $\tilde{\mathcal{F}}_k = (A_{e_i}, \Lambda_{e_i})$ be a neutrosophic soft cubic set, where Y is subalgebra. Then $\tilde{\mathcal{F}}_k$ is NSCMSU under binary operation $*$ where $t_1, t_2 \in Y$ and $\alpha, \beta \in [0,1]$ if it fulfills these conditions:

$A^{\varrho}_{e_i}((t_1 * \alpha) * (t_2 * \beta)) \succeq \mathrm{rmin}\{A^{\varrho}_{e_i}(t_1 * \alpha), A^{\varrho}_{e_i}(t_2 * \beta)\}$ and $\lambda^{\varrho}_{e_i}((t_1 * \alpha) * (t_2 * \beta)) \preceq \max\{\lambda^{\varrho}_{e_i}(t_1 * \alpha), \lambda^{\varrho}_{e_i}(t_2 * \beta)\}$.

3. ɤ-Multiplication of Neutrosophic Cubic Normal Ideal and Closed Normal Ideal

Definition 3.1. Let $Ƕ = (H_{T,I,F}, \lambda_{T,I,F})$ be a NCS of X and $ɤ \in [0,1]$. An object of the form $Ƕ^{M}_{ɤ} = ({}^{M}_{ɤ}H^{Ƕ}_{T,I,F}, {}^{M}_{ɤ}\lambda^{Ƕ}_{T,I,F})$ is called neutrosophic cubic ɤ multiplication of Ƕ X if it fulfills following axioms:

${}^{M}_{ɤ}H^{Ƕ}_{T}(x) = ɤ.\, H^{Ƕ}_{T}(x)$, ${}^{M}_{ɤ}\lambda^{Ƕ}_{T}(x) = ɤ.\, \lambda^{Ƕ}_{T}(x)$,

${}^{M}_{ɤ}H^{Ƕ}_{I}(x) = ɤ.\, H^{Ƕ}_{I}(x)$, ${}^{M}_{ɤ}\lambda^{Ƕ}_{I}(x) = ɤ.\, \lambda^{Ƕ}_{I}(x)$,

${}^{M}_{ɤ}H^{Ƕ}_{F}(x) = ɤ.\, H^{Ƕ}_{F}(x)$, ${}^{M}_{ɤ}\lambda^{Ƕ}_{F}(x) = ɤ.\, \lambda^{Ƕ}_{F}(x)$.

For convinience we use ${}^{M}_{ɤ}H^{Ƕ}_{T,I,F} = ɤ.\, H^{Ƕ}_{T,I,F}(x)$ and ${}^{M}_{ɤ}\lambda^{Ƕ}_{T,I,F} = ɤ.\, \lambda^{Ƕ}_{T,I,F}(x)$.

Theorem 3.1 A ɤ-multilplication of NCCNID of B-algebra X is also a ɤ-multilplication of NCMSU of X.

Proof. Suppose $Ƕ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ be a NCCNID of X, then for any $ţ \in X$, we have ${}^{M}_{ɤ}H_{T,I,F}(0 * (ţ * \alpha)) = ɤ.\, H_{T,I,F}(0 * (ţ * \alpha)) \geq ɤ.\, H_{T,I,F}(ţ * \alpha)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(0 * (ţ * \alpha)) = ɤ.\, \lambda_{T,I,F}(0 * (ţ * \alpha)) \leq ɤ.\, \lambda_{T,I,F}(ţ * \alpha)$. Now by N4, N6, and through proposition 3.3 of article M subalgebra, we know that ${}^{M}_{ɤ}H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)) = ɤ.\, H_{T,I,F}((ţ * \alpha) * (ƀ * \beta)) \geq ɤ.\, \mathrm{rmin}\{H_{T,I,F}(((ţ * \alpha) * (ƀ * \beta)) * (0 * (ƀ * \beta))), H_{T,I,F}(0 * (ƀ * \beta))\} = ɤ.\, \mathrm{rmin}\{H_{T,I,F}(ţ * \alpha), H_{T,I,F}(0 * (ƀ * \beta))\} \geq ɤ.\, \mathrm{rmin}\{H_{T,I,F}(ţ * \alpha), H_{T,I,F}(ƀ * \beta)\} = \mathrm{rmin}\{ɤ.\, H_{T,I,F}(ţ * \alpha), ɤ.\, H_{T,I,F}(ƀ * \beta)\} = \mathrm{rmin}\{{}^{M}_{ɤ}H_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}H_{T,I,F}(ƀ * \beta)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}((ţ * \alpha) * (ƀ * \beta)) = ɤ.\, \lambda_{T,I,F}((ţ * \alpha) * (ƀ * \beta)) \leq ɤ.\, \max\{\lambda_{T,I,F}(((ţ * \alpha) * (ƀ * \beta)) * (0 * (ƀ * \beta))), \lambda_{T,I,F}(0 * (ƀ * \beta))\} = ɤ.\, \max\{\lambda_{T,I,F}(ţ * \alpha), \lambda_{T,I,F}(0 * (ƀ * \beta))\} \leq ɤ.\, \max\{\lambda_{T,I,F}(ţ * \alpha), \lambda_{T,I,F}(ƀ * \beta)\} = \max\{ɤ.\, \lambda_{T,I,F}(ţ * \alpha), ɤ.\, \lambda_{T,I,F}(ƀ * \beta)\} = \max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}\lambda_{T,I,F}(ƀ * \beta)\}$. Hence, ɤMNCCNID is ɤMNCMSU of X.

