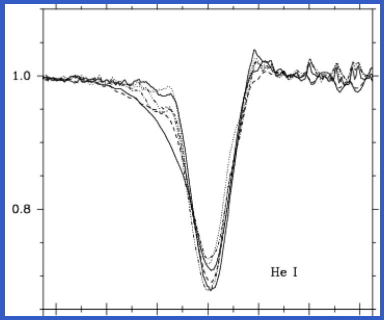
The Eddington limit in stars

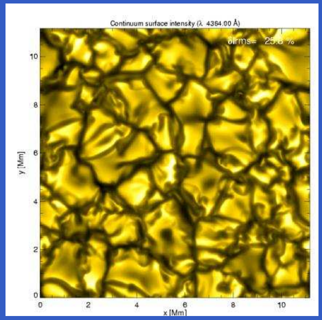
Norbert Langer (Bonn University)

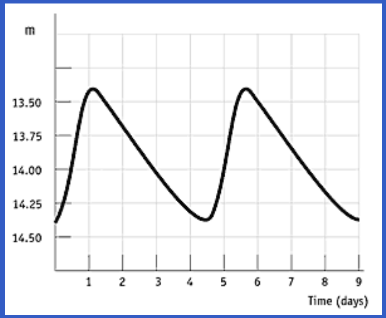
Stars evolve on nuclear time scale

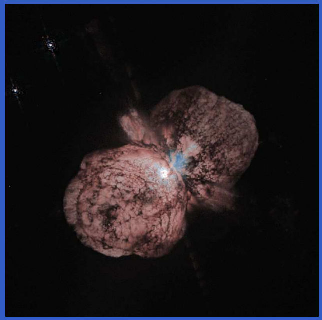
yet: we can watch them change

- spectral variability
- pulsations
- eruptions









⇒ stars have problems to "shine"

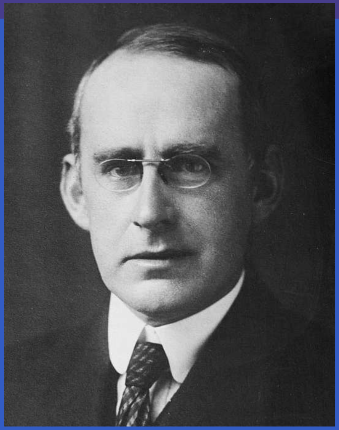
The stellar Eddington limit

acceleration through photon momentum balances gravity

$$\frac{\kappa F}{c} = g$$

$$\Rightarrow \boxed{L_{\mathrm{Edd}} = \frac{4\pi c G}{\kappa} M}$$

$$\Gamma = L/L_{\mathrm{Edd}}$$



A. Eddington (1882-1944)

$L > L_{\mathrm{Edd}} \Rightarrow$?

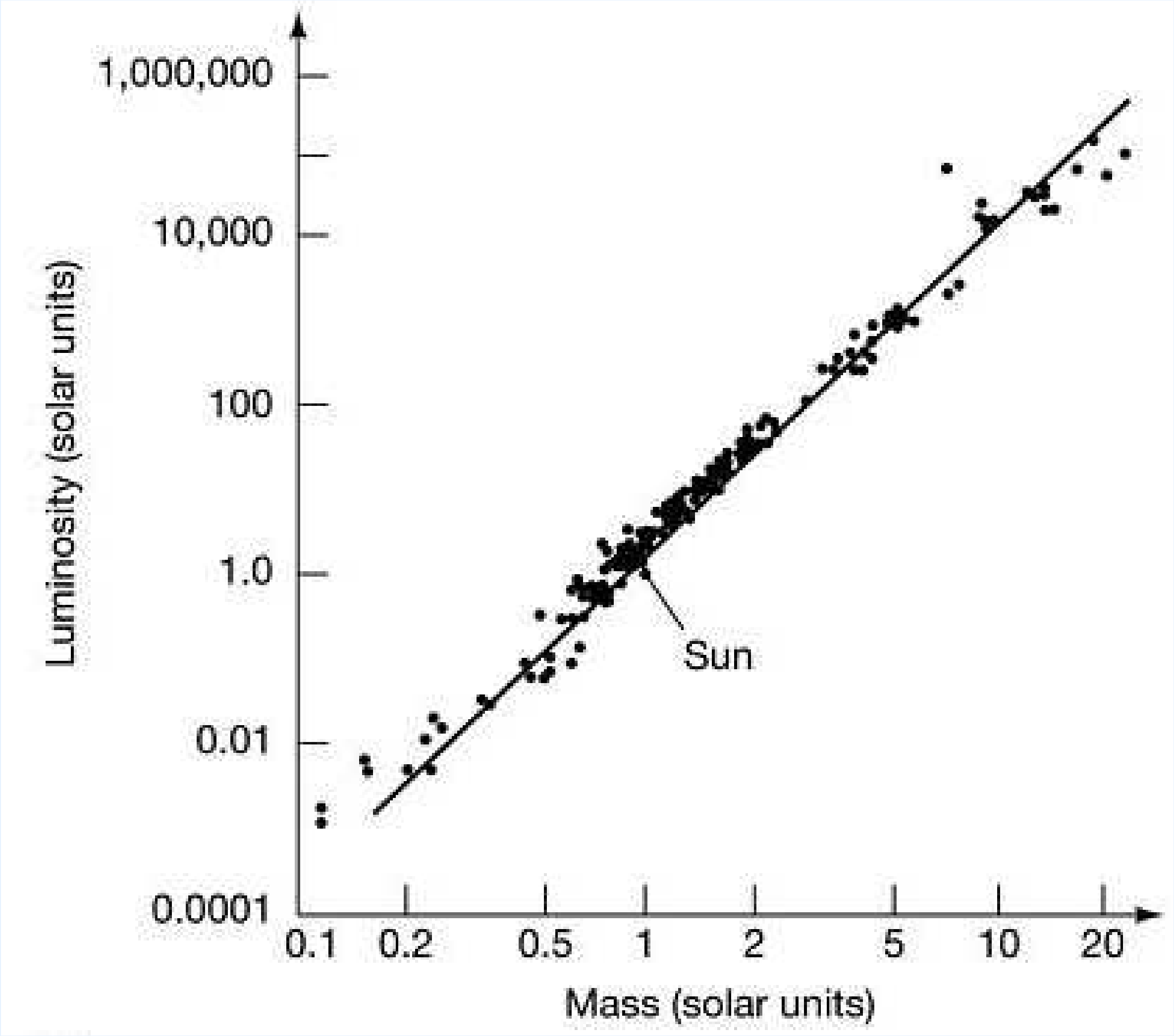
Mass-luminosity relation

$$L \sim M^{\alpha}\mu^{\delta}$$

- holds for main sequence stars (core H-burning)
- is observationally well tested
- $M \to \infty \Rightarrow \alpha \to 1$

(Kippenhahn & Weigert 1990)

