Constraint-based Gait Control Using Differential Inequalities

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***Abstract*— This paper proposes a bipedal walking controller based on quadratic programming and differential inequalities to keep the gait balance during the complete walking cycle. We design the differential inequalities constraints using geometric primitives within the dual quaternion algebra. Simulations on the lower body of the Poppy humanoid robot show the effectiveness of the approach.**

I. Introduction

Besides being a very complex mechanism, humanoids have larger workspace than fixed-base robots thanks to its mobility, which allows them to perform a wider range of tasks. However, this mobility also represents a challenge since the robot must maintain balance. The bipedal walking is divided into two parts for each step: the single (SSP) and the double support (DSP) phases [3]. In the first phase, one foot is fixed on the floor while the other one swings towards the desired pose; in the second phase, both feet are on the floor. Besides the feet pose control, balance must be taken into account to prevent the robot from falling.

Vukobratovic and Juricic [7] propose an implicit concept of Zero Moment Point (ZMP) to address the problem of stable biped locomotion. ZMP is the point where all the active forces acting on the mechanism can be replaced by a single force and the horizontal components of the resultant moment is zero. This strategy allows to perform balance tasks [6] and the control law must keep the ZMP inside the convex hull shaped by the supporting foot in the SSP or the area inside both feet outer borders in the DSP. Kajita et al. [4] compute the projection of the center of mass (CoM) reference through a predictive controller using the ZMP. However, the method is conservative since it limits the CoM to a single offline-generated trajectory, whereas the allowed area is the entire convex hull.

In this work, we propose a walking controller using quadratic programming (QP) with differential inequality constraints [5] to ensure that the projection of the CoM be always inside the support polygon while the feet moves to ensure the gait execution. The convex hull is modeled by geometric primitives, such as planes and infinite cylinders made of Plücker lines [1].

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II. Geometric primitives for the differential inequalities

To ensure that the projection of the CoM always stays inside the support polygon, we approximate it by simple geometric primitives that can later be associated to appropriate differential inequalities [5].

A. Single support phase

In the SSP, one foot stays rigidly attached to the ground with the whole sole in contact with the floor, while the other foot swings towards the desired pose. During the complete step, the projection of the CoM must be inside the convex hull associated with the foot sole on the ground. For that aim, we use a cylinder centered in the geometric center of the supporting foot, with a conservative radius r_{com} smaller than the foot boundaries, as shown in Fig. 1. The cylinder main axis is represented by a Plücker line and to keep the center of mass inside the allowable region, we impose a line-point distance constraint [5].

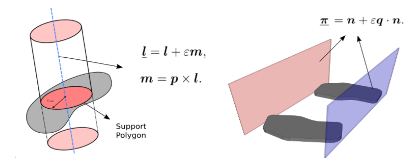

Fig. 1. Support polygon of each support phase. On the *right*, DSP support polygon, constrained to pink and blue planes. On the *left*, SSP support polygon with the projection of the cylinder of radius r_{com} inside the support foot sole.

B. Double support phase

During the DSP, both feet are in contact with the floor. The goal is to transfer the CoM from one foot to the next support foot. It is important to keep both feet in contact with the ground, while preventing them from sliding through the floor, but this latter restriction can be relaxed. Therefore, we allow the swinging foot to slide inside a very small area instead of being rigidly attached to the floor. In this sense, we define this area by using the intersection between two primitives: a plane on the floor, which is used with an equality constraint to ensure the foot contact with the ground, and a vertical cylinder, which is used with an inequality constraint to provide some freedom for the foot to slide.

The task also requires two other inequality constraints associated with vertical planes to keep the CoM inside the convex hull as shown in Fig. 1.

III. CONTROL STRATEGY

Our control strategy is based on a QP controller with differential inequalities, which are based on geometric primitives described within the dual quaternion algebra [5]. The SSP controller minimizes the foot pose error, $\tilde{\boldsymbol{x}}_{\text{foot}} \in \mathbb{R}^8$, where $\tilde{\boldsymbol{x}}_{\text{foot}} \triangleq \operatorname{vec}_8 \tilde{\underline{\boldsymbol{x}}}_{\text{foot}}$ [1] and $\tilde{\underline{\boldsymbol{x}}}_{\text{foot}} \in \text{Spin}(3) \ltimes \mathbb{R}^3$, whereas the DSP controller minimizes the projection of the CoM error, $\tilde{\boldsymbol{x}}_{\text{com}} \in \mathbb{R}^3$. Additionally, both controllers also minimize the velocity of the joints. Considering a robot with n degrees of freedom, the control input is given by

$$\begin{aligned} \boldsymbol{u} \in \underset{\dot{\boldsymbol{q}}}{\operatorname{argmin}} \quad & \|\boldsymbol{J}\dot{\boldsymbol{q}} + \eta\tilde{\boldsymbol{x}}\|_2^2 + \lambda^2 \|\dot{\boldsymbol{q}}\|_2^2 \\ \text{subject to} \quad & \boldsymbol{W}\dot{\boldsymbol{q}} \leq \boldsymbol{w} \\ & \boldsymbol{W}_{eq}\dot{\boldsymbol{q}} = \boldsymbol{w}_{eq}, \end{aligned} \tag{1}$$