Proposition 3.1 Every ɤ-multiplication of NCCNID is a ɤ-multiplication NCNID but the converse is not true in general.

Theorem 3.2 The R-intersection of any set of ɤMNCNIDs of X is also a ɤMNCNID of X.

Proof. Let $\text{Hʋ}_i = \{H^i_{T,I,F}, \lambda^i_{T,I,F}\}$, where i ∈ k, be a ɤMNCNID of X and $\text{ƫ}, \text{ƀ} \in X$. Then

$$(\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(0) = \text{rinf}\ {}^{M}_{\text{ɤ}}H^i_{T,I,F}(0) = \text{rinf}\ H^i_{T,I,F}(0).\text{ɤ}$$

$$\geq \text{rinf}H^i_{T,I,F}(\text{ƫ} * \alpha).\text{ɤ} = \text{rinf}\ {}^{M}_{\text{ɤ}}H^i_{T,I,F}(\text{ƫ} * \alpha)$$

$$= (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ƫ} * \alpha)$$

$$\Rightarrow (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(0) \geq (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ƫ} * \alpha)$$

and

$$(\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(0) = \text{sup}{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}(0) = \text{sup}\lambda^i_{T,I,F}(0)\,.\text{ɤ}$$

$$\leq \text{sup}\lambda^i_{T,I,F}(\text{ƫ} * \alpha)\,.\text{ɤ} = \text{sup}{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}(\text{ƫ} * \alpha)$$

$$= (\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ƫ} * \alpha)$$

$$\Rightarrow (\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(0) \leq (\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ƫ} * \alpha),$$

now

$$(\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ƫ} * \alpha) = \text{rinf}{}^{M}_{\text{ɤ}}H^i_{T,I,F}(\text{ƫ} * \alpha) = \text{rinf}\ H^i_{T,I,F}(\text{ƫ} * \alpha)\,.\text{ɤ}$$

$$\geq \text{rinf}\{\text{rmin}\{H^i_{T,I,F}((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)), H^i_{T,I,F}(\text{ƀ} * \beta)\}\}\,.\text{ɤ}$$

$$= \text{rmin}\{\text{rinf}H^i_{T,I,F}((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)).\text{ɤ}, \text{rinf}H^i_{T,I,F}(\text{ƀ} * \beta).\text{ɤ}\}$$

$$= \text{rmin}\{\text{rinf}{}^{M}_{\text{ɤ}}H^i_{T,I,F}((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)), \text{rinf}{}^{M}_{\text{ɤ}}H^i_{T,I,F}(\text{ƀ} * \beta)\}$$

$$= \text{rmin}\{(\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)), (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})((\text{ƀ} * \beta))\} \Rightarrow (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ƫ} * \alpha) \geq \text{rmin}\{(\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)), (\cap\ {}^{M}_{\text{ɤ}}H^i_{T,I,F})(\text{ƀ} * \beta)\}$$

and

$$(\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ƫ} * \alpha) = \text{sup}{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}(\text{ƫ} * \alpha) = \text{sup}\lambda^i_{T,I,F}((\text{ƫ} * \alpha)\,.\text{ɤ}$$

$$\leq \text{sup}\{\max\{\lambda^i_{T,I,F}((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)), \lambda^i_{T,I,F}(\text{ƀ} * \beta)\}\}\,.\text{ɤ}$$

$$= \max\{\text{sup}\lambda^i_{T,I,F}((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)).\text{ɤ}, \text{sup}\lambda^i_{T,I,F}(\text{ƀ} * \beta).\text{ɤ}\}$$

$$= \max\{\text{sup}{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)), \text{sup}{}^{M}_{\text{ɤ}}\lambda^i_{T,I,F}(\text{ƀ} * \beta)\}$$

$$= \max\{(\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)), (\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ƀ} * \beta)\}$$

$$\Rightarrow (\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ƫ} * \alpha) \leq \max\{(\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})((\text{ƫ} * \alpha) * (\text{ƀ} * \beta)), (\vee {}^{M}_{\text{ɤ}}\lambda^i_{T,I,F})(\text{ƀ} * \beta)\},$$

which show that R-intersection is a ɤMNCNID of X.

Theorem 3.3. The R-intersection of any set of ɤMNCCNIDs of X is also a ɤ-multiplication of NCCNID of X.

Proof**.** We can prove this theorem as Theorem 3.2.

Theorem 3.4. Let $Ƕ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ be a NCS of X. Then ɤMNCNID of Ƕ is a NCNID of X iff ${}^{M}_{ɤ}H^{-}_{T,I,F}$, ${}^{M}_{ɤ}H^{+}_{T,I,F}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}$ are fuzzy ideals of X.