Mass-luminosity relation



E.L.: only electron scattering

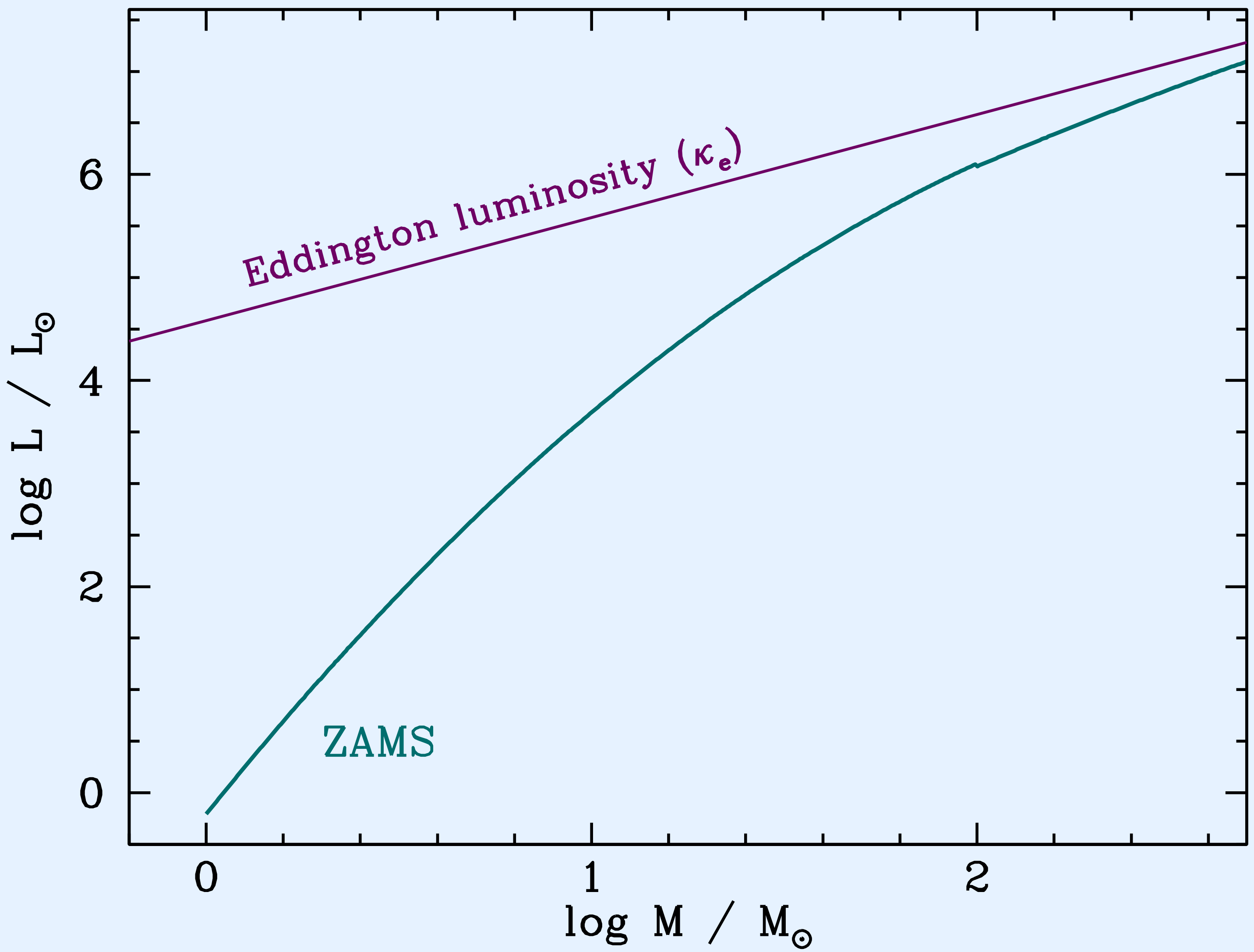
$$L_{\rm Edd} = \frac{4\pi c G}{\kappa} M$$

Eddington: $\kappa = \kappa_{\rm e} = 0.2(1+X) \simeq 0.34\,{\rm cm^2 g^{-1}}$

$$\Rightarrow \frac{L_{\rm Edd}}{L_\odot} \simeq 40000 \frac{M}{M_\odot}$$

- Sun: $L_\odot \rightarrow \frac{L}{L_{\rm Edd}} \simeq 0.00003$
- 10 $M_\odot$: $10^4 L_\odot \rightarrow \frac{L}{L_{\rm Edd}} \simeq 0.03$
- 100 $M_\odot$: $10^4 L_\odot \rightarrow \frac{L}{L_{\rm Edd}} \simeq 0.3$
- 10^5 $M_\odot$ $\rightarrow L = L_{\rm Edd}$ (S. Kato 1986)

$M - L_{\text{Eddington}}$ relation

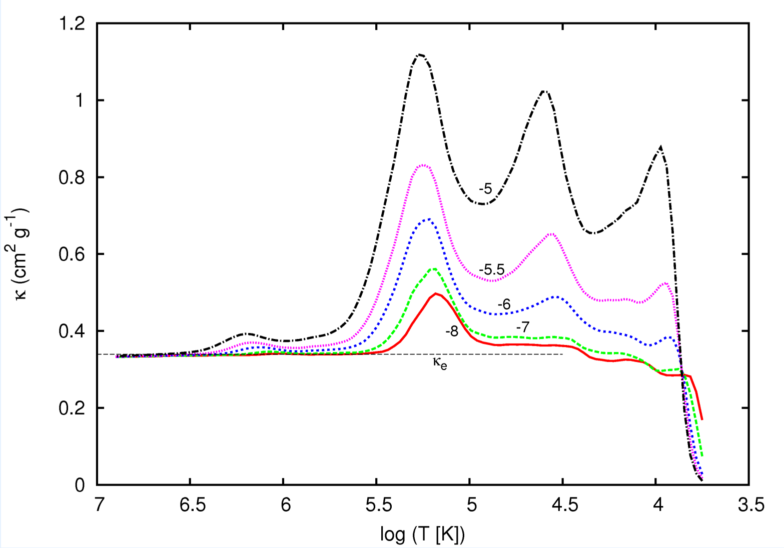


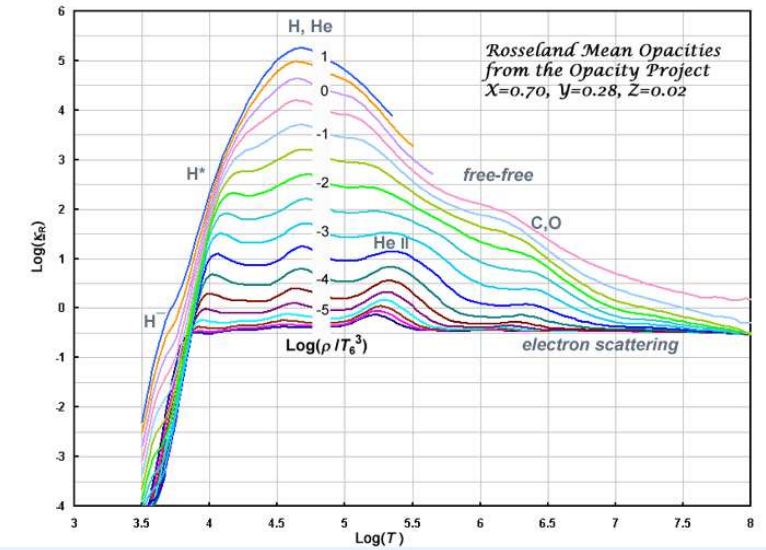
Opacity of stellar matter

- deep interior:
  $T \uparrow \Rightarrow$ complete ionisation $\Rightarrow \kappa_{\text{es}}$
- near the surface:
  $T \downarrow \Rightarrow$ (partial) recombination $\Rightarrow \kappa \uparrow$

