Neutrosophic soft sets and neutrosophic soft matrices based on decision making

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Abstract

Maji[32], firstly proposed neutrosophic soft sets can handle the indeterminate information and inconsistent information which exists commonly in belief systems. In this paper, we have firstly redefined complement, union and compared our definitions of neutrosophic soft with the definitions given by Maji. Then, we have introduced the concept of neutrosophic soft matrix and their operators which are more functional to make theoretical studies in the neutrosophic soft set theory. The matrix is useful for storing an neutrosophic soft set in computer memory which are very useful and applicable. Finally, based on some of these matrix operations a efficient methodology named as NSM-decision making has been developed to solve neutrosophic soft set based group decision making problems.

Keywords: Soft sets, Soft matrix, neutrosophic sets, neutrosophic soft sets, neutrosophic soft matrix, decision making

1. Introduction

In recent years a number of theories have been proposed to deal with uncertainty, imprecision, vagueness and indeterminacy. Theory of probability, fuzzy set theory[54], intuitionistic fuzzy sets [4], interval valued intuitionistic fuzzy sets [3], vague sets[26], rough set theory[41], neutrosophic theory[46],

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interval neutrosophic theory[52] etc. are consistently being utilized as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. However, each of these theories has its inherent difficulties as pointed out by Molodtsov[39]. The reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theories. Later on, many interesting results of soft set theory have been obtained by embedding the idea of fuzzy set, intuituionstic fuzzy set, vague set, rough set, interval intuitionistic fuzzy set, intuitionistic neutrosophic set, interval neutrosophic set, neutrosophic set and so on. For example, fuzzy soft set[34], intuitionistic fuzzy soft set[17, 31], rough soft set[24, 25], interval valued intuitionistic fuzzy soft set[27, 53, 55], neutrosophic soft set[32, 33], generalized neutrosophic soft set[7], intuitionstic neutrosophic soft set[8], interval valued neutrosophic soft set[20]. The theories has developed in many directions and applied to wide variety of fields such as on soft decision making[12, 56], fuzzy soft decision making[18, 19, 30, 45] ,on relation of fuzzy soft set[50, 51], on relation on intuiotionstic fuzzy soft set[22, 40], on relation on neutrosophic soft set[21], on relation on interval neutrosophic soft set[20] and so on.

Researchers published several papers on fuzzy soft matrices and intuitionistic fuzzy soft matrices, and it has been applying in many fields of real life scenarios(see[28, 30, 29, 35]). Recently Cagman et al[13] introduced soft matrices and applied it in decision making problem. They also introduced introduced fuzzy soft matrices[15], Chetia and Das[11] defined intuitionistic fuzzy soft matrices with different products and properties on these products. Further Saikia et al[48] defined generalized fuzzy soft matrices with four different product of generalized intuitionstic fuzzy soft matrices and presented an application in medical diagnosis. Next, Broumi et al[9] studied fuzzy soft matrix based on reference function and defined some new operations such fuzzy soft complement matrix, trace of fuzzy soft matrix based on reference function a new fuzzy soft matrix decision method based on reference function is presented. Recently, Mondal et al[36, 37, 38] introduced fuzzy and intuitionstic fuzzy soft matrix and multicrita in desion making based on three basic t-norm operators. The matrices has differently developed in many directions and applied to wide variety of fields in [5, 6, 44, 47].

Our objective is to introduce the concept of neutrosophic matrices and its applications in decision making problem. The remaining part of this paper is organized as follows. Section 2 contains basic definitions and notations that are used in the remaining parts of the paper. we investigated redefined neutrosophic soft set and some operations and compared our definitions of neutrosophic soft with the definitions given Maji[32] in section 3. In section 4, we introduce the concept of neutrosophic matrices and presente some of theirs basic properties. In section 5, we present two special products of neutrosophic matrices. In section 6, we present a soft decision making method based on and-product of neutrsophic matrice. Finally ,conclusion is made in section 7.

2. Preliminary

In this section, we give the basic definitions and results of neutrosophic set theory [46], soft set theory [39] and soft matrix theory [13] that are useful for subsequent discussions.

Definition 1. *[46] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets(N-sets) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(u)$; $I_A(u)$ and $F_A(u)$ are real standard or nonstandard subsets of $[0, 1]$. It can be written as*

$$A = \{< u, (T_A(u), I_A(u), F_A(u)) >: u \in U, T_A(u), I_A(u), F_A(u) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(u)$; $I_A(u)$ and $F_A(u)$, so $0 \leq supT_A(u) + supI_A(u) + supF_A(u) \leq 3$.

Definition 2. *[39] Let U be a universe, E be a set of parameters that are describe the elements of U, and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a set valued function f_A representing a mapping*

$$f_A : E \to P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \in E - A \tag{1}$$

where f_A is called approximate function of the soft set F_A. In other words, the soft set is a parametrized family of subsets of the set U, and therefore it can be written a set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) = \emptyset \text{ if } x \in E - A\}$$

The subscript A in the f_A indicates that f_A is the approximate function of F_A. The value $f_A(x)$ is a set called *x-element* of the soft set for every $x \in E$.

Definition 3. *[13] Let F_A be a soft set over U. Then a subset of $U \times E$ is uniquely defined by*

$$R_A = \{((u, x)/(u, x)) : (u, x) \in U \times E\}$$

which is called a relation form of F_A. The characteristic function of R_A is written by

$$\chi_{R_A} : U \times E \to [0, 1], \quad \chi_{R_A}(u, x) = \begin{cases} 1, & \textit{if } (u, x) \in R_A, \\ 0, & \textit{if } (u, x) \notin R_A. \end{cases}$$

If $U = \{u_1, u_2, ..., u_m\}$, $E = \{x_1, x_2, ..., x_n\}$ and $A \subseteq E$, then the R_A can be presented by a table as in the following form

R_A	x_1	x_2	$\dots$	x_n
u_1	$\chi_{R_A}(u_1, x_1)$	$\chi_{R_A}(u_1, x_2)$	$\dots$	$\chi_{R_A}(u_1, x_n)$
u_2	$\chi_{R_A}(u_2, x_1)$	$\chi_{R_A}(u_2, x_2)$	$\dots$	$\chi_{R_A}(u_2, x_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\chi_{R_A}(u_m, x_1)$	$\chi_{R_A}(u_m, x_2)$	$\dots$	$\chi_{R_A}(u_m, x_n)$

If $a_{ij} = \chi_{R_A}(u_i, x_j)$, we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

which is called an $m \times n$ s-matrix of the soft set F_A over U.

From now on we shall delete the subscripts $m \times n$ of $[a_{ij}]_{m\times n}$, we use $[a_{ij}]$ instead of $[a_{ij}]_{m\times n}$ for $i = 1, 2, ...m$ and $j = 1, 2, ...n$.

Definition 4. *[13] Let $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. Then And-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by*

$$\wedge : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \wedge [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \min\{a_{ij}, b_{ik}\}$ such that $p = n(j - 1) + k$.

