ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

Solving Vlasov with FVM: Preliminary Results

QuESpace Science Club

29.1.2010

A. Sandroos & L. Daldorff

Outline

- **Introduction**
- **1D Advection**
- **1D1V tests**
- **Conclusions**

Introduction

Vlasov equation (1d1v): $\partial_t f + v\partial_x f + a(x,v,t)\partial_v f = 0$

- Advection
- If $f(t=0) \geqslant 0$, $f(t) > 0$, i.e. *f* remains non-negative.
- Mass, momentum, energy, entropy, etc. conserved.

Task: Compare FVM methods vs. Semi-Lagrangian ones.

- Conservative properties.
- Diffusive properties of solvers (accuracy).
- Feasibility for 3d3v simulations (complexity, time consumption, ...).

Introduction

There are many different FVM schemes, but they all work with the same principle:

•Volume averages gridded: $f_i = \frac{1}{\Delta x \Delta v} \int_{cell} f(x,v)\, dx\, dv$

•Reconstruct f_i by using a polynome P(x,v) of degree *n* in cell *ij*.

•Calculate fluxes *H* at cell boundaries using rec. values.

•Propagate: $$\frac{df_{ij}}{dt} = -\frac{H^x_{i+0.5} - H^x_{i-0.5}}{\Delta x} - \frac{H^v_{j+0.5} - H^v_{j-0.5}}{\Delta v}$$

•The mass is trivially conserved.

Introduction

Reconstruction made using polynomials

$$f(x) = A + Bx + Cx^2$$

$$f(x) = f_i + \frac{\epsilon^U}{6\Delta x}\left[2(x-x_i)(x-x_{i-3/2}) + (x-x_{i-1/2})(x-x_{i-1/2})\right](f_{i+1}-f_i)$$
$$- \frac{\epsilon^L}{6\Delta x}\left[2(x-x_i)(x-x_{i+3/2}) + (x-x_{i-1/2})(x-x_{i+1/2})\right](f_i-f_{i-1})$$

$$f(x) = f_i - \frac{1}{24}\left[\epsilon^U(f_{i+1}-f_i) - \epsilon^L(f_i-f_{i-1})\right] + \frac{1}{2}\left[\epsilon^U(f_{i+1}-f_i) + \epsilon^L(f_i-f_{i-1})\right]x + \frac{1}{2}[\cdots]x^2$$

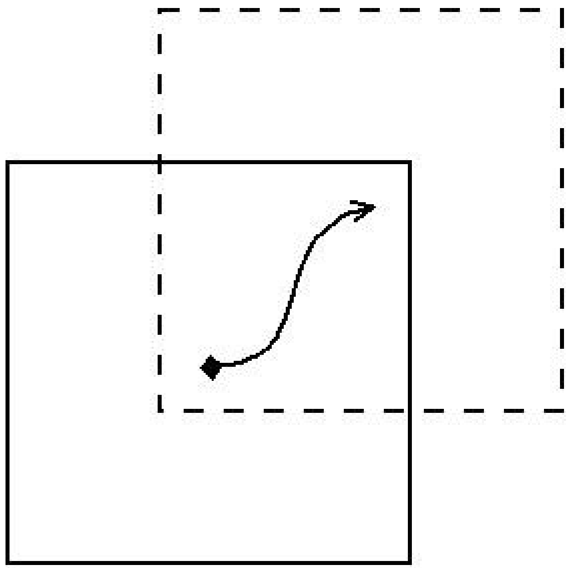
Introduction

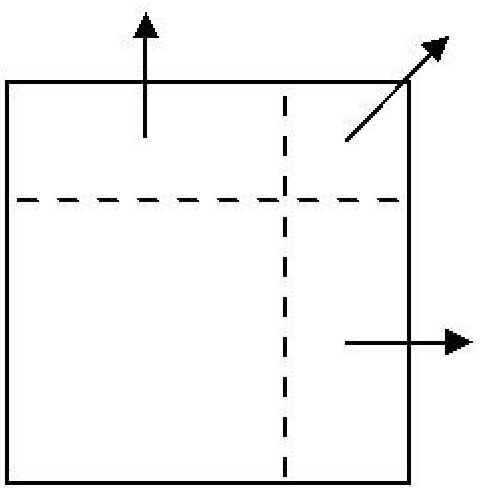
Corner Transport Method:

•Solve characteristics forward in time → *dx* & *dv*.

•Integrate *df* using reconstruction (polynomial integration).

