

# Denoising of PMU Measurements for Accurate Calculation of Transmission Line Parameters

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**Abstract**—Power system transmission line parameters are essential in monitoring and control applications. More specifically, the values of the transmission line parameters are used in state estimation, contingency analysis, and as settings in protection relays. However, the stored transmission line parameters in the control center database often deviate from their actual values, impacting negatively the applications of the power system control center. The use of PMU (Phasor Measurement Unit) measurements is the most convenient and simple way for refining the parameters of the transmission line; however, the contamination of the PMU measurements with noise from the instrument transformers and the communication channels could sometimes deteriorate the accuracy of the calculated parameters. In this paper, the denoising of the PMU measurements using wavelets is proposed prior to the calculation of the transmission line parameters. The proposed approach is used for calculating the transmission line parameters of the IEEE 14-bus system and the results are extensively discussed.

**Index Terms**—Discrete wavelet transform; noise; phasor measurement unit; signal denoising; transmission lines

## I. INTRODUCTION

The transmission line parameters are used in several monitoring and control applications of the control center and they are certainly one of the most important elements of the power system model. The line parameters are assumed time invariant and are usually stored in the control center database, which often contains parameters that deviate from the actual line parameters. Surveys have shown that the stored line parameters could have up to 30% deviation from the actual ones [1]. Such errors could exist in some control center databases due to the calculation of the parameters based on manufacturers data and ideal transmission line structures. Furthermore, environmental factors (i.e., ambient temperature, resistivity of the soil), modelling inaccuracies (i.e., parallel lines and ends of an overhead line that changes to underground cable), and human errors (i.e., line length miscalculation) also cause a considerable deviation of the stored line parameters from the actual ones.

The erroneous transmission line parameters impact negatively the monitoring and control application of power systems. For instance, the presence of several erroneous

transmission line parameters has more severe deterioration of the state estimation accuracy than the presence of noisy measurements [2], while the erroneous transmission line parameters could affect the outcomes of the voltage and angle stability analysis. In this sense, the regular refinement of the transmission line parameters is an important task that must be undertaken by the electric utilities.

There are several methodologies in the literature that deal with the identification of the lines that might be modelled with erroneous parameters. Usually, such methodologies are based on the outcome of a state estimator and the calculation of the normalized measurement residuals [3], [4]. However, the use of synchronized phasor measurements (voltage and current phasors) provided by PMUs (Phasor Measurement Units) is probably the most direct approach to refine erroneous transmission line parameters. The drawback with the use of synchronized phasor measurements is still the limited availability of the PMUs in the power system measurement layer, given that for the calculation of the line parameters two PMUs are required (one at each end of the line). Although such a case is not realistic today, measurement campaigns can be organized by the electric utilities where mobile PMUs can be installed at the two ends of a “suspicious” transmission line to collect the synchronized phasor measurements and then reinstall the PMUs at the ends of another line.

One can claim that the transmission line parameters can be calculated accurately since the PMU is considered a quite accurate measurement device. Even though this is true, the PMU measurements might not be as accurate as one expects in case that the instrument transformers that are connected to the PMU are not taken into consideration. In particular, the instrument transformers that are used to level down the voltage and the current, are categorized in accuracy classes according to the error that introduce to the measurand. Therefore, a low accuracy class transformer could introduce a large noise to the PMU measurements and therefore deteriorate the accuracy of the calculated line parameters [5].

In reality, several substations are equipped with low accuracy class instrument transformers and therefore it is not possible to obtain extremely accurate measurements from all the PMUs. In this paper, in order to avoid the miscalculation of the line parameters through PMU measurements due to noise, a

denoising technique is used for PMU measurements pre-processing. The PMU measurements denoising is performed using the discrete wavelet transformation, in which the signal (the measured data) are reconstructed through the wavelet coefficients. In this way, the level of noise in the reconstructed signal (measurement set) is much lower than the original one. It should be noted that the aim of this paper is to investigate the impact of denoising the PMU measurements prior to the calculation of the line parameters. In this sense, the use of other denoising techniques than the wavelet transformation is also possible.