Definition 5. *[13] Let* $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$*. Then Or-product of* $[a_{ij}]$ *and* $[b_{ik}]$ *is defined by*

$$\vee : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \vee [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \max\{a_{ij}, b_{ik}\}$ *such that* $p = n(j-1)+k$*.*

Definition 6. *[13] Let* $[c_{ip}] \in SM_{m\times n^2}$*,* $I_k = \{p : \exists i, c_{ip} \neq 0, (k-1)n < p \leqslant kn\}$ *for all* $k \in I = \{1, 2, ..., n\}$*. Then* fs*-max-min decision function, denoted* Mm*, is defined as follows*

$$Mm : FSM_{m\times n^2} \to FSM_{m\times 1}, \quad Mm[c_{ip}] = [d_{i1}] = [\max_k\{t_{ik}\}]$$

where

$$t_{ik} = \begin{cases} \min_{p\in I_k}\{c_{ip}\}, & \textit{if } I_k \neq \emptyset, \\ 0, & \textit{if } I_k = \emptyset. \end{cases}$$

The one column fs*-matrix* $Mm[c_{ip}]$ *is called max-min decision* fs*-matrix.*

Definition 7. *Let* $U = \{u_1, u_2, ..., u_m\}$ *be an initial universe and* $Mm[c_{ip}] = [d_{i1}]$*. Then a subset of* U *can be obtained by using* $[d_{i1}]$ *as in the following way*

$$opt_{[d_{i1}]}(U) = \{d_{i1}/u_i : u_i \in U, d_{i1} \neq 0\}$$

which is called an optimum fuzzy set on U*.*

Definition 8. *[32] Let* U *be a universe,* $N(U)$ *be the set of all neutrosophic sets on* U*,* E *be a set of parameters that are describe the elements of* U*, and* $A \subseteq E$*. Then, a neutrosophic soft set* N *over* U *is a set defined by a set valued function* f_N *representing a mapping*

$$f_N : A \to N(U)$$

where f_N *is called approximate function of the neutrosophic soft set* N*. In other words, the neutrosophic soft set is a parametrized family of some elements of the set* $P(U)$*, and therefore it can be written a set of ordered pairs*

$$N = \{(x, f_N(x)) : x \in A\}$$

Definition 9. *[32] Let* N_1 *and* N_2 *be two neutrosophic soft sets over neutrosophic soft universes* (U, A) *and* (U, B)*, respectively.*

1. N_1 is said to be neutrosophic soft subset of N_2 if $A \subseteq B$ and $T_{f_{N_1(x)}}(u) \leq T_{f_{N_2(x)}}(u)$, $I_{f_{N_1(x)}}(u) \leq I_{f_{N_2(x)}}(u)$,$F_{f_{N_1(x)}}(u) \geq F_{f_{N_2(x)}}(u)$, $\forall x \in A$, $u \in U$.

2. N_1 and N_2 are said to be equal if N_1 neutrosophic soft subset of N_2 and N_2 neutrosophic soft subset of N_2.

Definition 10. *[32] Let $E = \{e_1, e_2, ...\}$ be a set of parameters. The NOT set of E is denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, ...\}$ where $\neg e_i = not\, e_i, \forall i$.*

Definition 11. *[32] Let N_1 and N_2 be two neutrosophic soft sets over soft universes (U, A) and (U, B), respectively,*

1. The complement of a neutrosophic soft set N_1 denoted by N_1° and is defined by a set valued function f_N° representing a mapping $f_{N_1}^\circ : \neg E \to N(U)$

   $f_{N_1}^\circ = \{(u, < F_{f_{N_1(x)}}(u), I_{f_{N_1(x)}}(u), T_{f_{N_1(x)}}(u) >) : x \in \neg E, u \in U\}$.

2. Then the union of N_1 and N_2 is denoted by $N_1 \acute{\cup} N_2$ and is defined by $N_3(C = A \cup B)$, where the truth-membership, indeterminacy-membership and falsity-membership of N_3 are as follows: $\forall u \in U$,

$$T_{f_{N_3(x)}}(u) = \begin{cases} T_{f_{N_1(x)}}(u), & if x \in A - B \\ T_{f_{N_2(x)}}(u), & if x \in B - A \\ max\{T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)\}, & if x \in A \cap B \end{cases}$$

$$I_{f_{N_3(x)}}(u) = \begin{cases} I_{f_{N_1(x)}}(u), & if x \in A - B \\ I_{f_{N_2(x)}}(u), & if x \in B - A \\ \dfrac{(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))}{2}, & if x \in A \cap B \end{cases}$$

$$F_{f_{N_3(x)}}(u) = \begin{cases} F_{f_{N_1(x)}}(u), & if x \in A - B \\ F_{f_{N_2(x)}}(u), & if x \in B - A \\ min\{I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u)\}, & if x \in A \cap B \end{cases}$$

3. Then the intersection of N_1 and N_2 is denoted by $N_1 \acute{\cap} N_2$ and is defined by $N_3(C = A \cap B)$, where the truth-membership, indeterminacy-membership and falsity-membership of N_3 are as follows: $\forall u \in U$,

$T_{f_{N_3(x)}}(u) = min\{T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)\}$, $I_{f_{N_3(x)}}(u) = \frac{(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))}{2}$ *and* $F_{f_{N_3(x)}}(u) = max\{F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u)\}$, $\forall x \in C$.

Definition 12. *[23] t-norms are associative, monotonic and commutative two valued functions* t *that map from* $[0,1] \times [0,1]$ *into* $[0,1]$. *These properties are formulated with the following conditions:* $\forall a, b, c, d \in [0,1]$,

i. $t(0,0) = 0$ *and* $t(a,1) = t(1,a) = a$,

ii. *If* $a \leq c$ *and* $b \leq d$, *then* $t(a,b) \leq t(c,d)$

iii. $t(a,b) = t(b,a)$

iv. $t(a, t(b,c)) = t(t(a,b,c))$

Definition 13. *[23] t-conorms (s-norm) are associative, monotonic and commutative two placed functions* s *which map from* $[0,1] \times [0,1]$ *into* $[0,1]$. *These properties are formulated with the following conditions:* $\forall a, b, c, d \in [0,1]$,

i. $s(1,1) = 1$ *and* $s(a,0) = s(0,a) = a$,

ii. *if* $a \leq c$ *and* $b \leq d$, *then* $s(a,b) \leq s(c,d)$

iii. $s(a,b) = s(b,a)$

iv. $s(a, s(b,c)) = s(s(a,b,c)$

t-norm and t-conorm are related in a sense of lojical duality. Typical dual pairs of non parametrized t-norm and t-conorm are complied below:

i. *Drastic product:*

$$t_w(a,b) = \begin{cases} min\{a,b\}, & max\{ab\} = 1 \\ 0, & otherwise \end{cases}$$

ii. *Drastic sum:*

$$s_w(a,b) = \begin{cases} max\{a,b\}, & min\{ab\} = 0 \\ 1, & otherwise \end{cases}$$

iii. *Bounded product:*

$$t_1(a,b) = max\{0, a+b-1\}$$

iv. Bounded sum:

$$s_1(a, b) = min\{1, a + b\}$$

v. Einstein product:

$$t_{1.5}(a, b) = \frac{a.b}{2 - [a + b - a.b]}$$

vi. Einstein sum:

$$s_{1.5}(a, b) = \frac{a + b}{1 + a.b}$$

vii. Algebraic product:

$$t_2(a, b) = a.b$$

viii. Algebraic sum:

$$s_2(a, b) = a + b - a.b$$

ix. Hamacher product:

$$t_{2.5}(a, b) = \frac{a.b}{a + b - a.b}$$

x. Hamacher sum:

$$s_{2.5}(a, b) = \frac{a + b - 2.a.b}{1 - a.b}$$

xi. Minumum:

$$t_3(a, b) = min\{a, b\}$$

xii. Maximum:

$$s_3(a, b) = max\{a, b\}$$

3. Neutrosophic soft set and some operations redefined

Notion of the neutrosophic soft set theory is first given by Maji [32]. This section, we has modified the definition of neutrosophic soft set and operations as follows. Some of it is quoted from [1, 2, 14, 22, 32, 46].

