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RESEARCH ARTICLE

AN APPLICATION OF ESTIMATING CAPABILITY INDICES UNDER NON-NORMALITY USING S_B JOHNSON SYSTEM.

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Abstract

In discussion of process capability indices for normal and non-normal processes with symmetrical and asymmetrical tolerances it is noted that pertinent research work are based on either proposing new capability indices to purport better properties in certain circumstances or replacing existing by other procedures. In this paper an application is presented to estimate capability indices under non-normality using Johnson system for an earlier used data set. Numerous work have been published in literature to estimate these indices using Johnson system. This paper present an application of new learning approach to translate the measurements of non-normal process to Johnson distributions (S_B, S_U, S_L) countenanced the user to check foremost assumptions of estimating capability indices. In this approach the exact percentage points (0.135 lower and upper) are obtained acquiring the knowledge of density function before estimating capability indices under non-normality. Earlier these points are estimated from the process measurements come from non-normal process without knowledge of the density function. The procedure is illustrated by a data set which is transformed in S_B Johnson distribution and the percentage points are obtained from the proposed modified procedure results better while compare with existing procedure. The route is explained by flow chart and program is made in R-console.

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Introduction:-

Process capability indices PCIs always have been flourished for their uses and diversity. PCIs are the numerical quantities which raise the ability of a process related with process parameters and process preset values. For initially proposed PCIs process mean μ and process standard deviation σ are needed to develop functional relationship with process preset values as lower specification limit LSL, upper specification limit USL, target T, and mid-point m see for details Juran (1974), Kane (1986), Chan et al. (1988), Spiring (1991b), Pearn et al. (1992) among many others. It is also noted that these PCIs give false process fall out rates and erroneous results for measurements come from non-normal process because process parameters μ and σ are highly sensitive to departure from normality. For the process with asymmetrical tolerances ($T \neq m$) simply reflect the deviations from target and are less

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tolerable in one direction than other. For those processes the researchers observed the need of development of new PCIs which contains those process parameters that can characterize non-normal behavior sufficiently. More precisely two basic approaches are discussed so far in published literature to estimate PCIs for non-normal processes. First, estimate PCIs based on percentile method, most applied example is Clements percentile method (1989). He swapped \bar{X} by median M and 6σ , the difference of interval $(\mu - 3\sigma, \mu + 3\sigma)$ by 0.135th lower and upper quantiles for non-normal process from basic PCIs developed for normal processes such that the expected proportion of non-conforming NC should remain 0.27%. But Clements method needs the knowledge of coefficient of skewness and kurtosis which is not always easy to be obtained. Pearn, Kotz and Johnson (1992) discussed and improved the Clements method. Second approach based on translate non-normal process to normal using transformation methods few examples are Burr method (1942), Johnson transformation method (1949), Box-Cox power transformation method (1964) among others. This paper is an application of modified Johnson transformation method proposed by Safdar et al. (2019) for estimating capability indices for S_B distribution under non-normality. It is noted that this new learning and straightforward procedure prove indices while compared with earlier existing procedures of Pyzdek (1992) and Farnum (1997) under non-normality using Johnson system. The paper is organized as follows; Section 2 discussed shortly the capability indices and existing methods using families of distribution, Section 3 enlightened the new learning approach of estimating indices using Johnson method. Section 4 display a flow chart to explain the route of the algorithm made in R-Console. Section 5 illustrated new learning of estimating indices using Johnson System. Section 6 contains the discussion and conclusion about the new learning system.

Capability Indices and Existing Families of Distribution:

With individual families of distribution as Poisson, Binomial, Normal and many more there are variety of systems of distributions based on simple theory. Many systems of distribution have been discussed by numerous authors with their respective significance that will satisfactory represent observed data. Examples are Pearson System, Edgeworth Gram-Charlier Distributions and Johnson System of Distribution among others. Pearson system seeks to ascertain families of distribution that will satisfactory represent observed data. Various works have been proposed with the application of Pearson system to estimate the process capability indices for non-normal distributions see Bittani et al. (1998). The estimation of PCIs for non-normal processes using Pearson curves based on use of table giving the 0.00135 and 0.99865 percentiles of the standardized Pearson curves a functions of the values of skewness and kurtosis estimated from data but it suffers from two major drawbacks; first the table provides estimates for the required percentiles only over a limited range of skewness and kurtosis; second no information about the goodness of fit is available. Edgeworth (Gram-Charlier) Distributions approach seeks to represent a given density function as a series in the derivatives of the normal density function. Kocherlakota et al. (1992) established the distribution of \hat{C}_p when the process distribution $f(x)$ is a Gram Charlier Edgeworth series.