Proof. Suppose that $ʈ, ƫ \in X$. Since ${}^{M}_{ɤ}H^{-}_{T,I,F}(0) = H^{-}_{T,I,F}(0).ɤ \geq H^{-}_{T,I,F}(ʈ * \alpha).ɤ = {}^{M}_{ɤ}H^{-}_{T,I,F}(ʈ * \alpha)$, ${}^{M}_{ɤ}H^{+}_{T,I,F}(0) = H^{+}_{T,I,F}(0).ɤ \geq H^{+}_{T,I,F}(ʈ * \alpha).ɤ = {}^{M}_{ɤ}H^{-}_{T,I,F}(ʈ * \alpha)$, therefore, $H_{T,I,F}(0) \geq H_{T,I,F}(ʈ * \alpha)$, also ${}^{M}_{ɤ}\lambda_{T,I,F}(0) = \lambda_{T,I,F}(0).ɤ \leq \lambda_{T,I,F}(ʈ * \alpha).ɤ = {}^{M}_{ɤ}\lambda_{T,I,F}(ʈ * \alpha)$. Suppose that ${}^{M}_{ɤ}H^{-}_{T,I,F}$, ${}^{M}_{ɤ}H^{+}_{T,I,F}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}$ are ɤ-multiplication of fuzzy ideals of X. Then ${}^{M}_{ɤ}H_{T,I,F}(ʈ * \alpha) = H_{T,I,F}(ʈ * \alpha).ɤ = \{H^{-}_{T,I,F}(ʈ * \alpha), H^{+}_{T,I,F}(ʈ * \alpha)\}.ɤ \geq [\min\{H^{-}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), H^{-}_{T,I,F}((ƫ * \beta))\}, \min\{H^{+}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), H^{+}_{T,I,F}(ƫ * \beta)\}.ɤ = r\min\{[H^{-}_{T,I,F}\big((ʈ * \alpha) * (ƫ * \beta)\big), H^{+}_{T,I,F}\big((ʈ * \alpha) * (ƫ * \beta)\big)], [H^{-}_{T,I,F}\big((ƫ * \beta)\big), H^{+}_{T,I,F}(ƫ * \beta)]\}.ɤ = r\min\{H_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), H_{T,I,F}(ƫ * \beta)\}.ɤ = r\min\{H_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)).ɤ, H_{T,I,F}(ƫ * \beta).ɤ\} = r\min\{{}^{M}_{ɤ}H_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), {}^{M}_{ɤ}H_{T,I,F}(ƫ * \beta)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ʈ * \alpha) \leq \max\{{}^{M}_{ɤ}\lambda_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ * \beta)\}$. Therefore ɤMNCNID of Ƕ is a NCNID of X.

Conversely, assume that ɤMNCNID Ƕ is a NCNID of X. For any $ʈ, ƫ \in X$, we have $\{{}^{M}_{ɤ}H^{-}_{T,I,F}(ʈ * \alpha), {}^{M}_{ɤ}H^{+}_{T,I,F}(ʈ * \alpha)\} = \{H^{-}_{T,I,F}(ʈ * \alpha).ɤ, H^{+}_{T,I,F}(ʈ * \alpha).ɤ\} = \{H^{-}_{T,I,F}(ʈ * \alpha), H^{+}_{T,I,F}(ʈ * \alpha)\}.ɤ = H_{T,I,F}(ʈ * \alpha).ɤ = {}^{M}_{ɤ}H_{T,I,F}(ʈ * \alpha) = r\min$ $\{{}^{M}_{ɤ}H_{T,I,F}\big((ʈ * \alpha) * (ƫ * \beta)\big), {}^{M}_{ɤ}H_{T,I,F}(ƫ * \beta)\} = r\min\{[{}^{M}_{ɤ}H^{-}_{T,I,F}\big((ʈ * \alpha) * (ƫ * \beta)\big), {}^{M}_{ɤ}H^{+}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta))], [{}^{M}_{ɤ}H^{-}_{T,I,F}(ƫ * \beta), {}^{M}_{ɤ}H^{+}_{T,I,F}(ƫ * \beta)]\} = [\min\{H^{-}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)).ɤ, H^{-}_{T,I,F}(ƫ * \beta).ɤ\}, \min\{H^{+}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)).ɤ, H^{+}_{T,I,F}(ƫ * \beta).ɤ\} = [\min\{{}^{M}_{ɤ}H^{-}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), {}^{M}_{ɤ}H^{-}_{T,I,F}(ƫ * \beta)\}, \min\{{}^{M}_{ɤ}H^{+}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), {}^{M}_{ɤ}H^{+}_{T,I,F}(ƫ * \beta)\}]$. Thus, ${}^{M}_{ɤ}H^{-}_{T,I,F}(ʈ * \alpha) \geq \min\{{}^{M}_{ɤ}H^{-}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), {}^{M}_{ɤ}H^{-}_{T,I,F}(ƫ * \beta)\}, {}^{M}_{ɤ}H^{+}_{T,I,F}(ʈ * \alpha) \geq \min\{{}^{M}_{ɤ}H^{+}_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), {}^{M}_{ɤ}H^{+}_{T,I,F}(ƫ * \beta)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ʈ * \alpha) \leq \max\{{}^{M}_{ɤ}\lambda_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ * \beta)\}$. Hence, ${}^{M}_{ɤ}H^{-}_{T,I,F}$, ${}^{M}_{ɤ}H^{+}_{T,I,F}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}$ are fuzzy ideals of X.

Theorem 3.5. For a NCNID $Ƕ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ of X, the following statements are valid:

1. If $(ʈ * \alpha) * (ƫ * \beta) \leq z * \gamma$, then ${}^{M}_{ɤ}H_{T,I,F}(ʈ * \alpha) \geq r\min\{{}^{M}_{ɤ}H_{T,I,F}(ƫ * \beta), {}^{M}_{ɤ}H_{T,I,F}(z * \gamma)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ʈ * \alpha) \leq \max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ƫ * \beta), {}^{M}_{ɤ}\lambda_{T,I,F}(z * \gamma)\}$,

2. If $(ʈ * \alpha) \leq (ƫ * \beta)$, then ${}^{M}_{ɤ}H_{T,I,F}(ʈ * \alpha) \geq {}^{M}_{ɤ}H_{T,I,F}(ƫ * \beta)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ʈ * \alpha) \leq {}^{M}_{ɤ}\lambda_{T,I,F}(ƫ * \beta)$ $\forall$ $ʈ, ƫ, z \in X$ and $\alpha, \beta, \gamma \in [0,1]$.