The proposed approach which denoises the PMU measurement set prior to the calculation of the line parameters is tested in the IEEE 14-bus system. The rest of the paper is organized as follows: Section II briefly describes the line parameter calculation, while in Section III the sources of errors in a PMU measurement chain are described. Section IV introduces the concept of the wavelet and how wavelets can be used for denoising PMU measurements, while the results of the PMU measurement denoising in the calculation of the line parameters of the IEEE 14-bus system are shown in Section V. The paper concludes in Section VI.

## II. LINE PARAMETER CALCULATION USING PMU MEASUREMENTS

In this work the transmission line is represented as an equivalent pi-model. The corresponding model that is shown in Fig. 1, is described by four parameters, namely the series conductance ( $g_{sr}$ ), the series susceptance ( $b_{sr}$ ), the shunt conductance ( $g_{sh}$ ), and the shunt susceptance ( $b_{sh}$ ). Usually, the shunt conductance is very small and therefore it is neglected from the equivalent pi model of Fig. 1. With the presence of two PMUs at both ends of the line (Fig. 1), the synchronized current and voltage phasors can be used for calculating the series admittance  $y_{sr}$  and the shunt admittance  $y_{sh}$  as,

$$y_{sr} = \frac{\tilde{I}_s \tilde{V}_r + \tilde{V}_s \tilde{I}_r}{\tilde{V}_s^2 - \tilde{V}_r^2} \quad (1)$$

$$y_{sh} = \frac{\tilde{I}_s - \tilde{I}_r}{\tilde{V}_s + \tilde{V}_r} \quad (2)$$

where  $\tilde{V}_s$  and  $\tilde{V}_r$  are the voltage phasors of buses  $s$  and  $r$  respectively;  $\tilde{I}_s$  is the current phasor that flows from bus  $s$  and  $\tilde{I}_r$  is the current phasor that arrives to bus  $r$  as shown in Fig. 1.

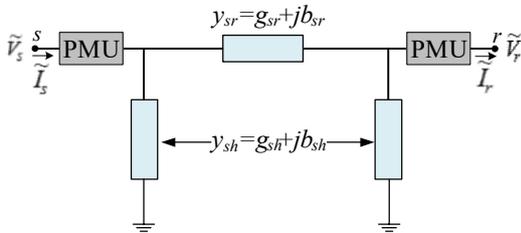


Fig. 1. Transmission line representation using a pi-model

## III. PMU MEASUREMENT CHAIN AND THE ASSOCIATED MEASUREMENT UNCERTAINTIES

The PMUs are installed in the power system substations through voltage and current transformers that level down the high voltage and current magnitude. A typical measurement chain consists of the instrument transformers (voltage and current), the PMU, and the cables/joints that connect the PMU to the instrument transformers. It should be noted that the PMU usually contains anti-aliasing filters, processing units, and analog to digital converters. The measurement chain is not ideal and therefore its components introduce a characteristic measurement uncertainty to the measurement, making the measurement to deviate from the actual one.

There are two ways for evaluating the measurement uncertainty, namely the Type A and Type B evaluation methods [6]. In the Type A evaluation method, the measurement uncertainty is approximated through the repeated measurements that are obtained from the measurement chain, under the same conditions; while, in the Type B evaluation method, the measurement uncertainty is calculated from the accuracy level of the measurement device specified by the manufacturers.

Since the operating condition of the power systems changes continuously, the Type A evaluation method is not appropriate for evaluating the measurement uncertainty in a PMU measurement chain. The Type B evaluation method also has a limitation: it can be applied in case that the manufacturer data specifies the accuracy level of the equipment. For some of the measurement chain components, i.e. anti-aliasing filters, A/D converters, and cables, it is not easy to approximate their contributions to the overall uncertainty of the measurement using the Type B evaluation method. This is not the case with the PMU and the instrument transformer where their measurement error is available in the manufacturer data sheet. In this sense, a simplified measurement chain shown in Fig. 2 is used for considering the measurement uncertainty in the PMU measurement chain.