Where $f_X(x) = \left[1 - \frac{1}{6}\lambda_3 D^3 + \frac{1}{24}\lambda_4 D^4 + \frac{1}{72}\lambda_3^2 D^6 \right] \phi(z; 0, 1)$ remains positive for all X Where D_j signifies j th

derivative with respect to X , and $\lambda_3 = \frac{\mu_3(X)}{\{\mu_2(X)\}^{\frac{3}{2}}} = \sqrt{\beta_1}$ and $\lambda_4 = \frac{\mu_4(X)}{\{\mu_2(X)\}^2} - 3 = \beta_2 - 3$ are

standardized measures of skewness and kurtosis respectively. For the distribution of \hat{C}_p

$$E[\hat{C}_p] = \frac{(n-1)^{\frac{1}{2}} \Gamma\left(\frac{1}{2}(n-2)\right)}{\sqrt{2} \Gamma\left(\frac{1}{2}(n-1)\right)} C_p \left\{ 1 + \frac{3}{8} g_n (\beta_2 - 3) - \frac{3}{8} h_n \beta_1 \right\} \quad \text{where } g_n = \frac{n-1}{n(n+1)} \quad h_n = \frac{(n-2)}{n(n+1)(n+3)}.$$

The pre-multiplier of C_p will be recognized as the bias correlation factor $bn-1$ so

$$E[\hat{C}_p] = E[\hat{C}_p | \text{normal}] \left\{ 1 + \frac{3}{8} g_n (\beta_2 - 3) - \frac{3}{8} h_n \beta_1 \right\} \text{ and}$$

$$v(\hat{C}_p) = \frac{n-1}{n(n-3)} C_p^2 \left[1 + \frac{1}{n} \left\{ 1 + g_n(\beta_2 - 3) - \frac{1}{9} h_n \beta_1 \right\} \right] - \{E[\hat{C}_p]\}^2. \text{ They also carried out an investigation}$$

for the distribution of the natural estimator of C_{pu} as $\hat{C}_{pu} = \frac{USL - \bar{X}}{3S}$ where USL is the upper specification limit,

\bar{X} and S are the sample mean and sample standard deviation of the given set of measurements. If the process is symmetrical ($\beta_1 = 0$), $E[\hat{C}_{pu}]$ is proportional to C_{pu} and very nearly proportional to C_{pu} even if $\beta_1 \neq 0$. Of

course if β_1 is large, it will not be possible to represent the distribution in the Edgeworth form.

Johnson System of frequency curve was first developed by Johnson (1963). Farnum (1997) has given a detailed description on the use of Johnson Curves. For a complete description of this system, see Bowman and Shenton (1983); Johnson, Kotz, and Balakrishnan (1993) and Stuart and Ord (1987). Concisely, there are three distributions

(S_B, S_U, S_L) of Johnson curves having two shape (γ and η), one location (ϵ) and one scale (λ) real parameters.

These distributions are generated by transformations of the form $z = \gamma + \eta k_i(x; \lambda, \epsilon)$ where $k_i(x; \lambda, \epsilon)$ are chosen to cover a wide range of possible shapes and z is a standard normal variable.

$$\text{For Bounded } S_B \text{ curve } Z = \gamma + \eta \log\left(\frac{x - \epsilon}{\lambda + \epsilon - x}\right) \quad (\epsilon < x < \epsilon + \lambda)$$

$$\text{For Unbounded } S_U \text{ curve } Z = \gamma + \eta \sinh^{-1}\left(\frac{x - \epsilon}{\lambda}\right) \quad (-\infty < x < \infty)$$

$$\text{For Lognormal } S_L \text{ } Z = \gamma + \eta \log(x - \epsilon) \quad (\epsilon < x)$$