Proof. 1. Assume that $ʈ, ƫ, z \in X$ such that $(ʈ * \alpha) * (ƫ * \beta) \leq (z * \gamma)$. Then $((ʈ * \alpha) * (ƫ * \beta)) * (z * \gamma) = 0$ and thus ${}^{M}_{ɤ}H_{T,I,F}(ʈ * \alpha) = H_{T,I,F}(ʈ * \alpha).ɤ \geq r\min\big\{H_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), H_{T,I,F}(ƫ * \beta)\big\}.ɤ \geq r\min\{r\min\{H_{T,I,F}(((ʈ * \alpha) * (ƫ * \beta)) * (z * \gamma)), H_{T,I,F}(z * \gamma)\}, H_{T,I,F}(ƫ * \beta)\}.ɤ = r\min\{r\min\{H_{T,I,F}(0), H_{T,I,F}(z * \gamma)\}, H_{T,I,F}(ƫ * \beta)\}.ɤ = r\min\{H_{T,I,F}(ƫ * \beta).ɤ, H_{T,I,F}(z * \gamma).ɤ\} = r\min\{{}^{M}_{ɤ}H_{T,I,F}(ƫ * \beta), {}^{M}_{ɤ}H_{T,I,F}(z * \gamma)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ʈ * \alpha) = \lambda_{T,I,F}(ʈ * \alpha).ɤ \leq \max\{\lambda_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), \lambda_{T,I,F}(ƫ * \beta)\}.ɤ \leq \max\{\max\{\lambda_{T,I,F}(((ʈ * \alpha) * (ƫ * \beta)) * (z * \gamma)), \lambda_{T,I,F}(z * \gamma)\}, \lambda_{T,I,F}(ƫ * \beta)\}.ɤ = \max\{\max\{\lambda_{T,I,F}(0), \lambda_{T,I,F}(z * \gamma)\}, \lambda_{T,I,F}(ƫ * \beta)\}.ɤ = \max\{\lambda_{T,I,F}(ƫ * \beta).ɤ, \lambda_{T,I,F}(z * \gamma).ɤ\} = \max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ƫ * \beta), {}^{M}_{ɤ}\lambda_{T,I,F}(z * \gamma)\}$.

2. Again, take $ʈ, ƫ \in X$ and $\alpha, \beta \in [0,1]$, such that $(ʈ * \alpha) \leq (ƫ * \beta)$. Then $(ʈ * \alpha) * (ƫ * \beta) = 0$ and thus ${}^{M}_{ɤ}H_{T,I,F}(ʈ * \alpha) = H_{T,I,F}(ʈ * \alpha).ɤ \geq rmin\{H_{T,I,F}((ʈ * \alpha) * (ƫ * \beta)), H_{T,I,F}(ƫ * \beta)\}.ɤ = rmin\{H_{T,I,F}(0), H_{T,I,F}(ƫ * \beta)\}.ɤ = H_{T,I,F}(ƫ * \beta).ɤ = {}^{M}_{ɤ}H_{T,I,F}(ƫ * \beta), so\ {}^{M}_{ɤ}H_{T,I,F}(ʈ * \alpha) \geq {}^{M}_{ɤ}H_{T,I,F}(ƫ * \beta)$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ʈ * \alpha) = \lambda_{T,I,F}(ʈ * \alpha).ɤ \leq$

$max\{\lambda_{T,I,F}((ţ * \alpha) * (ŧ * \beta)), \lambda_{T,I,F}(ŧ * \beta)\}.\curlyvee = \max\{\lambda_{T,I,F}(0), \lambda_{T,I,F}(ŧ * \beta)\}.\curlyvee = \lambda_{T,I,F}(ŧ * \beta).\curlyvee = {}^{M}_{\curlyvee}\lambda_{T,I,F}(ŧ * \beta)$, so ${}^{M}_{\curlyvee}\lambda_{T,I,F}(ţ * \alpha) \leq {}^{M}_{\curlyvee}\lambda_{T,I,F}(ŧ * \beta)$.

Theorem 3.6. Let ${}^{M}_{\curlyvee}Ꮀ$ of $Ꮀ = \{H_{T,I,F}, \lambda_{T,I,F}\}$ is a NCNID of X. $\forall$ ţ, ŧ $\in X$ and $\alpha, \beta \in [0,1]$, then Ꮀ is a NCMSU of X.