Definition 14. *Let U be a universe, $N(U)$ be the set of all neutrosophic sets on U, E be a set of parameters that are describe the elements of U Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping*

$$f_N : E \to N(U)$$

where f_N is called approximate function of the neutrosophic soft set N. For $x \in E$, the set $f_N(x)$ is called x-approximation of the neutrosophic soft set N which may be arbitrary, some of them may be empty and some may have a nonempty intersection. In other words, the neutrosophic soft set is a parametrized family of some elements of the set $N(U)$, and therefore it can be written a set of ordered pairs,

$$N = \{(x, \{< u, T_{f_{N(x)}}(u), I_{f_{N(x)}}(u), F_{f_{N(x)}}(u) >: x \in U\} : x \in E\}$$

where

$T_{f_{N(x)}}(u), I_{f_{N(x)}}(u), F_{f_{N(x)}}(u) \in [0, 1]$

Definition 15. *Let N_1 and N_2 be two neutrosophic soft sets. Then, the complement of a neutrosophic soft set N_1 denoted by N_1^c and is defined by*

$${N_1}^c = \{(x, \{< u, F_{f_{N_1(x)}}(u), I_{f_{N_1(x)}}(u), T_{f_{N_1(x)}}(u) >: x \in U\} : x \in E\}$$

Definition 16. *Let N_1 and N_2 be two neutrosophic soft sets. Then, the union of N_1 and N_2 is denoted by $N_3 = N_1 \tilde{\cup} N_2$ and is defined by*

$$N_3 = \{(x, \{< u, T_{f_{N_3(x)}}(u), I_{f_{N_3(x)}}(u), F_{f_{N_3(x)}}(u) >: x \in U\} : x \in E\}$$

where

$T_{f_{N_3(x)}}(u) = s(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u))$, $I_{f_{N_3(x)}}(u) = t(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))$ *and* $F_{f_{N_3(x)}}(u) = t(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u))$

Definition 17. *Let N_1 and N_2 be two neutrosophic soft sets. Then, the intersection of N_1 and N_2 is denoted by $N_4 = N_1 \tilde{\cap} N_2$ and is defined by*

$$N_4 = \{(x, \{< u, T_{f_{N_4(x)}}(u), I_{f_{N_4(x)}}(u), F_{f_{N_4(x)}}(u) >: x \in U\} : x \in E\}$$

where

$T_{f_{N_4(x)}}(u) = t(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u))$, $I_{f_{N_4(x)}}(u) = s(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))$ *and* $F_{f_{N_4(x)}}(u) = s(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u))$

Example 1. *Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3\}$. N_1 and N_2 be two neutrosophic soft sets as*

$$\begin{aligned} N_1 = \quad & \Big\{(x_1, \{< u_1, (0.4, 0.5, 0.8) >, < u_2, (0.2, 0.5, 0.1) >, < u_3, (0.3, 0.1, 0.4) >, \\ & < u_4, (0.4, 0.7, 0.7) >\}), (x_2, < u_1, (0.5, 0.7, 0.7) >, < u_2, (0.3, 0.6, 0.3) >, \\ & < u_3, (0.2, 0.6, 0.5) >, < u_4, (0.4, 0.5, 0.5) >\}), (x_3, \{< u_1, (0.7, 0.8, 0.6) >, \\ & < u_2, (0.5, 0.6, 0.7) >, < u_3, (0.7, 0.5, 0.8) >, < u_4, (0.2, 0.8, 0.5) >\})\Big\} \end{aligned}$$

and

$$N_2 = \Big\{(x_1, \{< u_1, (0.7, 0.6, 0.7) >, < u_2, (0.4, 0.2, 0.8) >, < u_3, (0.9, 0.1, 0.5) >,$$
$$< u_4, (0.4, 0.7, 0.7) >\}), (x_2, < u_1, (0.5, 0.7, 0.8) >, < u_2, (0.5, 0.9, 0.3) >,$$
$$< u_3, (0.5, 0.6, 0.8) >, < u_4, (0.5, 0.8, 0.5) >\}), (x_3, \{< u_1, (0.8, 0.6, 0.9) >,$$
$$< u_2, (0.5, 0.9, 0.9) >, < u_3, (0.7, 0.5, 0.4) >, < u_4, (0.3, 0.5, 0.6) >\})\Big\}$$

here;

$$N_1^c = \Big\{(x_1, \{< u_1, (0.8, 0.5, 0.4) >, < u_2, (0.1, 0.5, 0.2) >, < u_3, (0.4, 0.1, 0.3) >,$$
$$< u_4, (0.7, 0.7, 0.4) >\}), (x_2, < u_1, (0.7, 0.7, 0.5) >, < u_2, (0.3, 0.6, 0.3) >,$$
$$< u_3, (0.5, 0.6, 0.2) >, < u_4, (0.5, 0.5, 0.4) >\}), (x_3, \{< u_1, (0.6, 0.8, 0.7) >,$$
$$< u_2, (0.7, 0.6, 0.5) >, < u_3, (0.8, 0.5, 0.7) >, < u_4, (0.5, 0.8, 0.2) >\})\Big\}$$

Let us consider the t-norm $min\{a, b\}$ *and s-norm* $max\{a, b\}$. *Then,*

$$N_1 \tilde{\cup} N_2 = \Big\{(x_1, \{< u_1, (0.7, 0.5, 0.7) >, < u_2, (0.4, 0.2, 0.1) >, < u_3, (0.9, 0.1, 0.4) >,$$
$$< u_4, (0.4, 0.7, 0.7) >\}), (x_2, < u_1, (0.5, 0.7, 0.7) >, < u_2, (0.5, 0.6, 0.3) >,$$
$$< u_3, (0.5, 0.6, 0.5) >, < u_4, (0.5, 0.8, 0.5) >\}), (x_3, \{< u_1, (0.8, 0.6, 0.6) >,$$
$$< u_2, (0.5, 0.6, 0.7) >, < u_3, (0.7, 0.5, 0.4) >, < u_4, (0.3, 0.5, 0.5) >\})\Big\}$$

and

$$N_1 \tilde{\cap} N_2 = \Big\{(x_1, \{< u_1, (0.4, 0.6, 0.8) >, < u_2, (0.2, 0.5, 0.8) >, < u_3, (0.3, 0.1, 0.5) >,$$
$$< u_4, (0.4, 0.7, 0.7) >\}), (x_2, < u_1, (0.5, 0.7, 0.8) >, < u_2, (0.3, 0.9, 0.3) >,$$
$$< u_3, (0.2, 0.6, 0., 8) >, < u_4, (0.4, 0.8, 0.5) >\}), (x_3, \{< u_1, (0.7, 0.8, 0.9) >,$$
$$< u_2, (0.5, 0.9, 0.9) >, < u_3, (0.7, 0.5, 0.8) >, < u_4, (0.2, 0.8, 0.6) >\})\Big\}$$

Proposition 1. *Let* N_1, N_2 *and* N_3 *be any three neutrosophic soft sets. Then,*

1. $N_1 \widetilde{\cup} N_2 = N_2 \widetilde{\cup} N_1$
2. $N_1 \widetilde{\cap} N_2 = N_2 \widetilde{\cap} N_1$

3. $N_1 \widetilde{\cup} (N_2 \widetilde{\cup} N_3) = (N_1 \widetilde{\cup} N_2) \widetilde{\cup} N_3$

4. $N_1 \widetilde{\cap} (N_2 \widetilde{\cap} N_3) = (N_1 \widetilde{\cap} N_2) \widetilde{\cap} N_3$

Proof. The proofs can be easily obtained since the t-norm function and s-norm functions are commutative and associative.