Based on the simplified measurement chain, the PMU measurement is affected by the error of the instrument transformers and the error of the PMU. In particular, the voltage and current transformer are categorized into accuracy classes according to the error that is introduced to the measurand. The PMU itself is considered one of the most accurate measurement devices since its maximum error that is introduced to the voltage and current phasor measurements is quite small. However, in order to assess the accuracy of the PMU measurements one should consider both the instrument transformers and the PMU errors.

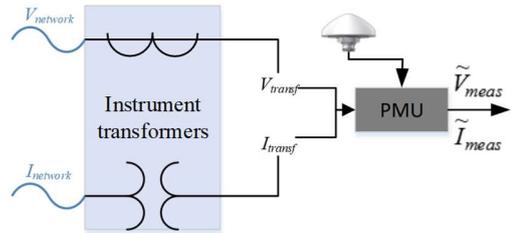


Fig. 2. Simplified measurement chain with a PMU

More specifically, it was shown in [7], that PMU measurements obtained through a measurement chain with a low accuracy class instrument transformer, deteriorate the state estimator accuracy no matter if PMU measurements are used in the state estimator. Further, in [5], it was shown that the accuracy class of the instrument transformer affects the accuracy of the calculated line parameters using PMU measurements. It should be noted that both in [5] and [7] as well as in this work, the effect of the PMU measurement chain is examined by considering only random errors (noise), assuming that any systematic errors (i.e., uncalibrated measurement device) are identified and corrected.

Based on the outcomes of [5], the use of PMU measurements does not guarantee accurate calculation of the line parameters especially in the case that low accuracy instrument transformers exist in the measurement chain. This is because the overall additive noise of the PMU measurements is quite large, and thus affect directly the calculated line parameters obtained through (1) and (2). Therefore, in this work the denoising of the PMU measurements prior to line parameter calculation is proposed. In this paper the discrete wavelet transformation is used for denoising PMU measurements, but other methods can also be used for measurement denoising.

#### IV. DENOISING USING THE DISCRETE WAVELET TRANSFORMATION

The family of wavelets consists of functions that have the following features: 1) they must integrate to zero when waving above and below the x axis, 2) they must be square integrable or equivalently have finite energy. The wavelets are used to decompose a signal to various wavelet coefficients. Unlike the Fourier transform that operates in the frequency domain and thus cannot provide simultaneously time and frequency information, wavelets are a time-frequency transformation. The wavelet transformation considers a mother wavelet (wavelet function) as the basis function and it localizes in the frequency domain through the dilations (magnitude scaling) of the mother wavelet, and in time through the translations (time shifting) of the mother wavelet. Another important advantage of the wavelet transform over the Fourier transform is its computational efficiency, where the Fourier transform has a complexity of  $O(n \log_2(n))$  while the wavelet transform has a complexity of  $O(n)$ .

The principle of the wavelet transformation is based on the dilations and translations of the mother wavelet. Assuming that the mother wavelet function in the time domain ( $t$ ) is  $\psi(t)$ , the translated and dilated versions of  $\psi(t)$  can be expressed as,

$$\psi_{jk}(t) = 2^{-j/2} \psi(2^{-j}t - k) \quad (3)$$

There are two types of wavelet transforms, namely the continuous and the discrete wavelet transforms. In the continuous wavelet transform, the signal  $x(t)$  is convolved with a set of basis functions obtained by translations and dilations of the mother wavelet. Actually, the mother wavelet function is first shifted across the signal and a correlation coefficient is obtained in each shift until the end of the signal. Then, the mother wavelet is scaled and shifted again across the whole signal for obtaining the next set of correlation coefficients. It is obvious that through the continuous wavelet transform there are infinite dilations of the mother wavelet.

On the contrary, the discrete wavelet transform uses a discrete set of scales and shifts to extract the wavelet coefficients. It is proved that when the dilations and translations are chosen based on powers of two (dyadic decimation), the discrete wavelet transform is much more efficient than the continuous wavelet transform and as accurate as the continuous wavelet transform [8]. The discrete wavelet transform can be viewed as a form of multiresolution analysis, which allows a multiscale analysis of a signal. More particularly, the signal is passed through a high pass and a low pass filter at each level while the signal is down-sampled as per the Nyquist criterion.