•Can use much larger time step than with flux-based methods

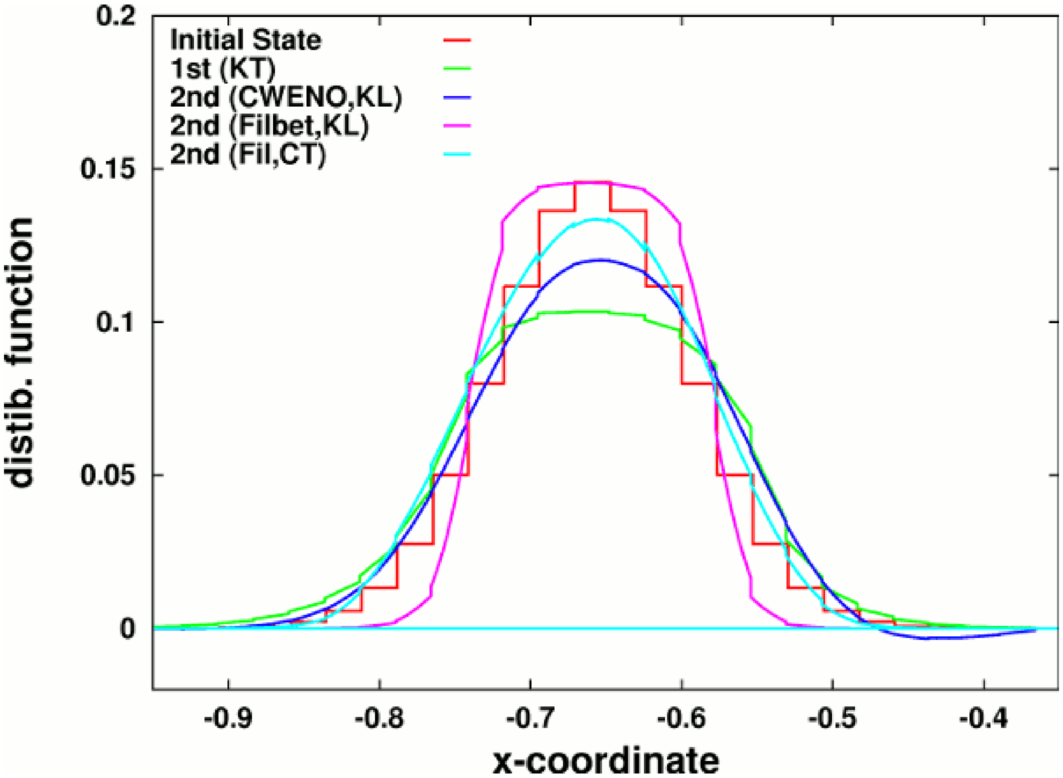




ILMATIETEEN LAITOS
METEOROLOGISKA INSTITUTET
FINNISH METEOROLOGICAL INSTITUTE

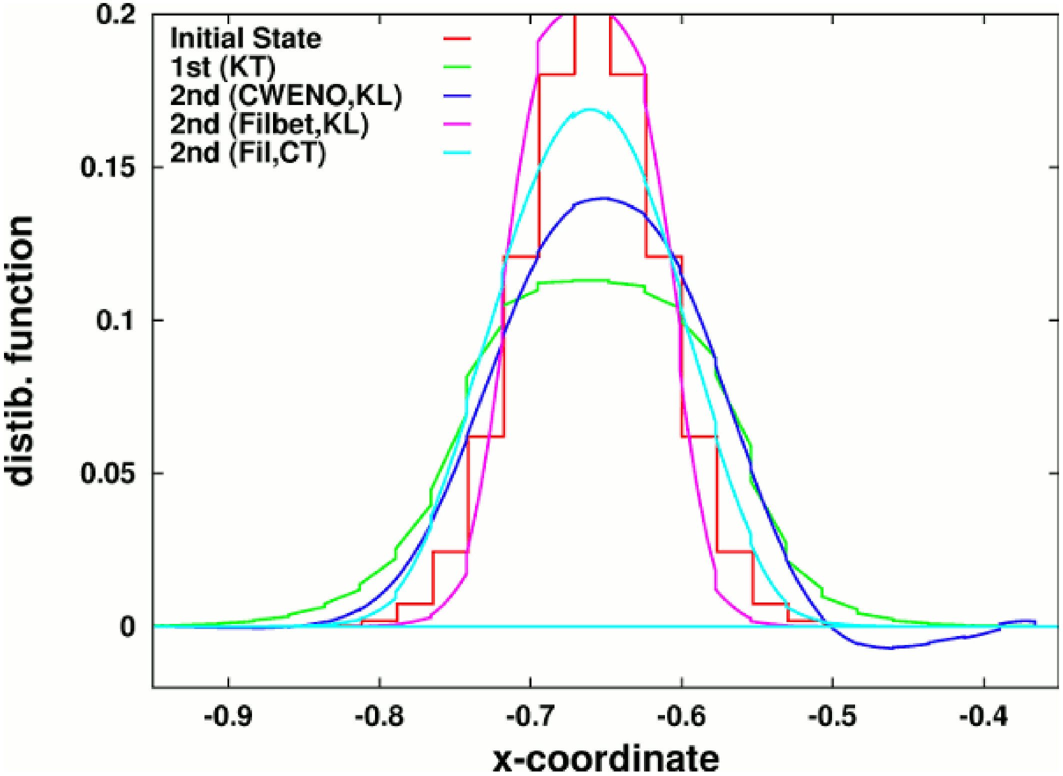
1D Advection

Gaussian pulse, 150 time steps, (v dt) / dx = 0.39



1D Advection

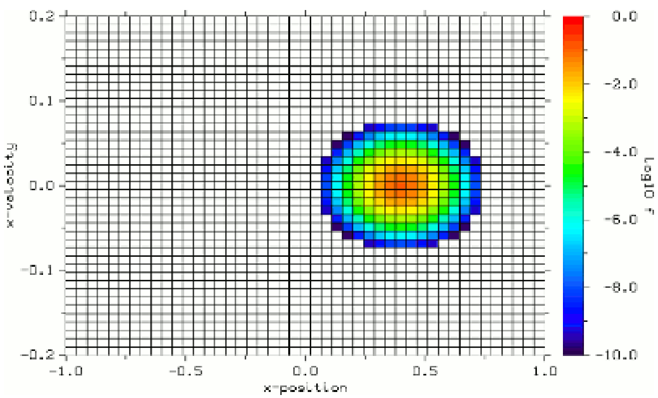
Gaussian pulse, 150 time steps, (v dt) / dx = 0.39



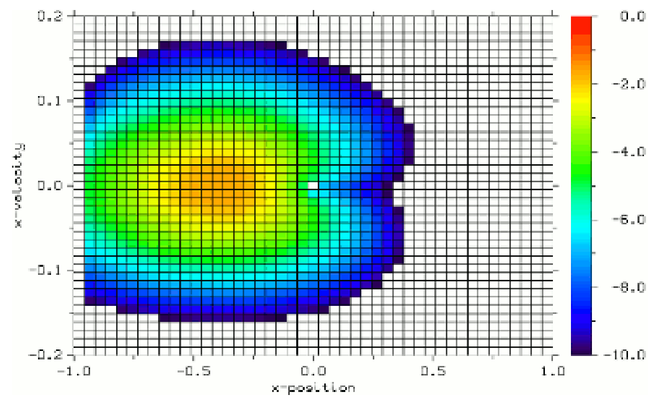
1D1V Harmonic Oscillator

Gaussian pulse, half period, 200 time steps.