3.1. *Comparision of the Definitions*

In this subsection, we compared our definitions of neutrosophic soft with the definitions given Maji[32] by inspiring from [14].

Let us compare our definitions of neutrosophic soft with the definitions given Maji[32] in Table 1.

In this paper our approach	in Maji
$N = \{(x, f_N(x)) : x \in E\}$	$N = \{(x, f_N(x)) : x \in A\}$
where	$N = \{(x, f_N(x)) : x \in A\}$
E parameter set and	$A \subseteq E$
$f_N : E \to N(U)$	$f_N : A \to N(U)\}$

Table 1

Let us compare our complement definitions of neutrosophic soft with the definitions given Maji[32] in Table 2.

In this paper our approach	in Maji
N_1^c	N_1°
$f_N^c : E \to N(U)$	$f_{N_1}^\circ : \neg E \to N(U)$
$T_{f_{N_1^c(x)}}(u) = F_{f_{N_1(x)}}(u)$	$T_{f_{N_1^\circ(x)}}(u) = F_{f_{N_1(x)}}(u)$
$I_{f_{N_1^c(x)}}(u) = 1 - I_{f_{N_1(x)}}(u)$	$I_{f_{N_1^\circ(x)}}(u) = I_{f_{N_1(x)}}(u)$
$F_{f_{N_1^c(x)}}(u) = T_{f_{N_1(x)}}(u)$	$F_{f_{N_1^\circ(x)}}(u) = T_{f_{N_1(x)}}(u)$

Table 2

Let us compare our union definitions of neutrosophic soft with the definitions given Maji[32] in Table 2.

In this paper our approach	in Maji
$N_3 = N_1 \tilde{\cup} N_2$	$N_3 = N_1 \acute{\cup} N_2$
$f_{N_3} : E \to N(U)$	$f_{N_3(x)} : A \to N(U)$
where	
$T_{f_{N_3(x)}}(u) = s(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u))$	$T_{f_{N_3(x)}}(u) = \begin{cases} T_{f_{N_1(x)}}(u), & x \in A - B \\ T_{f_{N_2(x)}}(u), & x \in B - A \\ max\{T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)\}, & x \in A \cap B \end{cases}$
$I_{f_{N_3(x)}}(u) = t(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))$	$I_{f_{N_3(x)}}(u) = \begin{cases} I_{f_{N_1(x)}}(u), & x \in A - B \\ I_{f_{N_2(x)}}(u), & x \in B - A \\ \dfrac{(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))}{2}, & x \in A \cap B \end{cases}$
$F_{f_{N_3(x)}}(u) = t(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u))$	$F_{f_{N_3(x)}}(u) = \begin{cases} F_{f_{N_1(x)}}(u), & x \in A - B \\ F_{f_{N_2(x)}}(u), & x \in B - A \\ min\{I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u)\}, & x \in A \cap B \end{cases}$

Table 2

Let us compare our intersection definitions of neutrosophic soft with the definitions given Maji[32] in Table 2.

In this paper our approach	in Maji
$N_3 = N_1 \tilde{\cap} N_2$	$N_3 = N_1 \acute{\cap} N_2$
$f_{N_3} : E \to N(U)$	$f_{N_3(x)} : A \to N(U)$
where	
$T_{f_{N_3(x)}}(u) = t(T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u))$	$T_{f_{N_3(x)}}(u) = min\{T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)\}$
$I_{f_{N_3(x)}}(u) = s(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))$	$I_{f_{N_3(x)}}(u) = \frac{(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))}{2}$
$F_{f_{N_3(x)}}(u) = s(F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u))$	$F_{f_{N_3(x)}}(u) = max\{F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u)\}$

Table 3

4. Neutrosophic Soft Matrices

In this section, we presented neutrosophic soft matrices which are representative of the neutrosophic soft sets. The matrix is useful for storing an neutrosophic soft set in computer memory which are very useful and applicable. Some of it is quoted from [13, 15, 5].

This section are an attempt to extend the concept of soft matrices[13], fuzzy soft matrices[15], intuitionistic fuzzy soft matrices[5].

Definition 18. *Let N be an neutrosophic soft set over $N(U)$. Then a subset of $N(U) \times E$ is uniquely defined by*

$R_N = \{(f_N(x), x) : x \in E, f_N(x) \in N(U)\}$ *which is called a relation form of (N, E). The characteristic function of R_N is written by*

$$\Theta_{R_N} : N(U) \times E \to [0,1]\times[0,1]\times[0,1], \Theta_{R_N}(u,x) = (T_{f_N(x)}(u), I_{f_N(x)}(u), F_{f_N(x)}(u))$$

where $T_{f_N(x)}(u)$, $I_{f_N(x)}(u)$ and $F_{f_N(x)}(u)$ is the truth-membership, indeterminacy-membership and falsity-membership of $u \in U$, respectively.

Definition 19. *Let $U = \{u_1, u_2, \ldots, u_m\}$, $E = \{x_1, x_2, \ldots, x_n\}$ and N be an neutrosophic soft set over $N(U)$. Then*

R_N	$f_N(x_1)$	$f_N(x_2)$	$\cdots$	$f_N(x_n)$
u_1	$\Theta_{R_N}(u_1, x_1)$	$\Theta_{R_N}(u_1, x_2)$	$\cdots$	$\Theta_{R_N}(u_1, x_n)$
u_2	$\Theta_{R_N}(u_2, x_1)$	$\Theta_{R_N}(u_2, x_2)$	$\cdots$	$\Theta_{R_N}(u_2, x_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\Theta_{R_N}(u_m, x_1)$	$\Theta_{R_N}(u_m, x_2)$	$\cdots$	$\Theta_{R_N}(u_m, x_n)$

If $a_{ij} = \Theta_{R_N}(u_i, x_j)$, we can define a matrix

$$[a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

such that $a_{ij} = (T_{f_N(x_j)}(u_i), I_{f_N(x_j)}(u_i), F_{f_N(x_j)}(u_i)) = (T^a_{ij}, I^a_{ij}, F^a_{ij})$, which is called an $m \times n$ neutrosophic soft matrix (or namely NS-matrix) of the neutrosophic soft set N over $N(U)$.