Pyzdek (1992) worked on S_B distribution and found capability index C_p , Farnum (1997) obtained indices C_p and C_{pk} using Johnson system and fitted S_U curve, Chung et al. (2007) proposed decision making rule using p-value for C_{pm} index by fitting S_L distribution. Safdar et al. (2019) presented a quick and straightforward modified procedure for non-normal process to estimate capability indices using Johnson system and a work example for S_U distribution was illustrated. This paper is an application of the procedure for an earlier used data set to estimate capability indices using S_B distribution. In section 3 the new learning system of estimating capability indices under non-normality using Johnson system is presented

Modified Procedure to Estimate Capability Indices using Johnson System:

In our procedure basic capability indices are first estimated using Vannman (1995) superstructure defined as the ratio of actual spread (based on preset values) and allowable spread (based on process parameters μ and σ which are sensitive to departure from normality and cannot characterize non-normal process).

$$C_p(u, v) = \frac{d - u|\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}} \quad u \geq 0; v \geq 0 \quad (1)$$

Such that,

$$C_p(0, 0) = C_p, C_p(1, 0) = C_{pk}, C_p(0, 1) = C_{pm} \text{ and } C_p(1, 1) = C_{pmk}.$$

Where T is the target value, d is half-length of specification interval i.e. $d = \frac{1}{2}(USL - LSL)$; and

$$m = \frac{1}{2}(USL + LSL); \text{ the midpoint between upper and lower specification limits.}$$

The proportion of non-conforming NC and process yield for each basic PCIs are also obtained for industrial and manufacturing purpose showing whether NC and process yields are tolerable for production.

Process measurements are then transformed to the best fitted Johnson system and curve is drawn to the given set of measurements come from non-normal process to check the adequacy of the fitted model. Samples (X -variates of Johnson System) of size $n=49, 99, 199, 499, 999, 1499$ and 1999 are simulated. Transformed x -variate and preset

values to z-variate of Jonson system. For x-variate and z-variate of Johnson system, test the normality assumption by drawing NPP and applying Shapiro-Wilk normality test.

For estimating capability indices using Johnson system, Pearn and Chan (1997) superstructure is used as under;

$$C_p(u, v) = \frac{d - u|\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}} \quad u \geq 0; v \geq 0 \quad (2)$$

Such that $C_{NP}(0,0) = C_{NP}$, $C_{NP}(1,0) = C_{Npk}$, $C_{NP}(0,1) = C_{Npm}$ and $C_{NP}(1,1) = C_{Npmk}$.

Where $F_{0.135}$, Median M and $F_{0.99865}$ are the 0.135th, 0.5th and 0.99865th percentage points of non-normal processes. Pyzdek (1992) and Farnum (1997) estimated these points from the process measurements by approximate method without acquiring the prior knowledge of density function exhibited by the data set. For our new capability calculations these points are obtained from the best fitted Johnson curve of the process measurements coming from non-normal process.

$$\begin{aligned} p(-\infty < X < \hat{F}_{0.135}) &= 0.00135; \\ p(-\infty < X < \hat{F}_{99.865}) &= 0.99865; \\ p(-\infty < X < M) &= 0.5 \end{aligned} \quad (3)$$

For our example we estimated capability indices using Equation (2) for each simulated sample drawn from S_B Johnson curve where the percentage points are obtained by approximate method (Pyzdek 1992, Farnum 1997) and named those indices as **John (Z)** and for our new learning procedure the percentage points are obtained using Equation (3) and named as **JPCI**.

Section 4 display a Flow chart to lustrate our procedure to estimate PCIs easily.