complete opacity

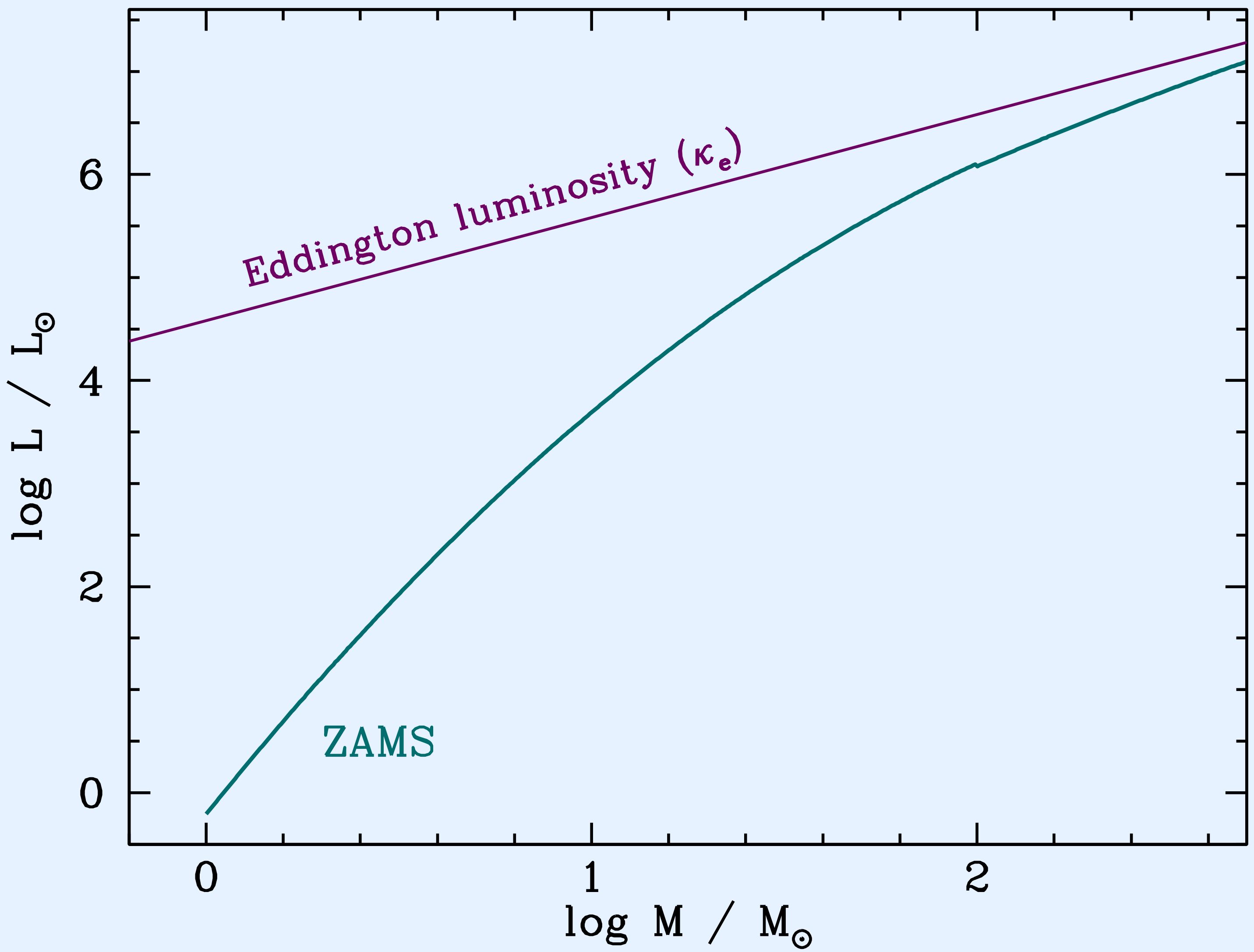
complete opacity: e^{-}-scattering + ff + bf + bb + . . .



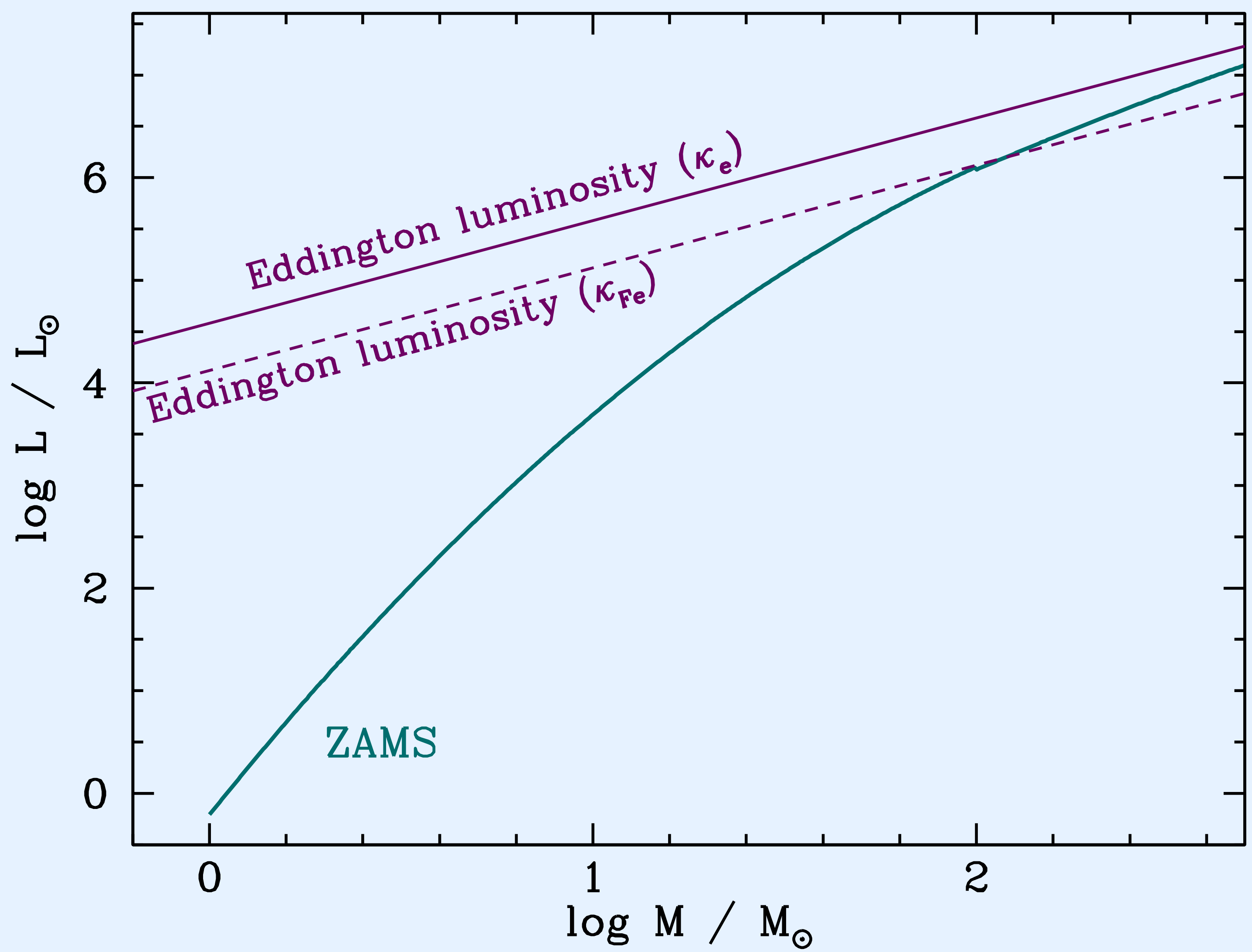


- hot stars: $\kappa_{\rm Fe} \simeq 2\kappa_{\rm e}$
- cool stars: $\kappa_{\rm H} \simeq 1000\kappa_{\rm e}$