According to this definition, an a neutrosophic soft set N is uniquely characterized by matrix $[a_{ij}]_{m\times n}$. Therefore, we shall identify any neutrosophic soft set with its soft NS-matrix and use these two concepts as interchangeable. The set of all $m \times n$ NS-matrix over $N(U)$ will be denoted by $\widetilde{N}_{m\times n}$. From now on we shall delete th subscripts $m \times n$ of $[a_{ij}]_{m\times n}$, we use $[a_{ij}]$ instead of $[a_{ij}]_{m\times n}$, since $[a_{ij}] \in \widetilde{N}_{m\times n}$ means that $[a_{ij}]$ is an $m \times n$ NS-matrix for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Example 2. *Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$. N_1 be a neutrosophic soft sets over neutrosophic as*

$$N = \Big\{(x_1, \{< u_1, (0.7, 0.6, 0.7) >, < u_2, (0.4, 0.2, 0.8) >, < u_3, (0.9, 0.1, 0.5) >\}),$$
$$(x_2, < u_1, (0.5, 0.7, 0.8) >, < u_2, (0.5, 0.9, 0.3) >, < u_3, (0.5, 0.6, 0., 8) >\}),$$
$$(x_3, \{< u_1, (0.8, 0.6, 0.9) >, < u_2, (0.5, 0.9, 0.9) >, < u_3, (0.7, 0.5, 0.4) >\})\Big\}$$

Then, the NS-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.6, 0.7) & (0.5, 0.7, 0.8) & (0.8, 0.6, 0.9) \\ (0.4, 0.2, 0.8) & (0.5, 0.9, 0.3) & (0.5, 0.9, 0.9) \\ (0.9, 0.1, 0.5) & (0.5, 0.6, 0.8) & (0.7, 0.5, 0.4) \end{bmatrix}$$

Definition 20. *A neutrosophic soft matrix of order $1 \times n$ i.e., with a single row is called a row-neutrosophic soft matrix. Physically, a row-neutrosophic soft matrix formally corresponds to an neutrosophic soft set whose universal set contains only one object.*

Example 3. *Let $U = \{u_1\}$, $E = \{x_1, x_2, x_3\}$. N_1 be a neutrosophic soft sets over neutrosophic as*

$$N = \Big\{(x_1, \{< u_1, (0.7, 0.6, 0.7) >\}), (x_2, < u_1, (0.5, 0.7, 0.8) >\}),$$
$$(x_3, \{< u_1, (0.8, 0.6, 0.9) >\})\Big\}$$

Then, the NS-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.6, 0.7) & (0.5, 0.7, 0.8) & (0.8, 0.6, 0.9) \end{bmatrix}.$$

Definition 21. *A neutrosophic soft matrix of order $m \times 1$ i.e., with a single column is called a column-neutrosophic soft matrix. Physically, a column-neutrosophic soft matrix formally corresponds to an neutrosophic soft set whose parameter set contains only one parameter.*

Example 4. *Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1\}$. N_1 be a neutrosophic soft sets over neutrosophic as*

$$N = \Big\{(x_1, \{< u_1, (0.7, 0.6, 0.7) >, < u_2, (0.4, 0.2, 0.8) >, < u_3, (0.9, 0.1, 0.5) >,$$
$$< u_4, (0.4, 0.7, 0.7) >\})\Big\}$$

Then, the NS-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.6, 0.7) \\ (0.4, 0.2, 0.8) \\ (0.9, 0.1, 0.5) \\ (0.4, 0.7, 0.7) \end{bmatrix}.$$

Definition 22. *A neutrosophic soft matrix of order $m \times n$ is said to be a square neutrosophic soft matrix if $m = n$ i.e., the number of rows and the number of columns are equal. That means a square-neutrosophic soft matrix is formally equal to an neutrosophic soft set having the same number of objects and parameters.*

Example 5. *Consider the Example 2. Here since the NS-matrix contains three rows and three columns, so it is a square-neutrosophic soft matrix.*

Definition 23. *A square neutrosophic soft matrix of order $m \times n$ is said to be a diagonal-neutrosophic soft matrix if all of its non-diagonal elements are $(0, 0, 1)$.*

Example 6. *Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3\}$. N_1 be a neutrosophic soft sets over neutrosophic as*

$$\begin{aligned} N = \Big\{ & (x_1, \{< u_1, (0.7, 0.6, 0.7) >, < u_2, (0.0, 1.0, 1.0) >, < u_3, (0.0, 1.0, 1.0) >\}), \\ & (x_2, < u_1, (0.0, 1.0, 1.0) >, < u_2, (0.0, 1.0, 1.0) >, < u_3, (0.0, 1.0, 1.0) >\}), \\ & (x_3, \{< u_1, (0.0, 1.0, 1.0) >, < u_2, (0.0, 1.0, 1.0) >, < u_3, (0.7, 0.5, 0.4) >\}) \Big\} \end{aligned}$$

Then, the NS-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.6, 0.7) & (0.0, 1.0, 1.0) & (0.0, 1.0, 1.0) \\ (0.0, 1.0, 1.0) & (0.0, 1.0, 1.0) & (0.0, 1.0, 1.0) \\ (0.0, 1.0, 1.0) & (0.0, 1.0, 1.0) & (0.7, 0.5, 0.4) \end{bmatrix}.$$

Definition 24. *The transpose of a square neutrosophic soft matrix $[a_{ij}]$ of order $m \times n$ is another square neutrosophic soft matrix of order $n \times m$ obtained from $[a_{ij}]$ by interchanging its rows and columns. It is denoted by $[a_{ij}^T]$. Therefore the neutrosophic soft set associated with $[a_{ij}^T]$ becomes a new neutrosophic soft set over the same universe and over the same set of parameters.*

Example 7. *Consider the Example 2. If the NS-matrix* $[a_{ij}]$ *is written by*

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.6, 0.7) & (0.5, 0.7, 0.8) & (0.8, 0.6, 0.9) \\ (0.4, 0.2, 0.8) & (0.5, 0.9, 0.3) & (0.5, 0.9, 0.9) \\ (0.9, 0.1, 0.5) & (0.5, 0.6, 0.8) & (0.7, 0.5, 0.4) \end{bmatrix}.$$

then, its transpose neutrosophic soft matrix as;

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.6, 0.7) & (0.4, 0.2, 0.8) & (0.9, 0.1, 0.5) \\ (0.5, 0.7, 0.8) & (0.5, 0.9, 0.3) & (0.5, 0.6, 0.8) \\ (0.8, 0.6, 0.9) & (0.5, 0.9, 0.9) & (0.7, 0.5, 0.4) \end{bmatrix}.$$

Definition 25. *A square neutrosophic soft matrix* $[a_{ij}]$ *of order* $n \times n$ *is said to be a symmetric neutrosophic soft matrix, if its transpose be equal to it, i.e., if* $[a_{ij}^T] = [a_{ij}]$*. Hence the neutrosophic soft matrix* $[a_{ij}]$*) is symmetric, if* $[a_{ij}] = [a_{ji}]$ $\forall i, j$.