Proof. Assume that ${}^{M}_{\curlyvee}Ꮀ$ is a NCNID of X, $\forall$ ţ, ŧ $\in X$ and $\alpha, \beta \in [0,1]$. Then $\curlyvee.H_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) = {}^{M}_{\curlyvee}H_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) \geq rmin\{{}^{M}_{\curlyvee}H_{T,I,F}((ŧ * \beta) * ((ţ * \alpha) * (ŧ * \beta))), {}^{M}_{\curlyvee}H_{T,I,F}(ŧ * \beta)\} = rmin\{{}^{M}_{\curlyvee}H_{T,I,F}(0), {}^{M}_{\curlyvee}H_{T,I,F}(ŧ * \beta)\} \geq rmin\{{}^{M}_{\curlyvee}H_{T,I,F}(ţ * \alpha), {}^{M}_{\curlyvee}H_{T,I,F}(ŧ * \beta)\} = rmin\{H_{T,I,F}(ţ * \alpha).\curlyvee, H_{T,I,F}(ŧ * \beta).\curlyvee\} = rmin\{H_{T,I,F}(ţ * \alpha), H_{T,I,F}(ŧ * \beta)\}.\curlyvee \Rightarrow H_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) \geq rmin\{H_{T,I,F}(ţ * \alpha), H_{T,I,F}(ŧ * \beta)\}$ and $\curlyvee.\lambda_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) = {}^{M}_{\curlyvee}\lambda_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) \leq max\{{}^{M}_{\curlyvee}\lambda_{T,I,F}((ŧ * \beta) * ((ţ * \alpha) * (ŧ * \beta))), {}^{M}_{\curlyvee}\lambda_{T,I,F}(ŧ * \beta)\} = max\{{}^{M}_{\curlyvee}\lambda_{T,I,F}(0), {}^{M}_{\curlyvee}\lambda_{T,I,F}(ŧ * \beta)\} \leq max\{{}^{M}_{\curlyvee}\lambda_{T,I,F}(ţ * \alpha), {}^{M}_{\curlyvee}\lambda_{T,I,F}(ŧ * \beta)\} = max\{\lambda_{T,I,F}(ţ * \alpha).\curlyvee, \lambda_{T,I,F}(ŧ * \beta).\curlyvee\} = max\{\lambda_{T,I,F}(ţ * \alpha), \lambda_{T,I,F}(ŧ * \beta)\}.\curlyvee \Rightarrow \lambda_{T,I,F}((ţ * \alpha) * (ŧ * \beta)) \leq max\{\lambda_{T,I,F}(ţ * \alpha), \lambda_{T,I,F}((ŧ * \beta))\}$. Hence, Ꮀ$\{H_{T,I,F}, \lambda_{T,I,F}\}$ is a NCMSU of X.

4. ɤ-MULTIPLICATION UNDER HOMOMORPHISM

Theorem 4.1. Suppose that $\Gamma|X \to Y$ is a homomorphic mapping of PS-algebra. If ${}^{M}_{\curlyvee}Ꮀ$ of $Ꮀ = (H_{T,I,F}, \lambda_{T,I,F})$ is a NCNID of Y, then pre-image $\Gamma^{-1}({}^{M}_{\curlyvee}Ꮀ) = (\Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F}), \Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F}))$ of ${}^{M}_{\curlyvee}Ꮀ$ under Γ of X is a NCNID of X.

Proof. For all ţ $\in X$ and $\alpha \in [0,1]$, $\Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F})(ţ * \alpha) = {}^{M}_{\curlyvee}H_{T,I,F}(\Gamma(ţ * \alpha)) = H_{T,I,F}(\Gamma(ţ * \alpha)).\curlyvee \leq H_{T,I,F}(\Gamma(0)).\curlyvee = {}^{M}_{\curlyvee}H_{T,I,F}(\Gamma(0)) = \Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F})(0)$ and $\Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F})(ţ * \alpha) = {}^{M}_{\curlyvee}\lambda_{T,I,F}(\Gamma(ţ * \alpha)) = \lambda_{T,I,F}(\Gamma(ţ * \alpha)).\curlyvee \geq \lambda_{T,I,F}(\Gamma(0)).\curlyvee = {}^{M}_{\curlyvee}\lambda_{T,I,F}(\Gamma(0)) = \Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F})(0)$.

Let ţ, ŧ $\in X$, $\Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F})(ţ * \alpha) = {}^{M}_{\curlyvee}H_{T,I,F}(\Gamma(ţ * \alpha)) = H_{T,I,F}(\Gamma(ţ * \alpha)).\curlyvee \geq rmin\{H_{T,I,F}(\Gamma(ţ * \alpha) * \Gamma(ŧ * \beta)), H_{T,I,F}(\Gamma(ŧ * \beta))\}.\curlyvee = rmin\{H_{T,I,F}(\Gamma((ţ * \alpha) * (ŧ * \beta))), H_{T,I,F}(\Gamma(ŧ * \beta))\}.\curlyvee = rmin\{\Gamma^{-1}(H_{T,I,F}((ţ * \alpha) * (ŧ * \beta)).\curlyvee), \Gamma^{-1}(H_{T,I,F}(ŧ * \beta).\curlyvee)\} = rmin\{\Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F}((ţ * \alpha) * (ŧ * \beta))), \Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F}(ŧ * \beta))\}$ and $\Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F})(ţ * \alpha) = {}^{M}_{\curlyvee}\lambda_{T,I,F}(\Gamma(ţ * \alpha)) = \lambda_{T,I,F}(\Gamma(ţ * \alpha)).\curlyvee \leq max\{\lambda_{T,I,F}(\Gamma(ţ * \alpha) * \Gamma(ŧ * \beta)), \lambda_{T,I,F}(\Gamma(ŧ * \beta))\}.\curlyvee = max\{\lambda_{T,I,F}(\Gamma((ţ * \alpha) * (ŧ * \beta))), \lambda_{T,I,F}(\Gamma(ŧ * \beta))\}.\curlyvee = max\{\Gamma^{-1}(\lambda_{T,I,F}((ţ * \alpha) * (ŧ * \beta)).\curlyvee), \Gamma^{-1}(\lambda_{T,I,F}(ŧ * \beta).\curlyvee)\} = max\{\Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F}((ţ * \alpha) * (ŧ * \beta)), \Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F}(ŧ * \beta))\}$. Hence, $\Gamma^{-1}({}^{M}_{\curlyvee}Ꮀ) = (\Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F}), \Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F}))$ is a NCNID of X.