where $\boldsymbol{J}$ is either the robot pose Jacobian, $\boldsymbol{J}_{\text{pose}} \in \mathbb{R}^{8\times n}$, in the SSP, or the robot CoM Jacobian, $\boldsymbol{J}_{\text{CoM}} \in \mathbb{R}^{3\times n}$, during the DSP. The vector $\boldsymbol{q} \in \mathbb{R}^n$ is the joints variables, $\eta \in (0, \infty)$ is the controller gain, and $\lambda \in (0, \infty)$ is the damping factor. Furthermore, the matrix $\boldsymbol{W} \in \mathbb{R}^{m\times n}$ and the vector $\boldsymbol{w} \in \mathbb{R}^m$ impose m inequality constraints to the quadratic controller whereas $\boldsymbol{W}_{eq} \in \mathbb{R}^{l\times n}$ and $\boldsymbol{w}_{eq} \in \mathbb{R}^l$ impose l equality constraints. Since we use differential inequalities written as $\dot{\tilde{d}}(\boldsymbol{q}) \leq -\eta\tilde{d}(\boldsymbol{q})$, then we always find the equivalent inequality $\boldsymbol{J}_d\dot{\boldsymbol{q}} \leq -\eta\tilde{d}(\boldsymbol{q})$, because $\dot{\tilde{d}}(\boldsymbol{q}) = \boldsymbol{J}_d\dot{\boldsymbol{q}}$; therefore, $\boldsymbol{W} \triangleq \boldsymbol{J}_d$ and $\boldsymbol{w} \triangleq \tilde{d}(\boldsymbol{q})$ [5].

IV. RESULTS

We validated our control strategy in simulation, which was performed with MATLAB software using the computational library DQ Robotics [2]. We used the model of the lower body part of the Poppy humanoid robot[1] and assumed that the CoM always starts inside the allowable region.

A. Single support phase

We model a subset of the support polygon as a vertical cylinder with a 5 cm radius and main axis passing through the foot centroid. To maintain balance, the CoM is enforced to be inside this cylinder by using a line-point constraint and its corresponding differential inequality [5]. Fig. 2 shows that the CoM always remained inside the cylinder, except at 39 s (iteration 48), when it trespassed the cylinder boundary by 5×10^{-4} m, due to discretization. This is not a problem because the region defined by the intersection of the cylinder and the floor plane is a conservative subset of the support polygon; therefore, such small violations do not affect the robot balance. Moreover, the foot pose converged to the desired pose, as shown by the left image in Fig. 2.

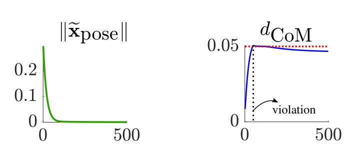

Fig. 2. Results of the SSP. On the *left*, convergence of the norm of the foot pose error. On the *right*, distance of the CoM to the cylinder main axis in *solid-blue* line and cylinder boundary in *dashed-red* line.

[1] https://www.poppy-project.org/en/robots/poppy-humanoid

B. Double support phase

During the DSP, the foot is constrained to the floor plane but is allowed to slide inside a circle of radius equal to 4 cm. Fig. 3 shows that the distance of the foot to the floor plane was 0.2 mm at $t = 0$ s, which is beyond the solver tolerance,[2] albeit very small, but the foot respected the line-point constraint and its reference point remained inside the allowable region composed of a circle with a 4 cm radius. Finally, we constrained the CoM between the two planes shown in Fig. 1 and Fig. 3 shows that the distance of the CoM to those planes were always greater than zero. Last, since $\tilde{\boldsymbol{x}}_{\text{CoM}}$ goes asymptotically to zero, the CoM converges to the desired value.

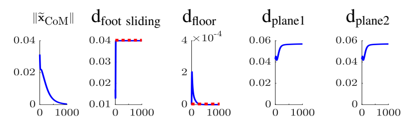

Fig. 3. Results of the DSP. From *left* to *right*: norm of the foot pose error; foot pose distance from cylinder main axis in *solid blue* and the corresponding boundary in *dashed-red* line; foot pose distance from the floor plane in *blue solid line;* distance of the CoM from plane 1; and CoM distance from plane 2.

V. CONCLUSION

We proposed a constraint-based controller to solve the problem of quasi-static gait control. The method has much simpler implementation than the one proposed by Kajita et al. [4] and also is less conservative because the CoM can move arbitrarily inside the support polygon, instead of being limited to a fixed trajectory. Both the CoM and the foot respected all imposed constraints during the entire step with the discretization problem playing a negligible role, which ensures a stable gait. The next step of this project is to implement this method on the Poppy humanoid robot.

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[2] The solver tolerance for equality constraints is 10^{-6}, whereas our violation was 2×10^{-4} m, which is very small but greater than the tolerance. Further investigations will be conducted in order to discover why the solver is returning a feasible solution when the violation is clearly greater than the tolerance.