In Fig. 3, a discrete wavelet transformation of a signal  $x(n)$  is shown. The signal consists of 512 samples when is convolved with a High Pass Filter (HPF) and a Low Pass Filter (LPF). The HPF is described by the wavelet function  $\psi_{jk}$  while the LPF by the scaling function  $\phi_k$  (according to the multiresolution analysis). The coefficients obtained from the convolution of the signal with the HPF are called detail coefficients, while the ones that are obtained from the LPF approximation coefficients. The coefficients are then down-sampled using dyadic decimation.

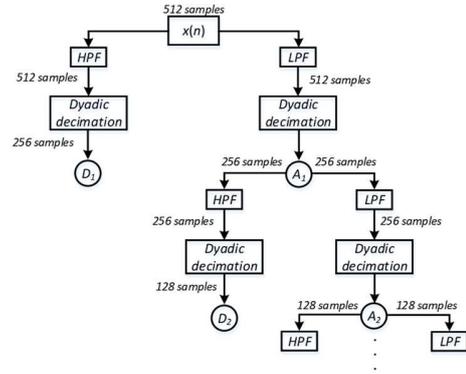


Fig. 3. Discrete wavelet transform through multiresolution analysis

As it is shown in Fig. 3, the approximation coefficients are further filtered to the second level resulting to another set of approximation coefficients ( $A_2$ ) and detail coefficients ( $D_2$ ). This multiresolution analysis can continue until the length of the signal data is equal to  $2^N$ , where  $N$  is the level of decomposition [9]. The signal can be reconstructed by taking the detail coefficients obtained by all the decomposition levels and the approximation coefficients at the last level of decomposition. Assuming that the resolution level is until level 2 (as in Fig. 3), the signal  $x(n)$  can be reconstructed as,

$$x(n) = \sum_{k=-\infty}^{\infty} A_{2k} \phi_{2k}(n) + \sum_{j=1}^2 \sum_{k=-\infty}^{\infty} D_{jk} \psi_{jk}(n) \quad (4)$$

According to (4), the reconstructed signal is highly dependent on the detail coefficients, therefore discarding the noise coefficients is very likely to denoise the reconstructed signal. The coefficients of the actual signal are very likely to have a large value while the noise coefficients have a small value. In this sense the denoising of the signal using the discrete wavelet transform can be performed by setting the coefficients that are smaller than a certain threshold to zero and using the large coefficients for the signal reconstruction. There are two ways of thresholding the detail coefficients, namely the soft and hard thresholding; in both cases the coefficients that are below

a certain threshold are set to zero. In the case of the soft thresholding the remaining coefficients (that are above the threshold) are shrunk closer to the threshold by subtracting the threshold value, while in the case of the hard thresholding the respective coefficients remain unchanged.

## V. ESTIMATION OF LINE PARAMETERS USING PMU MEASUREMENT DENOISING

In this work the denoising properties of the wavelet coefficients are applied to the PMU measurements that are used for transmission line parameters calculation. The IEEE 14-bus system is used as an example, but the methodology is not system specific. The actual line parameters (resistance and reactance) that are used in the DigSILENT software are assumed to vary with ambient temperature throughout the day, while the loading condition of the system varies throughout the day too, as it is shown in Fig. 4. The PMU measurements are generated by adding uniform error ( $U$ ) to the simulated phasor values obtained from DigSILENT considering the simplified measurement chain of Fig. 2 as,

$$V_{meas} = \underbrace{V_{network}(1+U(\bar{e}_{VT}^V))}_{V_{transf}} + V_{transf}U(\bar{e}_{PMU}^V) \quad (5)$$

$$I_{meas} = \underbrace{I_{network}(1+U(\bar{e}_{CT}^I))}_{I_{transf}} + I_{transf}U(\bar{e}_{PMU}^I) \quad (6)$$

$$\theta_{meas}^V = \theta_{network}^V + U(\bar{e}_{VT}^{\theta_V}) + U(\bar{e}_{PMU}^{\theta_V}) \quad (7)$$