Flow Chart:

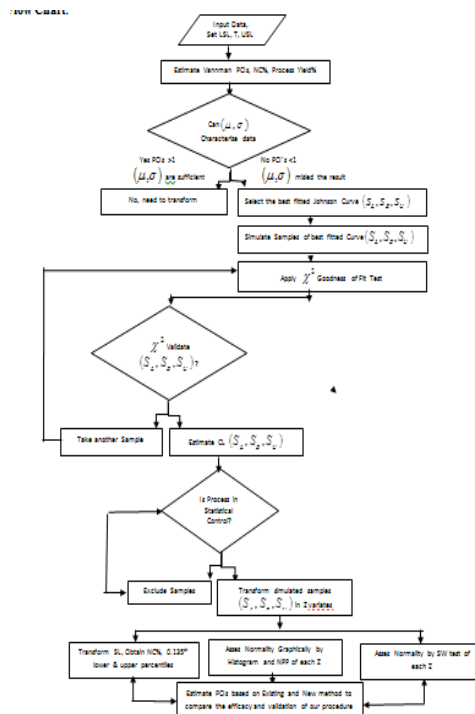


Fig 1:-Flow Chart explaining the New Learning Procedure of Estimating Capability Indices

Illustration:

Pearn and Chen (1997) estimated capability indices for non-normal process with an application in electrolytic capacitor manufacturing. He collected the data of capacitance of non-polarized with radial leads from an electronic

company of manufacturing of aluminum electrolytic capacitor. The collected sample data consisting of 100 observations of a non-normal distribution are displayed in Table 1. The preset specifications are LSL=285, T=300 USL=315

Table 1:-Non-Polarized (NP) with Radial Leads; Capacitance

292	293	294	294	294	294	294	294	295	295
295	295	295	295	295	296	296	296	297	297
297	297	297	298	298	298	298	298	298	298
299	299	299	300	300	300	300	300	301	301
301	301	302	302	302	302	302	302	302	303
303	303	303	303	303	304	304	304	304	304
305	305	305	305	305	305	306	306	306	306
306	306	307	307	307	308	308	308	308	309
309	309	309	309	309	310	310	310	311	312
312	313	313	313	313	315	316	319	320	324

First we estimate PCIs assuming that the observations come from normal distribution.

Table 2:-PCIs Assuming Normal Process

PCIs	Normal based PCIs	NC%	Process Yield%
C_p	0.759	2.28	97.77<99.73
C_{pk}	0.603	7.05	92.95<99.73
C_{pm}	0.687	3.93	96.07<99.73
C_{pmk}	0.545	10.2	89.80<99.73

Table 2 shows that if we run capability analysis for the experiment assuming normal distribution, large percentage of parts rejected were expected which cannot tolerate in any industrial or manufacturing production. There is a need to transform the data set.

With algorithm of estimating capability indices best fitted curve is found to be S_B (see Equation 4) after translating the data set under non-normality using Johnson system.

$$Z = 0.115 + 0.781 \log \left(\frac{x - 291.55}{24.37 + 291.55 - x} \right) (291.55 < x < 315.92) \quad (4)$$

The original data set and simulated samples 'x' are transformed in Z. For each simulated transformed sample 0.135th lower and upper quantiles are obtained by earlier and modified method.

Each simulated sample of x and z of Johnson system the foremost assumption of estimating capability indices is checked graphically by NPP and statistically assess by applying Shapiro-Wilk normality test.

Table 3 comprises the results and showed that percentage points are found different by old and new method.

Table 3:-Original& Transformed Specification Limits, NC (PPM) and Johnson Percentiles by Approximate and Modified Method

Size	(Lp, Up) (Farnum)	(Lp, Up) (Modified)
original data	291.99	291.8
	315.3	315.8
49	294	296.66
	316	315.99
99	293.4	297.06
	315.2	315.19

199	291.7	296.44
	318.8	318.79
499	291.8	296.27
	316.3	316.29
999	291.8	295.97
	315.8	315.83
1499	291.6	295.94
	315.6	315.63
1999	291.9	295.89
	315.3	315.27

We also noted the difference between the percentiles obtained by Farnum and our method in Table 3. For one of the assumption we plot the original data and superimpose the selected Johnson curve to know the adequacy of the fitted model.

