h8

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Part I

Computation of face value and derivative estimates for Slice3D Vlasov solver.

This sheet contains up to eight order estimates. In White et al. PQM paper up sixth order estimates are give. See that paper for more background.

"A high-order finite volume remapping scheme for nonuniform grids: The piecewise quartic method (PQM)", White L. and Adcroft A., J. Comp. Phys., 227, 7394–7422, 2008

In [3]:
```
from sympy import *
init_printing()
```

rho_hat is the density function, which when integrated gives us our density values in the cells.

x is the normalized coordinate, x=(v-v_{i-1/2})/dv, where v_{i-1/2} is the velocity at the left face of the center cell i

cf is center cell volume average p1f, p2f, p3f, ... are the cell volume averages of cells in the positive direction at a distance of 1, 2, 3 m1f, m2f, m3f, ... are the cell volume averages of cells in the negative direction at a distance of 1, 2, 38th order left-face value estimate: h8

In [4]:
```
a,b,c,d,e,f,g,h=symbols('a b c d e f g h ')
v,dv=symbols('v dv')
m4f,m3f,m2f, m1f,cf,p1f,p2f, p3f =symbols('m4f m3f m2f m1f cf p1f p2f p3f  ')

rho_hat=a+b*v+c*v**2 + d*v**3 + e*v**4 + f*v**5 + g*v**6 + h*v**7

ans_a=solve([
             integrate(rho_hat,(v,-4,-3))-m4f,
             integrate(rho_hat,(v,-3,-2))-m3f,
             integrate(rho_hat,(v,-2,-1))-m2f,
             integrate(rho_hat,(v,-1,0))-m1f,
             integrate(rho_hat,(v,0,1))-cf,
             integrate(rho_hat,(v,1,2))-p1f,
             integrate(rho_hat,(v,2,3))-p2f,
             integrate(rho_hat,(v,3,4))-p3f ] ,[a,b,c,d,e,f,g,h])

rho_hat_ans=rho_hat.subs([(a,ans_a[a]),
                          (b,ans_a[b]),
                          (c,ans_a[c]),
                          (d,ans_a[d]),
                          (e,ans_a[e]),
                          (f,ans_a[f]),
```

```
                                (g,ans_a[g]),
                                (h,ans_a[h])])
#collect(rho_hat_ans,v)

simplify(rho_hat_ans.subs(v,0))*840
```

Out [4]:

$$533cf + 533m1f - 139m2f + 29m3f - 3m4f - 139p1f + 29p2f - 3p3f$$

Now compute left-face 7:th order derivative estimates: dh7

In [63]:

```
d_rho_left=simplify(diff(rho_hat_ans,v).subs(v,0))
d_rho_left*5040
```

Out [63]:

$$7175cf - 7175m1f + 889m2f - 119m3f + 9m4f - 889p1f + 119p2f - 9p3f$$

H6 left face estimate

In [64]:

```
a,b,c,d,e,f=symbols('a b c d e f')
v,dv=symbols('v dv')
m3f,m2f, m1f,cf,p1f,p2f=symbols('m3f m2f m1f cf p1f p2f')

rho_hat=a+b*v+c*v**2 + d*v**3 + e*v**4 + f*v**5

ans_a=solve([
                 integrate(rho_hat,(v,-3,-2))-m3f,
          integrate(rho_hat,(v,-2,-1))-m2f,
          integrate(rho_hat,(v,-1,0))-m1f,
          integrate(rho_hat,(v,0,1))-cf,
          integrate(rho_hat,(v,1,2))-p1f,
          integrate(rho_hat,(v,2,3))-p2f ] ,[a,b,c,d,e,f])

rho_hat_ans=rho_hat.subs([(a,ans_a[a]),(b,ans_a[b]),(c,ans_a[c]),(d,ans_a[d]),(e,ans_a
#collect(rho_hat_ans,v)

simplify(rho_hat_ans.subs(v,0))
```

Out [64]:

$$\frac{37cf}{60} + \frac{37m1f}{60} - \frac{2m2f}{15} + \frac{m3f}{60} - \frac{2p1f}{15} + \frac{p2f}{60}$$

Now compute dh5 left estimate (depends on what order rho_hat ans is, h6 gives dh5)

In [68]:

```
d_rho_left=simplify(diff(rho_hat_ans,v).subs(v,0))
d_rho_left*180
```

Out [68]:

$$245cf - 245m1f + 25m2f - 2m3f - 25p1f + 2p2f$$

H4 face value estimate

In [71]:

```
a,b,c,d=symbols('a b c d ')
v,dv=symbols('v dv')
m2f, m1f,cf,p1f=symbols(' m2f m1f cf p1f ')

rho_hat=a+b*v+c*v**2 + d*v**3

ans_a=solve([
          integrate(rho_hat,(v,-2,-1))-m2f,
          integrate(rho_hat,(v,-1,0))-m1f,
          integrate(rho_hat,(v,0,1))-cf,
          integrate(rho_hat,(v,1,2))-p1f] ,[a,b,c,d])

rho_hat_ans=rho_hat.subs([(a,ans_a[a]),(b,ans_a[b]),(c,ans_a[c]),(d,ans_a[d])])
#collect(rho_hat_ans,v)

simplify(rho_hat_ans.subs(v,0))
```

Out [71]:

$$\frac{7cf}{12} + \frac{7m1f}{12} - \frac{m2f}{12} - \frac{p1f}{12}$$

H3 face derivative estimate

In [72]:

```
d_rho_left=simplify(diff(rho_hat_ans,v).subs(v,0))
d_rho_left
```

Out [72]:

$$\frac{5cf}{4} - \frac{5m1f}{4} + \frac{m2f}{12} - \frac{p1f}{12}$$