$$\theta_{meas}^I = \theta_{network}^I + U(\bar{e}_{CT}^{\theta_I}) + U(\bar{e}_{PMU}^{\theta_I}), \quad (8)$$

where  $\bar{e}_{VT}^V$ ,  $\bar{e}_{CT}^I$ ,  $\bar{e}_{PMU}^V$ ,  $\bar{e}_{PMU}^I$ ,  $\bar{e}_{VT}^{\theta_V}$ ,  $\bar{e}_{CT}^{\theta_I}$ ,  $\bar{e}_{PMU}^{\theta_V}$ ,  $\bar{e}_{PMU}^{\theta_I}$  are the instrument transformers and the PMU magnitude and angle errors defined by the manufacturers and are shown in Tables I and II respectively. It should be noted that in this work the instrument transformers are assumed to belong to the 0.5 accuracy class. Further, the maximum errors are used as the bounds of the uniform distribution.

Since the line parameters can be calculated offline, the voltage and current phasor measurements for a whole day can be first collected and then each measurement set (i.e., voltage/current magnitude and angle) can be separately denoised. In this work, the Matlab wavelet toolbox (*wden* function) is used for PMU measurement denoising. In this function, the wavelet, the level of decomposition, and the type of thresholding (i.e., hard or soft thresholding) should be defined. In this work, the Daubechies 2 wavelet (db2) was chosen since the waveform of this wavelet is similar to the one-day PMU measurements waveform [8], while the level of decomposition is set to 6 with a soft thresholding.

In order to assess the impact of denoising PMU measurements on the calculated line parameters, the average calculation error (over one day) of the line parameters when they are calculated from: 1) *denoised PMU measurements* and 2) *PMU measurements without denoising* is obtained as,

$$avererror_{par}(\%) = \frac{1}{N} \sum_{i=1}^N \left| \frac{par_{act}^i - par_{cal}^i}{par_{act}^i} \right| \times 100 \quad (9)$$

where,  $par_{act}^i$  is the actual value of each of the three parameters at time instant  $i$ ,  $par_{cal}^i$  is the value of the corresponding parameter calculated by PMU measurements (with or without

denoising) at time instant  $i$ , and  $N$  is the number of time instants that the parameters are calculated over one day (in this work  $N=96$ ). The average errors of the three parameters (series conductance and susceptance, and shunt susceptance) of each line when calculated from PMU measurements with or without denoising are shown in Figs. 5, 6, and 7 respectively.

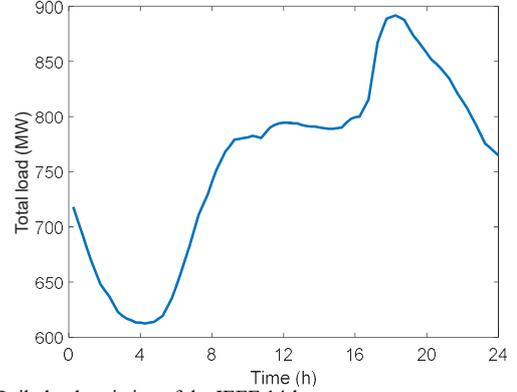


Fig. 4. Daily load variation of the IEEE 14-bus system

TABLE I  
PMU MAXIMUM ERRORS [10]

Voltage magnitude PMU (%)	Current magnitude PMU (%)	Phase angle PMU (degrees)
±0.02	±0.03	±0.54

TABLE II  
MAXIMUM ERRORS-0.5 ACCURACY CLASS INSTRUMENT TRANSFORMERS [11], [12]

Voltage transformers		Current transformers									
Voltage magnitude error %	Phase displacement (degrees)	Current error at percentage of rated current %				Phase displacement at percentage of rated current (degrees)					
		1	5	20	100	120	1	5	20	100	120
±0.5	±0.333	-	1.5	0.75	0.5	0.5	-	1.5	0.75	0.5	0.5

It should be noted that only the lines that have non-zero parameter values are considered in the three graphs. For instance, only 7 transmission lines are considered as long lines in the IEEE 14-bus system (i.e., they have non-zero shunt susceptance) and therefore Fig. 7 shows the average calculation error for only 7 lines. As it is shown in the three figures the average error of three parameters when calculated by denoised PMU measurements is much smaller than the case of the calculated parameters from PMU measurements without denoising.