Theorem 4.2. Let $\Gamma|X \to Y$ be a homomorphic mapping of B-algebra. If ${}^{M}_{\curlyvee}Ꮀ_i$ of $Ꮀ_i = (H^{i}_{T,I,F}, \lambda^{i}_{T,I,F})$ is a NCNID of Y where $i \in k$, then the pre-image $\Gamma^{-1}(\bigcap_{i\in k} {}^{M}_{\curlyvee}Ꮀ^{i}_{T,I,F}) = (\Gamma^{-1}(\bigcap_{i\in k} {}^{M}_{\curlyvee}H^{i}_{T,I,F}), \Gamma^{-1}(\bigcap_{i\in k} {}^{M}_{\curlyvee}\lambda^{i}_{T,I,F}))$ is a NCNID of X.

Proof. We can prove this theorem through Theorem 3.2 and Theorem 4.1.

Theorem 4.3. Let $\Gamma|X \to Y$ is an epimorphic mapping of B-algebra.Then ${}^{M}_{\curlyvee}Ꮀ = ({}^{M}_{\curlyvee}H_{T,I,F}, {}^{M}_{\curlyvee}\lambda_{T,I,F})$ is a NCNID of Y, if pre-image $\Gamma^{-1}({}^{M}_{\curlyvee}Ꮀ) = (\Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F}), \Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F}))$ of ${}^{M}_{\curlyvee}Ꮀ$ under Γ of X is a NCNID of X

Proof. For any ŧ $\in Y$, ţ $\in X$ and $\alpha, \beta \in [0,1]$ such that $(ŧ * \beta) = \Gamma(ţ * \alpha)$. Then ${}^{M}_{\curlyvee}H_{T,I,F}(ŧ * \beta) = {}^{M}_{\curlyvee}H_{T,I,F}(\Gamma(ţ * \alpha)) = \Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F})(ţ * \alpha) = \Gamma^{-1}(H_{T,I,F})(ţ * \alpha).\curlyvee \geq \Gamma^{-1}(H_{T,I,F})(0).\curlyvee = H_{T,I,F}(\Gamma(0)).\curlyvee = H_{T,I,F}(0).\curlyvee = {}^{M}_{\curlyvee}H_{T,I,F}(0)$ and ${}^{M}_{\curlyvee}\lambda_{T,I,F}(ŧ * \beta) = {}^{M}_{\curlyvee}\lambda_{T,I,F}(\Gamma(ţ * \alpha)) = \Gamma^{-1}({}^{M}_{\curlyvee}\lambda_{T,I,F})(ţ * \alpha) = \Gamma^{-1}(\lambda_{T,I,F})(ţ * \alpha).\curlyvee \leq \Gamma^{-1}(\lambda_{T,I,F})(0).\curlyvee = \lambda_{T,I,F}(\Gamma(0)).\curlyvee = \lambda_{T,I,F}(0).\curlyvee = {}^{M}_{\curlyvee}\lambda_{T,I,F}(0)$.

Assume $ŧ_1, ŧ_2 \in Y$. Then $\Gamma(ţ_1 * \alpha) = ŧ_1 * \beta$ and $\Gamma(ţ_2 * \alpha) = ŧ_2 * \beta$ for some $ţ_1, ţ_2 \in X$ and $\alpha, \beta \in [0,1]$. Thus ${}^{M}_{\curlyvee}H_{T,I,F}(ŧ_1 * \beta) = {}^{M}_{\curlyvee}H_{T,I,F}(\Gamma(ţ_1 * \alpha)) = \Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F})(ţ_1 * \alpha) = \Gamma^{-1}(H_{T,I,F})(ţ_1 * \alpha).\curlyvee \geq rmin\{\Gamma^{-1}({}^{M}_{\curlyvee}H_{T,I,F})((ţ_1 *$

$\alpha) * (ţ_2 * \alpha)), \Gamma^{-1}({}^{M}_{ɤ}H_{T,I,F})(ţ_2 * \alpha)\}.ɤ = rmin\{{}^{M}_{ɤ}H_{T,I,F}(\Gamma((ţ_1 * \alpha) * (ţ_2 * \alpha))), H_{T,I,F}(\Gamma(ţ_2 * \alpha))\}.ɤ = rmin\{H_{T,I,F}(\Gamma(ţ_1 * \alpha) * \Gamma(ţ_2 * \alpha)), H_{T,I,F}(\Gamma(ţ_2 * \alpha))\} = rmin\{H_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), H_{T,I,F}(ŧ_2 * \beta)\}.ɤ = rmin\{H_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)).ɤ, H_{T,I,F}(ŧ_2 * \beta).ɤ\} = rmin\{{}^{M}_{ɤ}H_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), {}^{M}_{ɤ}H_{T,I,F}(ŧ_2 * \beta)\}$ and ${}^{M}_{ɤ}\lambda_{T,I,F}(ŧ_1 * \beta) = {}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma(ţ_1 * \alpha)) = \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(ţ_1 * \alpha) = \Gamma^{-1}(\lambda_{T,I,F})(ţ_1 * \alpha).ɤ \leq max\{\Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})((ţ_1 * \alpha) * (ţ_2 * \alpha)), \Gamma^{-1}({}^{M}_{ɤ}\lambda_{T,I,F})(ţ_2 * \alpha)\}.ɤ = max\{{}^{M}_{ɤ}\lambda_{T,I,F}(\Gamma((ţ_1 * \alpha) * (ţ_2 * \alpha))), \lambda_{T,I,F}(\Gamma(ţ_2 * \alpha))\}.ɤ = max\{\lambda_{T,I,F}(\Gamma(ţ_1 * \alpha) * \Gamma(ţ_2 * \alpha)), \lambda_{T,I,F}(\Gamma(ţ_2 * \alpha))\} = max\{\lambda_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), \lambda_{T,I,F}(ŧ_2 * \beta)\}.ɤ = max\{\lambda_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)).ɤ, \lambda_{T,I,F}(ŧ_2 * \beta).ɤ\} = max\{{}^{M}_{ɤ}\lambda_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), {}^{M}_{ɤ}\lambda_{T,I,F}(ŧ_2 * \beta)\}$.