Example 8. *Let* $U = \{u_1, u_2, u_3\}$*,* $E = \{x_1, x_2, x_3\}$*.* N_1 *be a neutrosophic soft sets as*

$$N = \Big\{(x_1, \{< u_1, (0.7, 0.6, 0.7) >, < u_2, (0.4, 0.2, 0.8) >, < u_3, (0.9, 0.1, 0.5) >\}),$$
$$(x_2, < u_1, (0.4, 0.2, 0.8) >, < u_2, (0.5, 0.9, 0.3) >, < u_3, (0.5, 0.9, 0.9) >\}),$$
$$(x_3, \{< u_1, (0.9, 0.1, 0.5) >, < u_2, (0.5, 0.9, 0.9) >, < u_3, (0.7, 0.5, 0.4) >\})\Big\}$$

Then, the symmetric neutrosophic matrix $[a_{ij}]$ *is written by*

$$[a_{ij}] = \begin{bmatrix} (0.7, 0.6, 0.7) & (0.4, 0.2, 0.8) & (0.9, 0.1, 0.5) \\ (0.4, 0.2, 0.8) & (0.5, 0.9, 0.3) & (0.5, 0.9, 0.9) \\ (0.9, 0.1, 0.5) & (0.5, 0.9, 0.9) & (0.7, 0.5, 0.4) \end{bmatrix}.$$

Definition 26. *Let* $[a_{ij}] \in \widetilde{N}_{m \times n}$*. Then* $[a_{ij}]$ *is called*

i. A zero NS-matrix, denoted by $[\tilde{0}]$*, if* $a_{ij} = (0, 1, 1)$ *for all* i *and* j*.*

ii. A universal NS-matrix, denoted by $[\tilde{1}]$*, if* $a_{ij} = (1, 0, 0)$ *for all* i *and* j*.*

Example 9. *Let* $U = \{u_1, u_2, u_3\}$*,* $E = \{x_1, x_2, x_3\}$*. Then, a zero NS-matrix* $[a_{ij}]$ *is written by*

$$[a_{ij}] = \begin{bmatrix} (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \\ (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \\ (0, 1, 1) & (0, 1, 1) & (0, 1, 1) \end{bmatrix}.$$

and a universal NS-matrix $[a_{ij}]$ *is written by*

$$[a_{ij}] = \begin{bmatrix} (1,0,0) & (1,0,0) & (1,0,0) \\ (1,0,0) & (1,0,0) & (1,0,0) \\ (1,0,0) & (1,0,0) & (1,0,0) \end{bmatrix}.$$

Definition 27. *Let* $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$*. Then*

- *i.* $[a_{ij}]$ *is an NS-submatrix of* $[b_{ij}]$*, denoted,* $[a_{ij}]\tilde{\subseteq}[b_{ij}]$*, if* $T^b_{ij} \geq T^a_{ij}$*,* $I^a_{ij} \geq I^b_{ij}$ *and* $F^a_{ij} \geq F^b_{ij}$*, for all* i *and* j*.*
- *ii.* $[a_{ij}]$ *is a proper NS-submatrix of* $[b_{ij}]$*, denoted,* $[a_{ij}]\tilde{\subset}[b_{ij}]$*, if* $T^a_{ij} \geq T^b_{ij}$*,* $I^a_{ij} \leq I^b_{ij}$ *and* $F^a_{ij} \leq F^b_{ij}$ *for at least* $T^a_{ij} > T^b_{ij}$ *and* $I^a_{ij} < I^b_{ij}$ *and* $F^a_{ij} < F^b_{ij}$ *for all* i *and* j*.*
- *iii.* $[a_{ij}]$ *and* $[b_{ij}]$ *are IFS equal matrices, denoted by* $[a_{ij}] = [b_{ij}]$*, if* $a_{ij} = b_{ij}$ *for all* i *and* j*.*

Definition 28. *Let* $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$*. Then*

- *i. Union of* $[a_{ij}]$ *and* $[b_{ij}]$*, denoted,* $[a_{ij}]\tilde{\cup}[b_{ij}]$*, if* $c_{ij} = (T^c_{ij}, I^c_{ij}, F^c_{ij})$*, where* $T^c_{ij} = \max\{T^a_{ij}, T^b_{ij}\}$*,* $I^c_{ij} = \min\{I^a_{ij}, I^b_{ij}\}$ *and* $F^c_{ij} = \min\{F^a_{ij}, F^b_{ij}\}$ *for all* i *and* j*.*
- *ii. Intersection of* $[a_{ij}]$ *and* $[b_{ij}]$*, denoted,* $[a_{ij}]\tilde{\cap}[b_{ij}]$*, if* $c_{ij} = (T^c_{ij}, I^c_{ij}, F^c_{ij})$*, where* $T^c_{ij} = \min\{T^a_{ij}, T^b_{ij}\}$*,* $I^c_{ij} = \max\{I^a_{ij}, I^b_{ij}\}$ *and* $F^c_{ij} = \max\{F^a_{ij}, F^b_{ij}\}$ *for all* i *and* j*.*
- *iii. Complement of* $[a_{ij}]$*, denoted by* $[a_{ij}]^c$*, if* $c_{ij} = (F^a_{ij}, 1 - I^a_{ij}, T^a_{ij})$ *for all* i *and* j*.*

Example 10. *Consider the Example 1. Then,*

$$[a_{ij}]\tilde{\cup}[b_{ij}] = \begin{bmatrix} (0.7,0.5,0.7) & (0.5,0.7,0.7) & (0.8,0.6,0.6) \\ (0.4,0.2,0.1) & (0.5,0.6,0.3) & (0.5,0.6,0.7) \\ (0.9,0.1,0.4) & (0.5,0.6,0.5) & (0.7,0.5,0.4) \\ (0.4,0.7,0.7) & (0.5,0.8,0.5) & (0.3,0.5,0.5) \end{bmatrix},$$

$$[a_{ij}]\tilde{\cap}[b_{ij}] = \begin{bmatrix} (0.4,0.6,0.8) & (0.5,0.7,0.8) & (0.7,0.8,0.9) \\ (0.2,0.5,0.8) & (0.3,0.9,0.3) & (0.5,0.9,0.9) \\ (0.3,0.1,0.5) & (0.2,0.6,0.8) & (0.7,0.5,0.8) \\ (0.4,0.7,0.7) & (0.4,0.8,0.5) & (0.2,0.8,0.6) \end{bmatrix}.$$

and

$$[a_{ij}]^c = \begin{bmatrix} (0.8, 0.5, 0.4) & (0.7, 0.3, 0.5) & (0.6, 0.2, 0.7) \\ (0.1, 0.5, 0.2) & (0.3, 0.4, 0.3) & (0.7, 0.4, 0.5) \\ (0.4, 0.9, 0.3) & (0.5, 0.4, 0.2) & (0.8, 0.5, 0.7) \\ (0.7, 0.3, 0.4) & (0.5, 0.5, 0.4) & (0.5, 0.2, 0.2) \end{bmatrix}.$$

Definition 29. *Let* $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$. *Then* $[a_{ij}]$ *and* $[b_{ij}]$ *are disjoint, if* $[a_{ij}]\tilde{\cap}[b_{ij}] = [\tilde{0}]$ *for all* i *and* j.

Proposition 2. *Let* $[a_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

i. $\big([a_{ij}]^c\big)^c = [a_{ij}]$

ii. $[\tilde{0}]^c = [\tilde{1}]$.