Further, Fig. 8 depicts the mean average error of the series susceptances for different mother wavelets, decomposition levels, and types of thresholding. It can be concluded that the use of soft type for the threshold denoise better the PMU measurements, while the decomposition level for effective denoising should be between 5 to 8. Regarding the mother wavelet that is used in the wavelet transformation, it seems that the average error is slightly smaller for a wavelet from the Daubechies or Symlets family; however, the average error is affected more by the decomposition level.

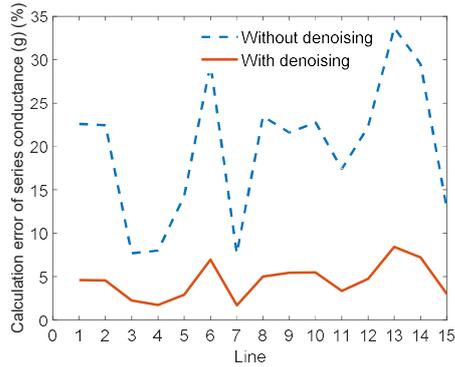


Fig. 5. Series conductance error

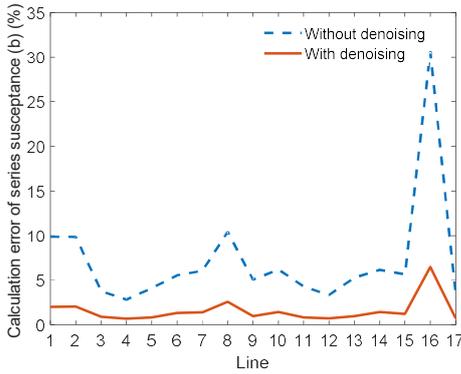


Fig. 6. Series susceptance error

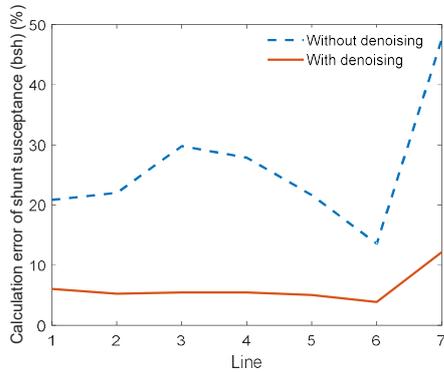


Fig. 7. Shunt susceptance error

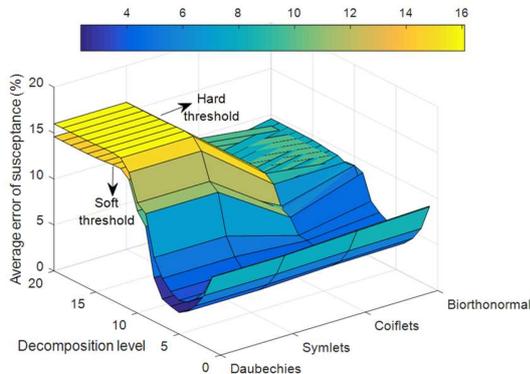


Fig. 8. Average susceptance error varying with decomposition level, wavelet, and type of threshold

## VI. CONCLUSIONS

This paper proposes the denoising of the PMU measurements through the discrete wavelet transformation before calculating transmission line parameters. As it is shown in the simulation results, the calculation error of the line parameters that are obtained through denoised PMU measurements is kept low and below 10% for all the transmission lines. On the other hand, the calculation error of the parameters obtained without denoising the PMU measurement can be higher than 30% for some transmission lines. Such large error deteriorates the performance of several monitoring and control applications, thus the consideration of noise level in the PMU measurements in case of low accuracy class instrument transformers is essential. As it was shown in this paper, even in the case of large noise level, the use of discrete wavelet transform for PMU measurement denoising can be an effective solution for decreasing the calculation error of the line parameters. Consequently, the accurate line parameters will improve the accuracy of several monitoring and control applications and will enable the power system operators to refine the protection settings of the impedance relays.

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