Hence, ${}^{M}_{ɤ}Ƕ = ({}^{M}_{ɤ}H_{T,I,F}, {}^{M}_{ɤ}\lambda_{T,I,F})$ is a NCNID of Y.

5. ɤ-MULTIPLICATION OF CARTESIAN PRODUCT

Theorem 5.1. Let ${}^{M}_{ɤ}Ƕ = ({}^{M}_{ɤ}H_{T,I,F}, {}^{M}_{ɤ}\lambda_{T,I,F})$ and ${}^{M}_{ɤ}Ƒ = ({}^{M}_{ɤ}F_{T,I,F}, {}^{M}_{ɤ}\mu_{T,I,F})$ are NCNIDs of X and Y respectively. Then ${}^{M}_{ɤ}Ƕ \times {}^{M}_{ɤ}Ƒ$ is a neutrosophic cubic normal ideal of $X \times Y$.

Proof. For any $(ţ, ŧ) \in X \times Y$ and $\alpha, \beta \in [0,1]$. We have $({}^{M}_{ɤ}H_{T,I,F} \times {}^{M}_{ɤ}F_{T,I,F})(0,0) = ɤ.(H_{T,I,F} \times F_{T,I,F})(0,0) = ɤ.rmin\{H_{T,I,F}(0), F_{T,I,F}(0)\} \geq ɤ.rmin\{H_{T,I,F}(ţ * \alpha), F_{T,I,F}(ŧ * \beta)\} = rmin\{H_{T,I,F}(ţ * \alpha).ɤ, F_{T,I,F}(ŧ * \beta).ɤ\} = rmin\{{}^{M}_{ɤ}H_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}F_{T,I,F}(ŧ * \beta)\} = ({}^{M}_{ɤ}H_{T,I,F} \times {}^{M}_{ɤ}F_{T,I,F})(ţ * \alpha, ŧ * \beta)$ and $({}^{M}_{ɤ}\lambda_{T,I,F} \times {}^{M}_{ɤ}\mu_{T,I,F})(0,0) = ɤ.(\lambda_{T,I,F} \times \mu_{T,I,F})(0,0) = ɤ.max\{\lambda_{T,I,F}(0), \mu_{T,I,F}(0)\} \leq ɤ.max\{\lambda_{T,I,F}(ţ * \alpha), \mu_{T,I,F}(ŧ * \beta)\} = max\{\lambda_{T,I,F}(ţ * \alpha).ɤ, \mu_{T,I,F}(ŧ * \beta).ɤ\} = max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}\mu_{T,I,F}(ŧ * \beta)\} = ({}^{M}_{ɤ}\lambda_{T,I,F} \times {}^{M}_{ɤ}\mu_{T,I,F})(ţ * \alpha, ŧ * \beta)$.