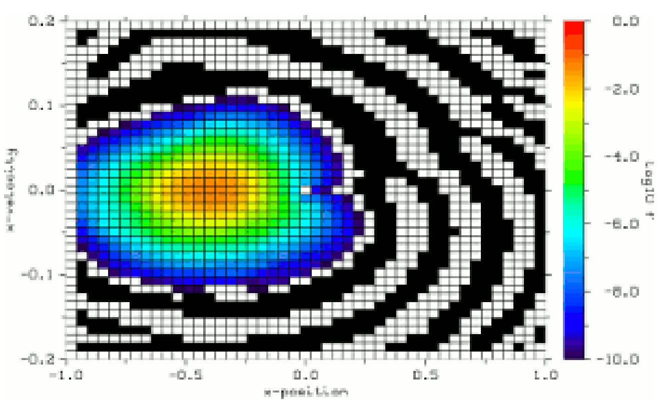
Init. state



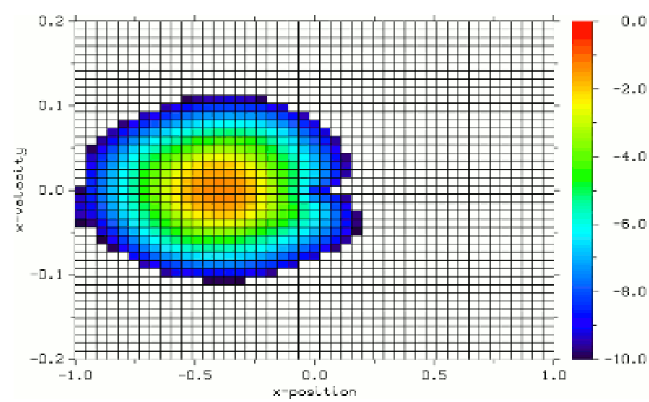
1st order (KT)



2nd (Filbet)



2nd (CT)



Conclusions

•Reconstruction will be time consuming, same for FVM and semi-Lagrangian methods.

•*f* > 0 only if reconstruction is positive, problem for all solvers.

•Have to use at least 3rd order method.

•Considerable diffusion → acceleration.

•CT method seems to be the most promising one, can use rather large time steps.