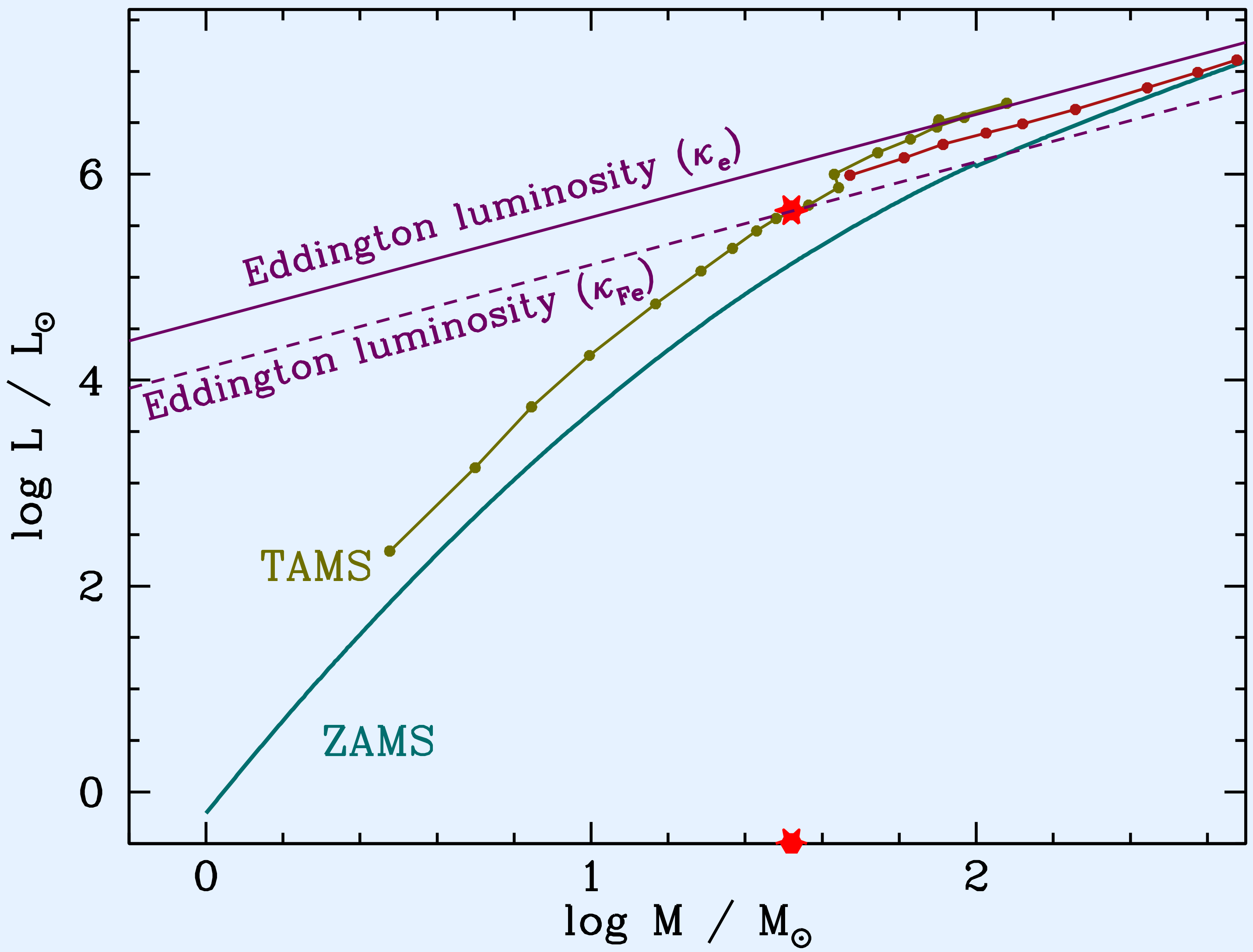
$M - L_{\rm Eddington}$ relation



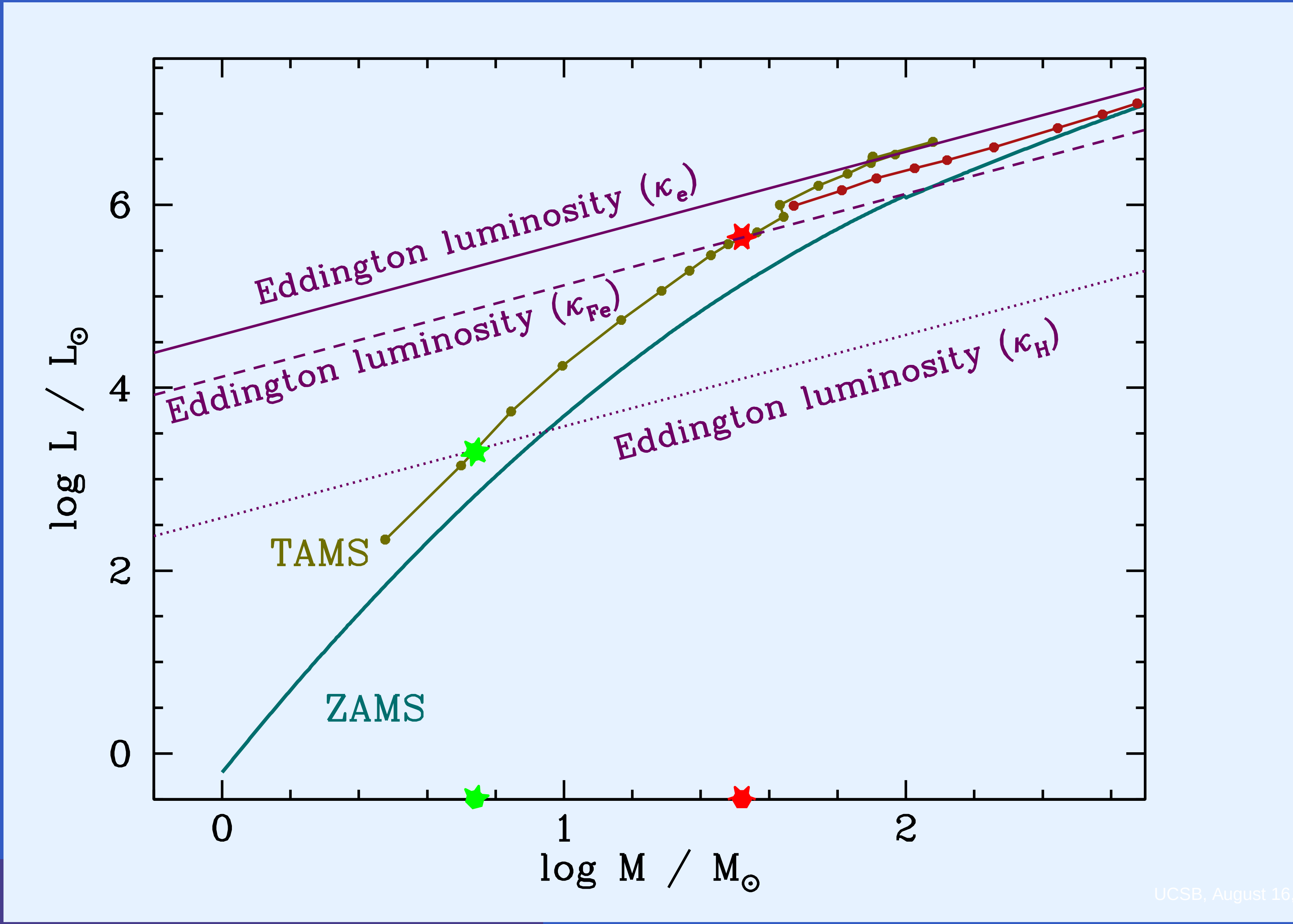
$M - L_{\text{Eddington}}$ relation



$M - L_{\rm Eddington}$ relation



$M - L_{\text{Eddington}}$ relation

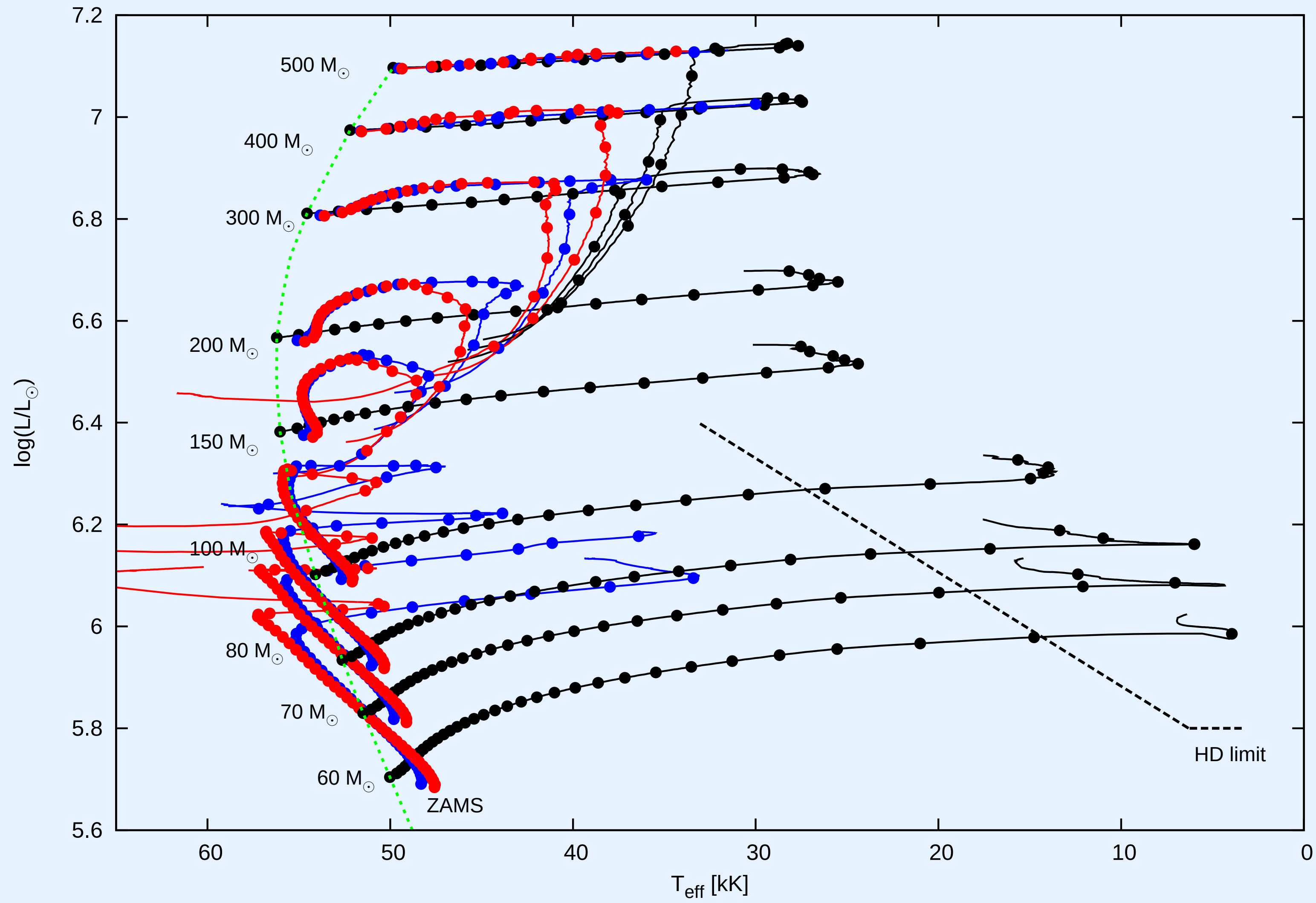


The Eddington factor

- only electron scattering:
  $\Gamma_{\mathrm{es}} = L/L_{\mathrm{Edd}} = \frac{1}{4\pi cG}\kappa_{\mathrm{es}}\frac{L}{M}$
- full opacity:
  $\Gamma = \frac{1}{4\pi cG}\kappa\frac{L}{M}$
- in the interior:
  $\Gamma(r) = \frac{1}{4\pi cG}\kappa\frac{L_{\mathrm{rad}}}{M}$
  $\Gamma(r) \leq 1$ is is a sufficient but not a necessary stability criterion
  $\Gamma_{\mathrm{max}} = max\left(\Gamma(r)\right)$