Proposition 3. *Let* $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

i. $[a_{ij}] \subseteq [\tilde{1}]$

ii. $[\tilde{0}]\tilde{\subseteq}[a_{ij}]$

iii. $[a_{ij}]\tilde{\subseteq}[a_{ij}]$

iv. $[a_{ij}]\tilde{\subseteq}[b_{ij}]$ *and* $[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow [a_{ij}]\tilde{\subseteq}[c_{ij}]$

Proposition 4. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

i. $[a_{ij}] = [b_{ij}]$ *and* $[b_{ij}] = [c_{ij}] \Leftrightarrow [a_{ij}] = [c_{ij}]$

ii. $[a_{ij}]\tilde{\subseteq}[b_{ij}]$ *and* $[b_{ij}]\tilde{\subseteq}[a_{ij}] \Leftrightarrow [a_{ij}] = [b_{ij}]$

Proposition 5. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

i. $[a_{ij}]\tilde{\cup}[a_{ij}] = [a_{ij}]$

ii. $[a_{ij}]\tilde{\cup}[\tilde{0}] = [a_{ij}]$

iii. $[a_{ij}]\tilde{\cup}[\tilde{1}] = [\tilde{1}]$

iv. $[a_{ij}]\tilde{\cup}[b_{ij}] = [b_{ij}]\tilde{\cup}[a_{ij}]$

v. $([a_{ij}]\tilde{\cup}[b_{ij}])\tilde{\cup}[c_{ij}] = [a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cup}[c_{ij}])$

Proposition 6. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

i. $[a_{ij}]\tilde{\cap}[a_{ij}] = [a_{ij}]$

ii. $[a_{ij}]\tilde{\cap}[\tilde{0}] = [\tilde{0}]$

iii. $[a_{ij}]\tilde{\cap}[\tilde{1}] = [a_{ij}]$

iv. $[a_{ij}]\tilde{\cap}[b_{ij}] = [b_{ij}]\tilde{\cap}[a_{ij}]$

v. $([a_{ij}]\tilde{\cap}[b_{ij}])\tilde{\cap}[c_{ij}] = [a_{ij}]\tilde{\cap}([b_{ij}]\tilde{\cap}[c_{ij}])$

Proposition 7. *Let* $[a_{ij}], [b_{ij}] \in \widetilde{N}_{m\times n}$. *Then De Morgan's laws are valid*

i. $([a_{ij}]\tilde{\cup}[b_{ij}])^c = [a_{ij}]^c\tilde{\cap}[b_{ij}]^c$

ii. $([a_{ij}]\tilde{\cap}[b_{ij}])^c = [a_{ij}]^c\tilde{\cup}[b_{ij}]^c$

Proof. *i.*

$$\begin{aligned}
([a_{ij}]\tilde{\cup}[b_{ij}])^c &= ([(T^a_{ij}, I^a_{ij}, F^a_{ij})]\tilde{\cup}[(T^b_{ij}, I^b_{ij}, F^b_{ij})])^c \\
&= [(\max\{T^a_{ij}, T^b_{ij}\}, \min\{I^a_{ij}, I^b_{ij}\}, \min\{F^a_{ij}, F^b_{ij}\})]^c \\
&= [(\min\{F^a_{ij}, F^b_{ij}\}, \max\{1 - I^a_{ij}, 1 - I^b_{ij}\}, \max\{T^a_{ij}, T^b_{ij}\}))] \\
&= [(F^a_{ij}, I^a_{ij}, T^a_{ij})]\tilde{\cap}[(F^b_{ij}, I^b_{ij}, T^b_{ij})] \\
&= [a_{ij}]^c\tilde{\cap}[b_{ij}]^c
\end{aligned}$$

i.

$$\begin{aligned}
([a_{ij}]\tilde{\cap}[b_{ij}])^c &= ([(T^a_{ij}, I^a_{ij}, F^a_{ij})]\tilde{\cap}[(T^b_{ij}, I^b_{ij}, F^b_{ij})])^c \\
&= [(\min\{T^a_{ij}, T^b_{ij}\}, \max\{I^a_{ij}, I^b_{ij}\}, \max\{F^a_{ij}, F^b_{ij}\})]^c \\
&= [(\max\{F^a_{ij}, F^b_{ij}\}, \min\{1 - I^a_{ij}, 1 - I^b_{ij}\}, \min\{T^a_{ij}, T^b_{ij}\}))] \\
&= [(F^a_{ij}, I^a_{ij}, T^a_{ij})]\tilde{\cup}[(F^b_{ij}, I^b_{ij}, T^b_{ij})] \\
&= [a_{ij}]^c\tilde{\cup}[b_{ij}]^c
\end{aligned}$$

Proposition 8. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then*

i. $[a_{ij}]\tilde{\cap}([b_{ij}]\tilde{\cup}[c_{ij}]) = ([a_{ij}]\tilde{\cap}([b_{ij}])\tilde{\cup}([a_{ij}]\tilde{\cap}[c_{ij}])$

ii. $[a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cap}[c_{ij}]) = ([a_{ij}]\tilde{\cup}([b_{ij}])\tilde{\cap}([a_{ij}]\tilde{\cup}[c_{ij}])$

5. Products of NS-Matrices

In this section, we define two special products of NS-matrices to construct soft decision making methods.

Definition 30. *Let* $[a_{ij}], [b_{ik}] \in \widetilde{N}_{m\times n}$. *Then* And-*product of* $[a_{ij}]$ *and* $[b_{ij}]$ *is defined by*

$$\wedge : \widetilde{N}_{m\times n} \times \widetilde{N}_{m\times n} \to \widetilde{N}_{m\times n^2} \qquad [a_{ij}] \wedge [b_{ik}] = [c_{ip}] = (T^c_{ip}, I^c_{ip}, F^c_{ip})$$

where

$T^c_{ip} = t(T^a_{ij}, T^b_{jk})$, $I^c_{ip} = s(I^a_{ij}, I^b_{jk})$ *and* $F^c_{ip} = s(F^a_{ij}, F^b_{jk})$ *such that* $p = n(j-1)+k$

Definition 31. *Let* $[a_{ij}], [b_{ik}] \in \widetilde{N}_{m\times n}$. *Then* And-*product of* $[a_{ij}]$ *and* $[b_{ij}]$ *is defined by*

$$\vee : \widetilde{N}_{m\times n} \times \widetilde{N}_{m\times n} \to \widetilde{N}_{m\times n^2} \qquad [a_{ij}] \vee [b_{ik}] = [c_{ip}] = (T^c_{ip}, I^c_{ip}, F^c_{ip})$$

where

$T^c_{ip} = s(T^a_{ij}, T^b_{jk})$, $I^c_{ip} = t(I^a_{ij}, I^b_{jk})$ *and* $F^c_{ip} = t(F^a_{ij}, F^b_{jk})$ *such that* $p = n(j-1)+k$

Example 11. *Assume that* $[a_{ij}], [b_{ik}] \in \widetilde{N}_{3\times 2}$ *are given as follows*

$$[a_{ij}] = \begin{bmatrix} (1.0, 0.1, 0.1) & (1.0, 0.4, 0.1) \\ (1.0, 0.2, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.7, 0.1) \end{bmatrix}$$

$$[b_{ij}] = \begin{bmatrix} (1.0, 0.7, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.5, 0.1) \end{bmatrix}$$

$$[a_{ij}] \wedge [b_{ij}] = \begin{bmatrix} (1.0, 0.7, 0.1) & (1.0, 0.1, 0.1) & (1.0, 0.7, 0.1) & (1.0, 0.4, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) & (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.8, 0.1) & (1.0, 0.7, 0.1) & (1.0, 0.7, 0.1) \end{bmatrix}$$

$$[a_{ij}] \vee [b_{ij}] = \begin{bmatrix} (1.0, 0.1, 0.1) & (1.0, 0.1, 0.1) & (1.0, 0.4, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.2, 0.1) & (1.0, 0.2, 0.1) & (1.0, 0.1, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.8, 0.1) & (1.0, 0.5, 0.1) & (1.0, 0.5, 0.1) \end{bmatrix}$$