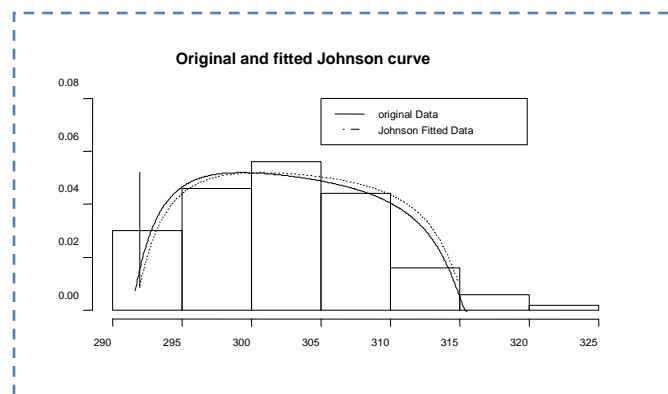


Fig 2:-Original and best fitted Johnson curve with probability Histogram

We have noted that the selected Johnson curve adequately fits the data specially on tails of the distribution. For normality assumption before estimating capability indices shapiro-Wilk test is performed and results are summarized in table 4.

Table 4:-Statistical Assessment of Normality using Shapiro-Wilk Test

Sample Size	Type	SW Statistics	P-value
Original data	X	0.965	0.010
	Z	0.990	0.698
49	X	0.948	0.036
	Z	0.981	0.627
99	X	0.971	0.029
	Z	0.988	0.545
199	X	0.969	0.000
	Z	0.987	0.076
499	X	0.971	0.000
	Z	0.994	0.061
999	X	0.972	0.000
	Z	0.996	0.015
1499	X	0.971	0.000
	Z	0.996	0.001

1999	X	0.970	0.0000
	Z	0.998	0.025

Preceding the above findings, finally we focus on the estimation of new capability calculations for indices using Existing and method with our modified method.

Table gives details of estimating capability indices while the percentage points are obtained by existing and modified methods.

Table 5:-PCIs under Johnson Distribution

Samples	Methods	C_p	C_{pk}	C_{pm}	C_{pmk}
Original Data	NPCI (Vannman)	0.759	0.603	0.687	0.546
	John(Z) (Pyzdek& Farnum)	1.288	0.974	0.944	0.718
	JPCI(Modified)	1.286	0.974	1.038	0.842
49	John(Z)	1.364	0.909	0.806	0.538
	JPCI	1.552	1.289	1.219	1.012
99	John(Z)	1.376	0.982	0.888	0.634
	JPCI	1.655	1.267	1.079	0.826
199	John(Z)	1.107	0.721	0.726	0.474
	JPCI	1.342	1.121	1.118	0.934
499	John(Z)	1.224	0.893	0.864	0.628
	JPCI	1.499	1.192	1.103	0.878
999	John(Z)	1.250	0.933	0.906	0.677
	JPCI	1.511	1.215	1.129	0.908
1499	John(Z)	1.250	0.950	0.929	0.706
	JPCI	1.524	1.217	1.121	0.896
1999	John(Z)	1.282	0.974	0.942	0.716
	JPCI	1.547	1.227	1.116	0.885

Table 5 summarizes the results that our new capability calculation **JPCI** based on estimating percentiles using Johnson density function improve results with existing method under normality.

Concluding Remarks

This paper is an application of the modified procedure of estimating capability indices for non-normal process using Johnson system. A data set is taken to exemplify the procedure of new learning system of estimating capability indices under non-normality using Johnson system. The results shown that the modified procedure based on obtaining percentage points of Pearn and Chan superstructure by first finding the density function of the process measurements improve the indices while compared with existing procedure in which the percentage points are obtained from the process measurements without having prior knowledge of density function. The program is made in R-Console and follows the algorithm estimating capability indices under non-normality using Johnson system. This procedure also allow user to run a straightforward procedure which first estimate capability indices, proportion of non-conforming and process yields assuming normal process to show that basic indices misled the result for non-normal process and transformation is needed. For the process measurements come from non-normal process the algorithm select the best fitted Johnson curve, draw fitted curve on process measurements to check the adequacy, simulate samples from estimated parameters of the selected Johnson distribution. The route of this new learning system check the assumptions before estimating capability indices of simulated samples. The tolerance region of process measurement is specified, normality assumption is checked graphically by drawing NPP and applying Shapiro-Wilk normality test of all simulated samples. The algorithm can also exclude those simulated samples which are beyond the limits of the selected Johnson curve to make the process in statistical control. After validating the assumptions before estimating capability indices the algorithm obtained lower and upper 0.135th percentage points for existing and new method under Johnson system. The results shows that the new modified procedure improve capability indices for each simulated sample.

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