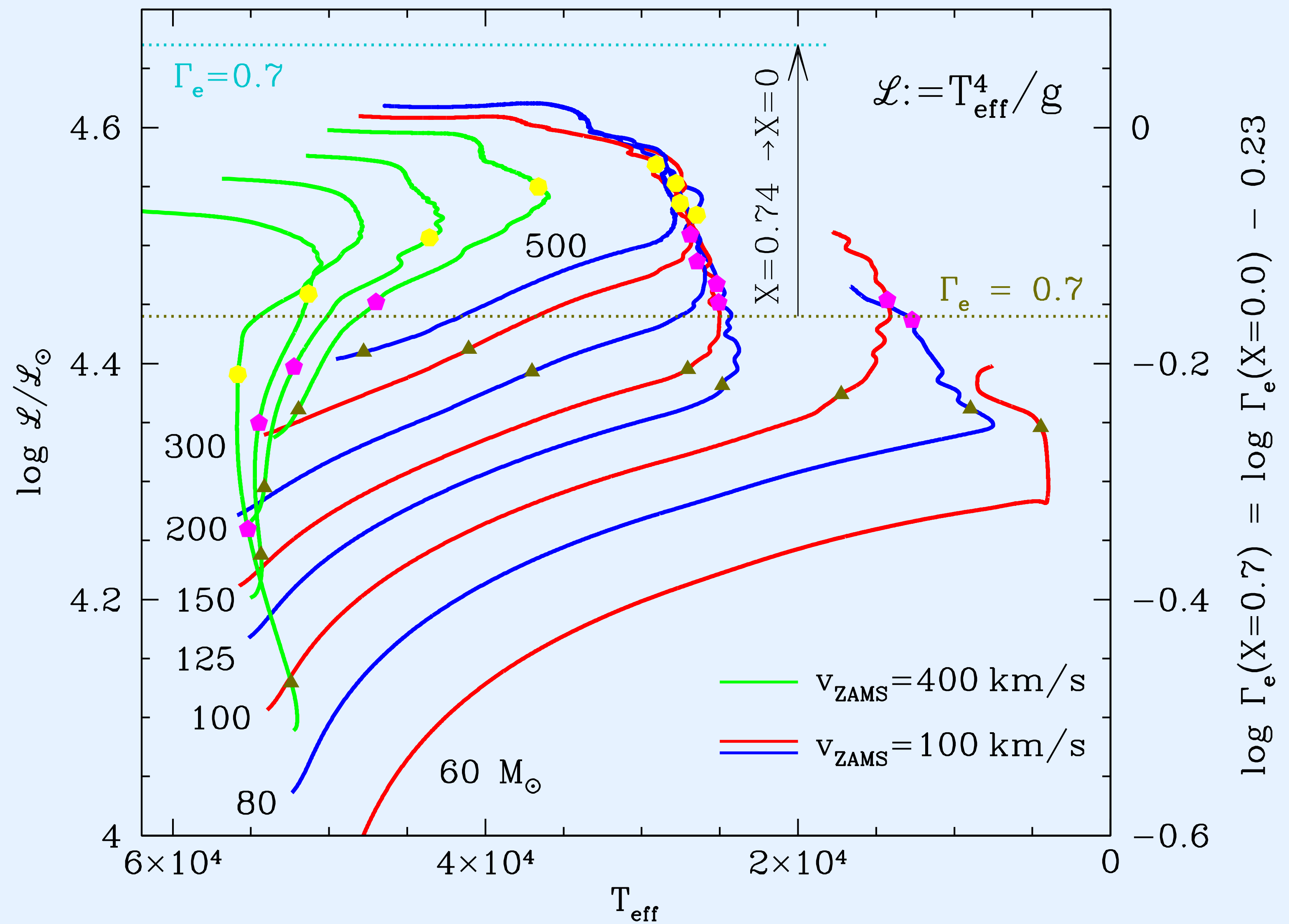
Start Minilab 1

Massive (hot) stars: HR diagram



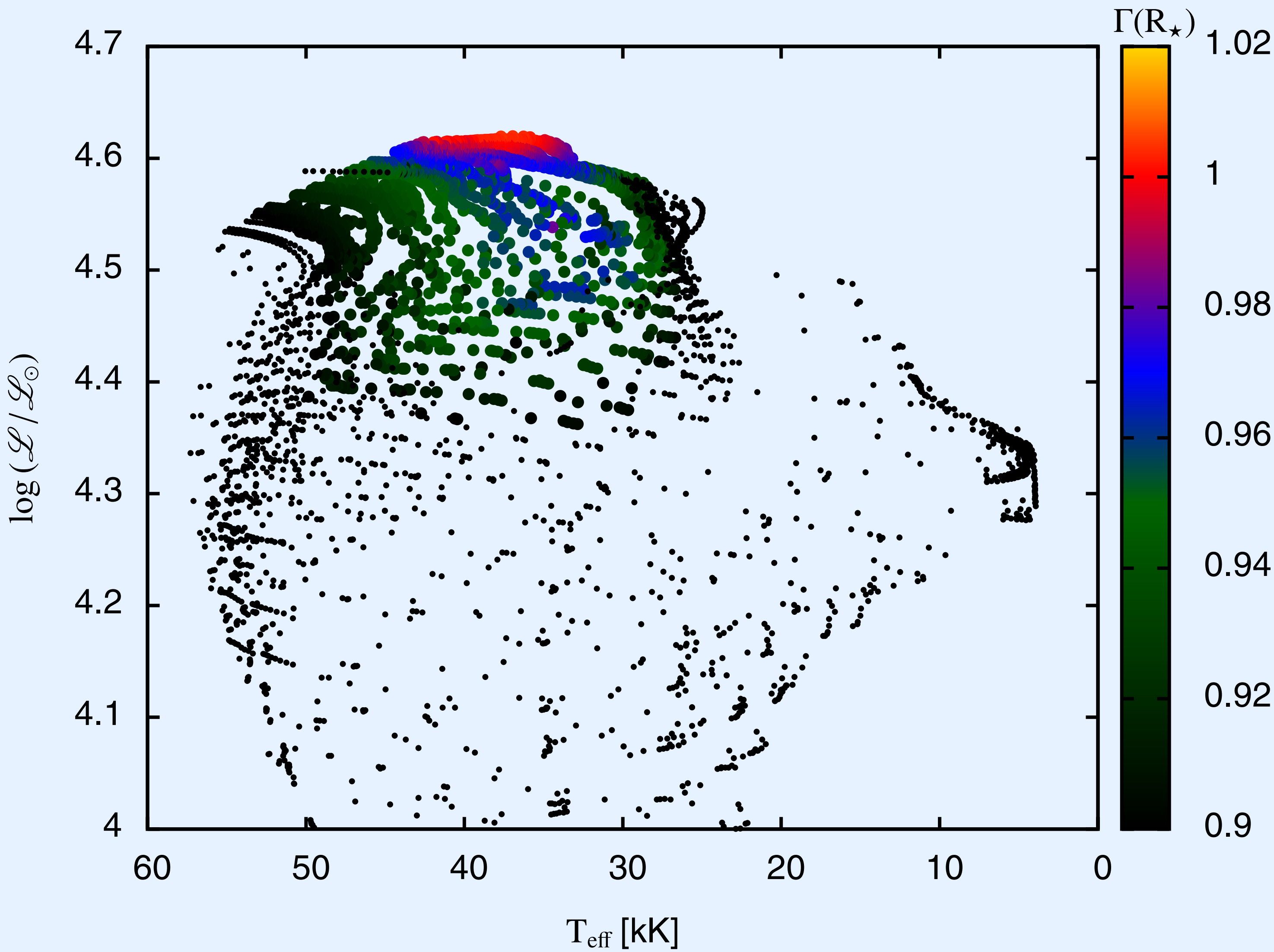
Köhler et al. 2015

Massive (hot) stars: sHR diagram



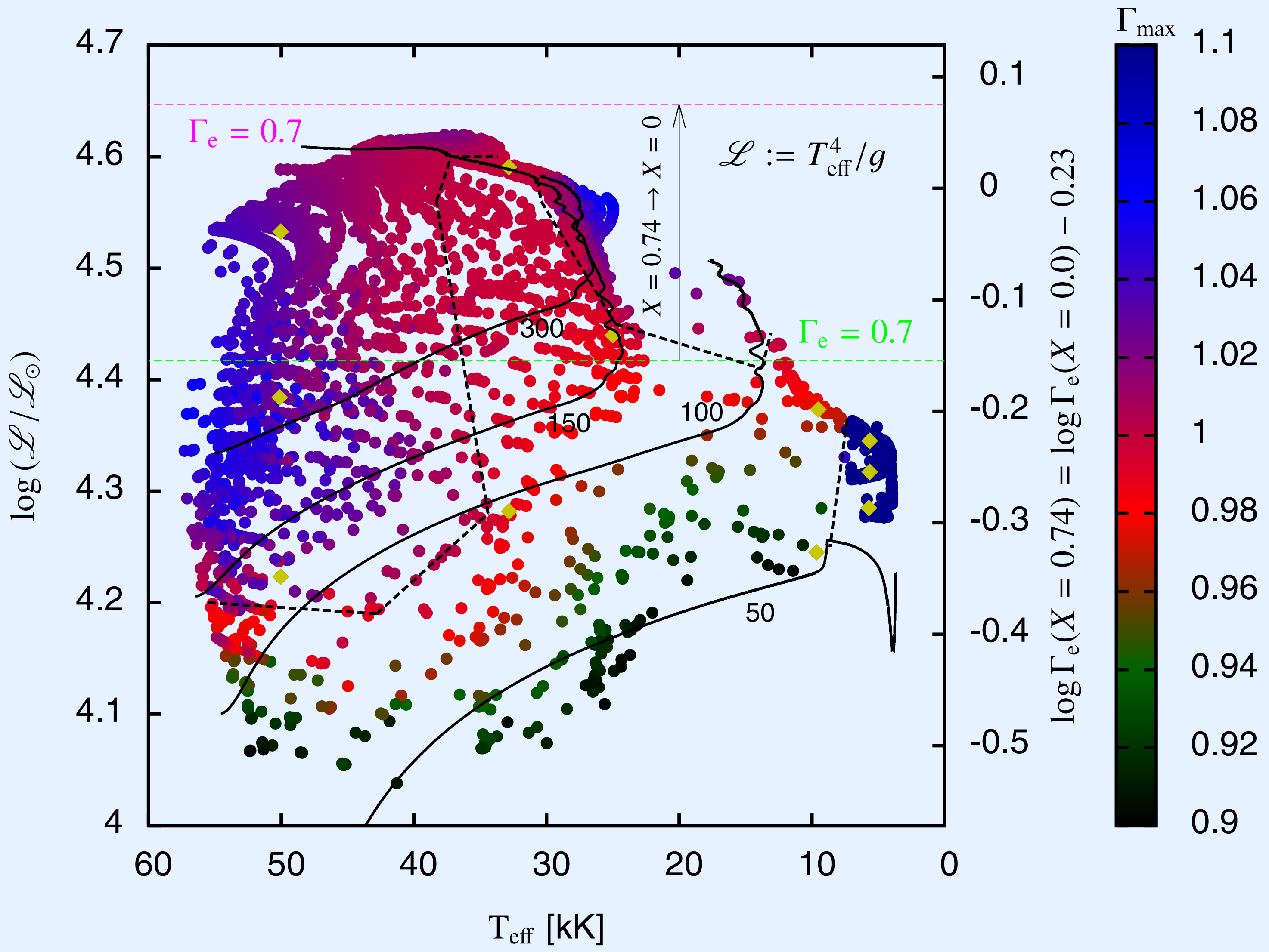
Langer & Kudritzki 2014

Surface Eddington factor



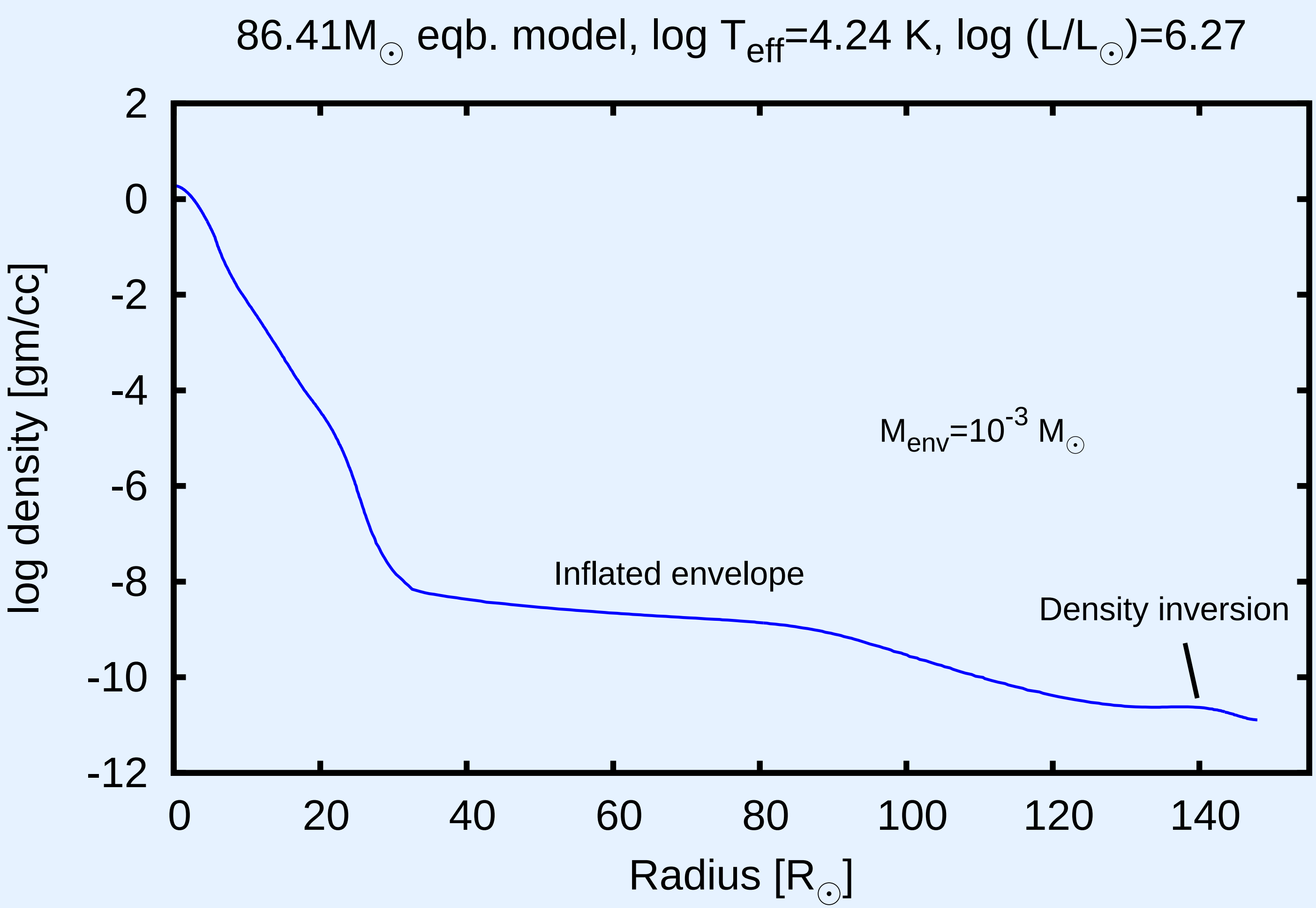
Sanyal et al. 2015

Sub-surface Eddington factor



Sanyal et al. 2015

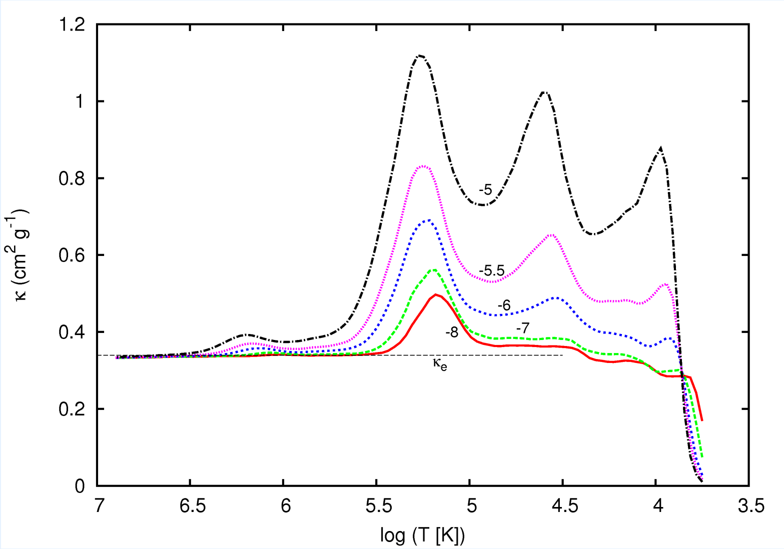
Inflation: 1D

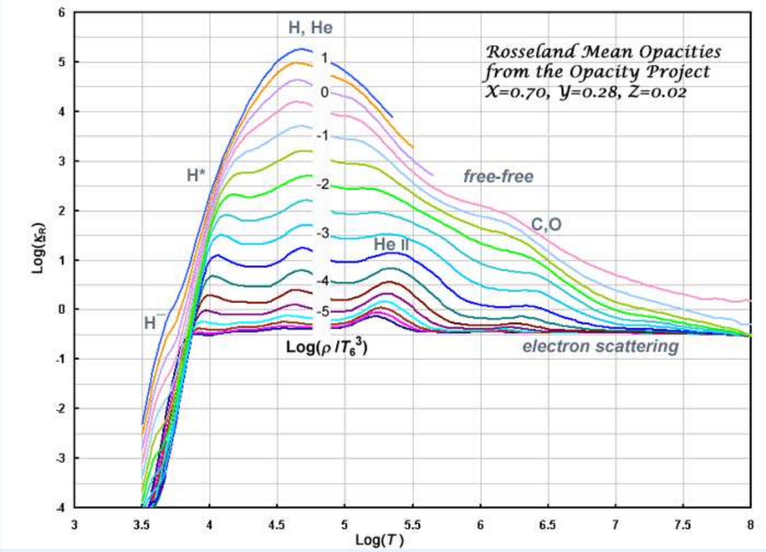


Sanyal et al. 2015

complete opacity

complete opacity: e^{-}-scattering + ff + bf + bb + . . .





- atomic opacity increases with density

Analytics

$$\frac{dP}{dr} = \frac{dP_{\rm gas}}{dr} + \frac{dP_{\rm rad}}{dr} = -\rho g \qquad (1)$$

$$\frac{dP_{\rm rad}}{dr} = \frac{a}{3}\frac{dT^4}{dr} = \frac{a}{3}\frac{3\kappa\rho}{ac}\frac{1}{4\pi r^2}L_{\rm rad} = \frac{\rho G M}{r^2}\frac{\kappa L_{\rm rad}}{4\pi c G M}$$
$$= -g\rho\Gamma$$

$$\Rightarrow \boxed{\frac{1}{g\rho}\frac{dP_{\rm gas}}{dr} = \Gamma - 1} \qquad (2)$$

$$\Rightarrow \Gamma > 1 \rightarrow \frac{dP_{\rm gas}}{dr} > 0 \rightarrow \frac{d\rho}{dr} > 0$$