Proposition 9. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in \widetilde{N}_{m\times n}$. *Then the De morgan's types of results are true.*

i. $([a_{ij}] \vee [b_{ij}])^c = [a_{ij}]^c \wedge [b_{ij}]^c$

ii. $([a_{ij}] \wedge [b_{ij}])^c = [a_{ij}]^c \vee [b_{ij}]^c$

6. Decision making problem using and-product of neutrosophic soft matrices

Definition 32. *[13] Let* $[(\mu_{ip}, \nu_{ip}, w_{ip})] \in NSM_{m\times n^2}$, $I_k = \{p : (\mu_{ip}, \nu_{ip}, w_{ip}) \neq 0$, *for some* $1 \leq i \leq m$, $(k-1)n < p \leqslant kn$ *for all* $k \in I = \{1, 2, ..., n\}$. *Then NS-max-min decision function, denoted* D_{mMM}, *is defined as follows*

$$D_{mMM} : NSM_{m\times n^2} \to NSM_{m\times 1},$$

$$D_{mMM} = [(\mu_{ip}, \nu_{ip}, w_{ip})] = [d_{i1}] = [(\max_k\{\mu'_{ipk}\}, \{\nu'_{ipk}\}, \min_k\{w'_{ipk}\})]$$

where

$$\mu'_{ipk} = \begin{cases} \max_{p\in I_k}\{\mu_{ipk}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

$$\nu'_{ipk} = \begin{cases} \min_{p\in I_k}\{\nu_{ipk}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

$$w'_{ipk} = \begin{cases} \min_{p\in I_k}\{w_{ipk}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

The one column fs*-matrix* $Mm[c_{ip}]$ *is called max-min decision* fs*-matrix.*

Definition 33. *Let* $U = \{u_1, u_2, u_3, u_m\}$ *be the universe and* $D_{mMM}(\mu_{ip}, \nu_{ip}, w_{ip}) = [d_{i1}]$. *Then the set defined by*

$$opt^m_{[d_{i1}]}(U) = \{u_i/d_i : u_i \in U, d_i = max\{s_i\}\},$$

where $s_i = \mu_{ip} - \nu_{ip}, w_{ip}, d_{i1} \neq 0$ *which is called an optimum fuzzy set on U.*

Algorithm
The algorithm for the solution is given below
Step 1: Choose feasible subset of the set of parameters.
Step 2: Construct the neutrosophic matrices for each parameter.
Step 3: Choose a product of the neutrosophic matrices ,
Step 4: Find the method min-max-max decision N-matrices.
Step 5: Find an optimum fuzzy set on U.

Remark 1. *We can also define NS-matrices max-min-min decision making methods. One of them may be more useful than the others according to the type of problem.*

Case study: Assume that , a car dealer stores three different types of cars $U = \{u_1, u_2, u_3\}$ which may be characterize by the set of parameters $E = \{e_1, e_2\}$ where e_1 stands for costly , e_2 stands for fuel effciency. Then we consider the following example. Suppose a couple Mr. X and Mrs. X come to the dealer to buy a car before Dugra Puja. If each partner has to consider his/her own set of parameters, then we select the car on the basis of partner's parameters by using NS-matrices min-max-max decision making as follow.

Step 1: First Mr.X and Mrs.X have to chose the sets of their parameters $A = \{e_1, e_2\}$ and $B = \{e_1, e_2\}$, respectively.

Step 2:Then we construct the NS-matrices $[a_{ij}]$ and $[b_{ij}]$ according to their set of parameters A and B, respectively, as follow:

$$[a_{ij}] = \begin{bmatrix} (1.0, 0.1, 0.1) & (1.0, 0.4, 0.1) \\ (1.0, 0.2, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.7, 0.1) \end{bmatrix}$$

and

$$[b_{ij}] = \begin{bmatrix} (1.0, 0.7, 0.1) & (1.0, 0.1, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.5, 0.1) \end{bmatrix}$$

Step 3: Now ,we can find the And-product of the NS-matrices $[a_{ij}]$ and $[b_{ij}]$ as follow:

$$[a_{ij}] \wedge [b_{ij}] = \begin{bmatrix} (1.0, 0.7, 0.1) & (1.0, 0.1, 0.1) & (1.0, 0.7, 0.1) & (1.0, 0.4, 0.1) \\ (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) & (1.0, 0.5, 0.1) & (1.0, 0.2, 0.1) \\ (1.0, 0.8, 0.1) & (1.0, 0.8, 0.1) & (1.0, 0.7, 0.1) & (1.0, 0.7, 0.1) \end{bmatrix}$$

Step 4: Now,we calculate; for $i = \{1, 2, 3\}$

$$[d_{i1}] = \begin{bmatrix} (\mu_{11}, \nu_{11}, w_{11}) \\ (\mu_{21}, \nu_{21}, w_{21}) \\ (\mu_{31}, \nu_{31}, w_{31}) \end{bmatrix}$$

To demonstrate, let us find d_{21} for $i = 2$. Since $i = 2$ and $k \in \{1, 2\}$ so $d_{21} = (\mu_{21}, \nu_{21}, w_{21})$.

Let $t_{2k} = \{t_{21}, t_{22}\}$, where $t_{2k} = (\mu_{2p}, \nu_{2p}, w_{2p})$ then,

we have to find t_{2k} for all $k \in \{1, 2\}$. First to find t_{21}, $I_1 = \{p : 0 < p \leq 2\}$ for $k = 1$ and $n = 2$.

We have $t_{21} = (min\{\mu_{2p}\}, max\{\nu_{2p}\}, max\{w_{2p}\})$,
here $p = 1, 2$ $(min\{\mu_{21}, \mu_{22}\}, max\{\nu_{21}, \nu_{22}\}, max\{w_{21}, w_{22}\})$
$= (min\{1, 1\}, max\{0.5, 0.2\}, max\{0.1, 0.1\}) = (1, 0.5, 0.1)$ and
$t_{22} = (min\{\mu_{2p}\}, max\{\nu_{2p}\}, max\{w_{2p}\})$,
here $p = 3, 4$ $(min\{\mu_{23}, \mu_{24}\}, max\{\nu_{23}, \nu_{24}\}, max\{w_{23}, w_{24}\})$
$= (min\{1, 1\}, max\{0.5, 0.5\}, max\{0.1, 0.1\}) = (1, 0.5, 0.1)$
Similarly ,we can find d_{11} and d_{31} as $d_{11} = (1, 0.7, 0.1)$, $d_{31} = (1, 0.8, 0.1)$,

$$[d_{i1}] = \begin{bmatrix} (1, 0.7, 0.1) \\ (1, 0.5, 0.1) \\ (1, 0.8, 0.1) \end{bmatrix}$$

$$max[s_i] = \begin{bmatrix} 0.95 \\ 0.93 \\ 0.92 \end{bmatrix}$$

where $s_i = \mu_{11} - \nu_{11}.w_{11}$

Step 5: Finally , we can find an optimum fuzzy set on U as:

$$opt^2_{[d_{i1}]}(U) = \{u_1/0.95, u_2/0.93, u_3/0.92\}$$

Thus u_1 has the maximum value. Therefore the couple may decide to buy the car u_1.

7. Conclusion

In this paper we have redefine the notion of neutrosophic set in a new way and proposed the concept of neutrosophic soft matrix and after that different types of matrices in neutrosophic soft theory have been defiend.then we have introduced some new operations and properties on these matrices.

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