Let $(ţ_1, ŧ_1), (ţ_2, ŧ_2) \in X \times Y$ and $\alpha, \beta \in [0,1]$. Then $({}^{M}_{ɤ}H_{T,I,F} \times {}^{M}_{ɤ}F_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) = ɤ.(H_{T,I,F} \times F_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) = ɤ.rmin\{H_{T,I,F}(ţ_1 * \alpha), F_{T,I,F}(ŧ_1 * \beta)\} \geq ɤ.rmin\{rmin\{H_{T,I,F}((ţ_1 * \alpha) * (ţ_2 * \alpha)), H_{T,I,F}(ţ_2 * \alpha)\}, rmin\{F_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), F_{T,I,F}(ŧ_2 * \beta)\}\} = ɤ.rmin\{rmin\{H_{T,I,F}((ţ_1 * \alpha) * (ţ_2 * \alpha)), F_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta))\}, rmin\{H_{T,I,F}(ţ_2 * \alpha), F_{T,I,F}(ŧ_2 * \beta)\}\} = ɤ.rmin\{(H_{T,I,F} \times F_{T,I,F})((ţ_1 * \alpha) * (ţ_2 * \alpha), (ŧ_1 * \beta) * (ŧ_2 * \beta)), (H_{T,I,F} \times F_{T,I,F})((ţ_2 * \alpha), (ŧ_2 * \beta))\} = rmin\{(H_{T,I,F} \times F_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)).ɤ, (H_{T,I,F} \times F_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta).ɤ\} = rmin\{({}^{M}_{ɤ}R_{T,I,F} \times {}^{M}_{ɤ}F_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)), ({}^{M}_{ɤ}R_{T,I,F} \times {}^{M}_{ɤ}F_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta)\}$ and $({}^{M}_{ɤ}\lambda_{T,I,F} \times {}^{M}_{ɤ}\mu_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) = ɤ.(\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_1 * \alpha, ŧ_1 * \beta) = ɤ.max\{\lambda_{T,I,F}(ţ_1 * \alpha), \mu_{T,I,F}(ŧ_1 * \beta)\} \leq ɤ.max\{max\{\lambda_{T,I,F}((ţ_1 * \alpha) * (ţ_2 * \alpha)), \lambda_{T,I,F}(ţ_2 * \alpha)\}, max\{\mu_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta)), F_{T,I,F}(ŧ_2 * \beta)\}\} = ɤ.max\{max\{\lambda_{T,I,F}((ţ_1 * \alpha) * (ţ_2 * \alpha)), \mu_{T,I,F}((ŧ_1 * \beta) * (ŧ_2 * \beta))\}, max\{\lambda_{T,I,F}(ţ_2 * \alpha), \mu_{T,I,F}(ŧ_2 * \beta)\}\} = ɤ.max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_1 * \alpha) * (ţ_2 * \alpha), (ŧ_1 * \beta) * (ŧ_2 * \beta)), (\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_2 * \alpha), (ŧ_2 * \beta))\} = max\{(\lambda_{T,I,F} \times \mu_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)).ɤ, (\lambda_{T,I,F} \times \mu_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta).ɤ\} = max\{({}^{M}_{ɤ}\lambda_{T,I,F} \times {}^{M}_{ɤ}\mu_{T,I,F})((ţ_1 * \alpha, ŧ_1 * \beta) * (ţ_2 * \alpha, ŧ_2 * \beta)), ({}^{M}_{ɤ}\lambda_{T,I,F} \times {}^{M}_{ɤ}\mu_{T,I,F})(ţ_2 * \alpha, ŧ_2 * \beta)\}$. Hence, ${}^{M}_{ɤ}Ƕ \times {}^{M}_{ɤ}Ƒ$ is a neutrosophic cubic normal ideal of $X \times Y$.

Theorem 5.2. Let ${}^{M}_{ɤ}Ƕ = ({}^{M}_{ɤ}H_{T,I,F}, {}^{M}_{ɤ}\lambda_{T,I,F})$ and ${}^{M}_{ɤ}Ƒ = ({}^{M}_{ɤ}F_{T,I,F}, {}^{M}_{ɤ}\mu_{T,I,F})$ are two ɤ-multiplications of neutrosophic cubic closed normal ideals of X and Y respectively. Then ${}^{M}_{ɤ}Ƕ \times {}^{M}_{ɤ}Ƒ$ is a NCCNID of $X \times Y$.

Proof. By Proposition 3.1 and Theorem 5.1, ${}^{M}_{ɤ}Ƕ \times {}^{M}_{ɤ}Ƒ$ is NCNID. Now, $({}^{M}_{ɤ}H_{T,I,F} \times {}^{M}_{ɤ}F_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)) = (H_{T,I,F} \times F_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)).ɤ = (H_{T,I,F} \times F_{T,I,F})(0 * (ţ * \alpha), 0 * (ŧ * \beta)).ɤ = ɤ.rmin\{H_{T,I,F}(0 * (ţ * \alpha)), F_{T,I,F}(0 * (ŧ * \beta))\} \geq ɤ.rmin\{H_{T,I,F}(ţ * \alpha), F_{T,I,F}(ŧ * \beta)\} = rmin\{H_{T,I,F}(ţ * \alpha).ɤ, F_{T,I,F}(ŧ * \beta).ɤ\} = rmin\{{}^{M}_{ɤ}H_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}F_{T,I,F}(ŧ * \beta)\} = ({}^{M}_{ɤ}H_{T,I,F} \times {}^{M}_{ɤ}F_{T,I,F})(ţ * \alpha, ŧ * \beta)$ and $({}^{M}_{ɤ}\lambda_{T,I,F} \times {}^{M}_{ɤ}\mu_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)) = (\lambda_{T,I,F} \times \mu_{T,I,F})((0,0) * (ţ * \alpha, ŧ * \beta)).ɤ = (\lambda_{T,I,F} \times \mu_{T,I,F})(0 * (ţ * \alpha), 0 * (ŧ * \beta)).ɤ = ɤ.max\{\lambda_{T,I,F}(0 * (ţ * \alpha)), \mu_{T,I,F}(0 * (ŧ * \beta))\} \leq ɤ.max\{\lambda_{T,I,F}(ţ * \alpha), \mu_{T,I,F}(ŧ * \beta)\} = max\{\lambda_{T,I,F}(ţ * \alpha).ɤ, \mu_{T,I,F}(ŧ * \beta).ɤ\} = max\{{}^{M}_{ɤ}\lambda_{T,I,F}(ţ * \alpha), {}^{M}_{ɤ}\mu_{T,I,F}(ŧ * \beta)\} = ({}^{M}_{ɤ}\lambda_{T,I,F} \times {}^{M}_{ɤ}\mu_{T,I,F})(ţ * \alpha, ŧ * \beta)$. Hence, ${}^{M}_{ɤ}Ƕ \times {}^{M}_{ɤ}Ƒ$ is a neutrosophic cubic closed normal ideal of $X \times Y$.

6. Conclusion

In this paper, the notion of ɤ-multiplication of neutrosophic cubic set was introduced and ɤ-multiplication was studies by several useful results. This study will provide the base for further work like t-neutrosophic soft cubic and intuitionistic soft cubic set etc

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