Joss et al. 1973, Paxton et al. 2013

Analytics II

(1) / (2)

$$\Rightarrow \frac{1}{\rho g}\frac{dP_{\mathrm{gas}}}{dP} = -\frac{\Gamma-1}{\rho g}$$

$$\beta := \frac{P_{\mathrm{gas}}}{P}$$

$$\Rightarrow \frac{d(\beta P)}{dP} = \beta + P\frac{d\beta}{dP} = 1-\Gamma$$

$$\beta = \mathrm{const.}$$

$$\Rightarrow \boxed{\beta = 1-\Gamma} \qquad (3)$$

Gräfener & Owocki 2012, Sanyal et al. 2016

Analytics III

$$H_\rho := \frac{d \ln \rho}{dr}$$ and (2)

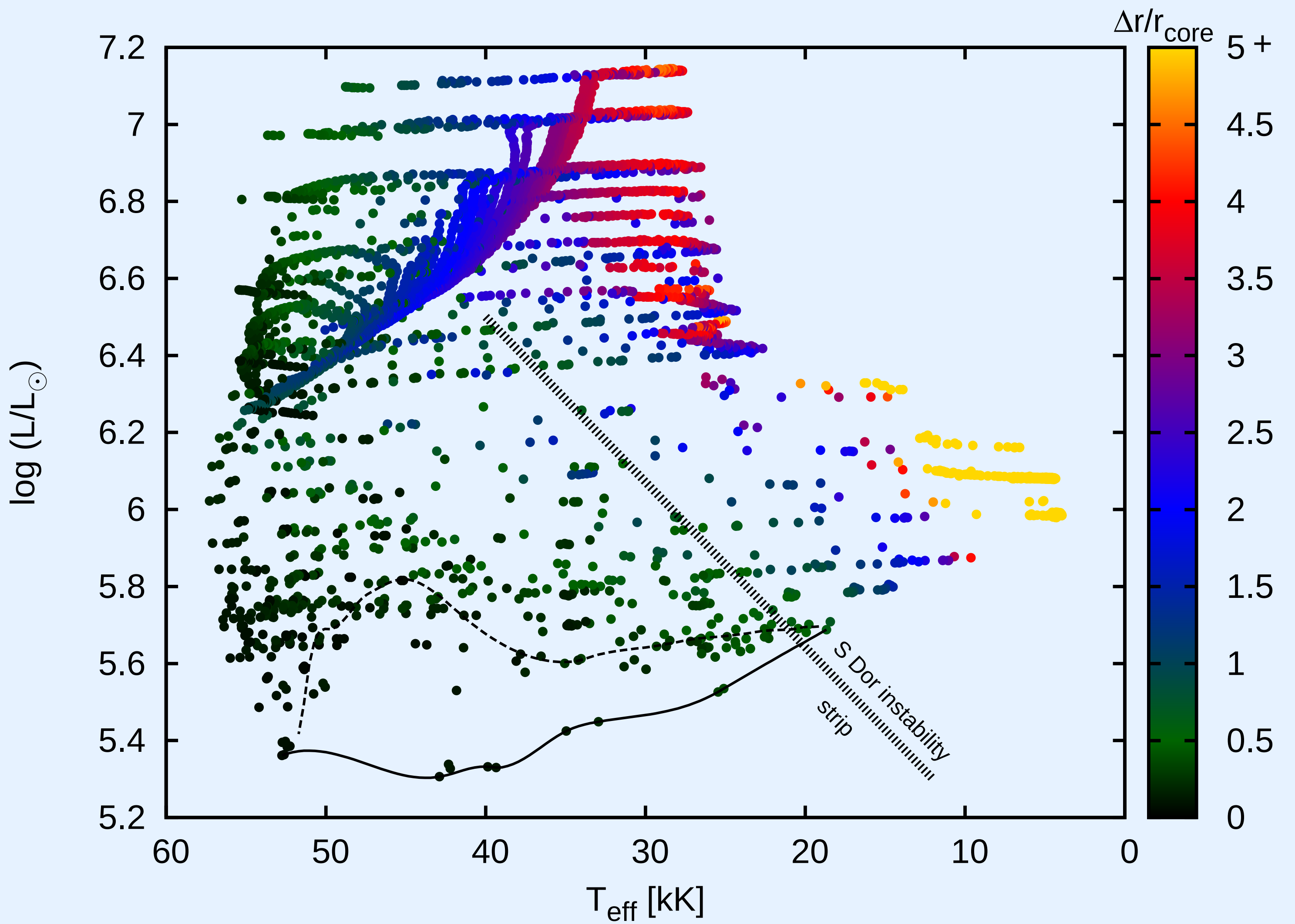
$$\Rightarrow \frac{1}{H_\rho} = \frac{g\mu}{\Re T}(\Gamma - 1 + \beta\nabla)$$

(3) $$\Rightarrow \frac{1}{H_\rho} = \frac{g\mu}{\Re T}(\nabla + 1)(\Gamma - 1)$$

$$\Rightarrow \boxed{\Gamma \to 1 \Longrightarrow H_\rho \to \infty}$$

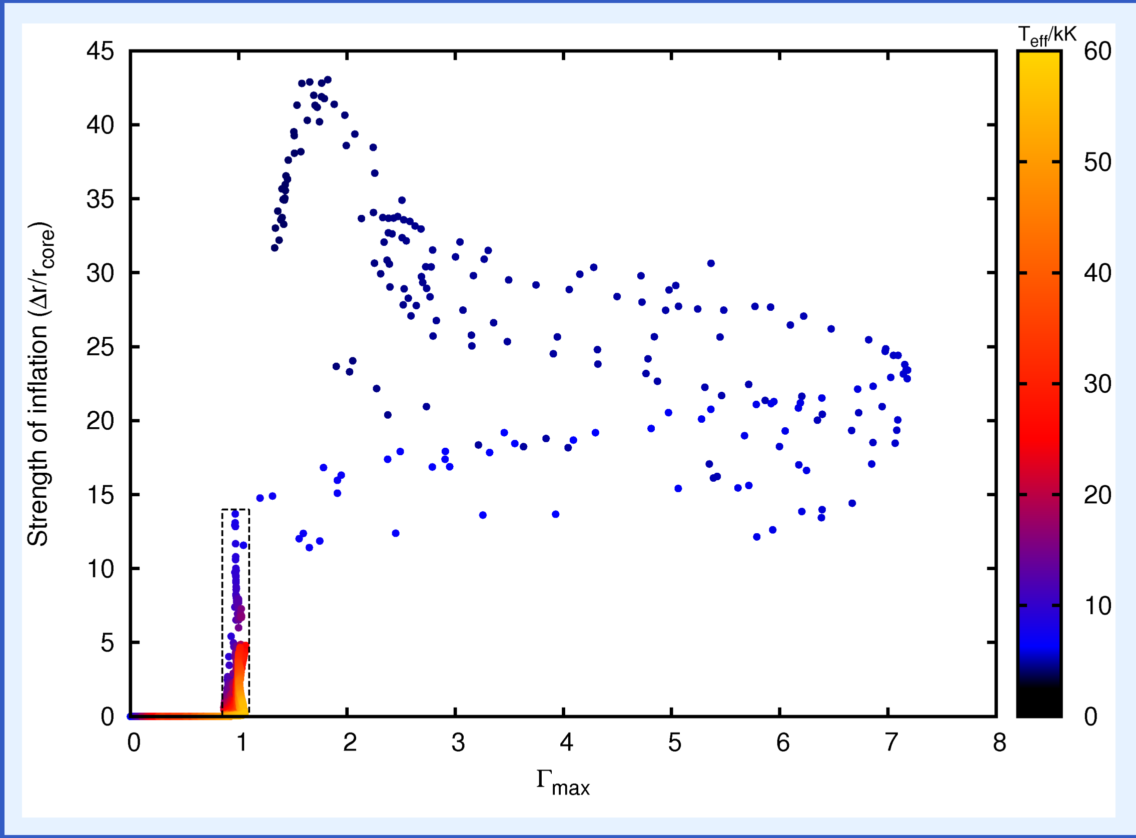
Sanyal et al. 2016

Eddington-limit → Inflation



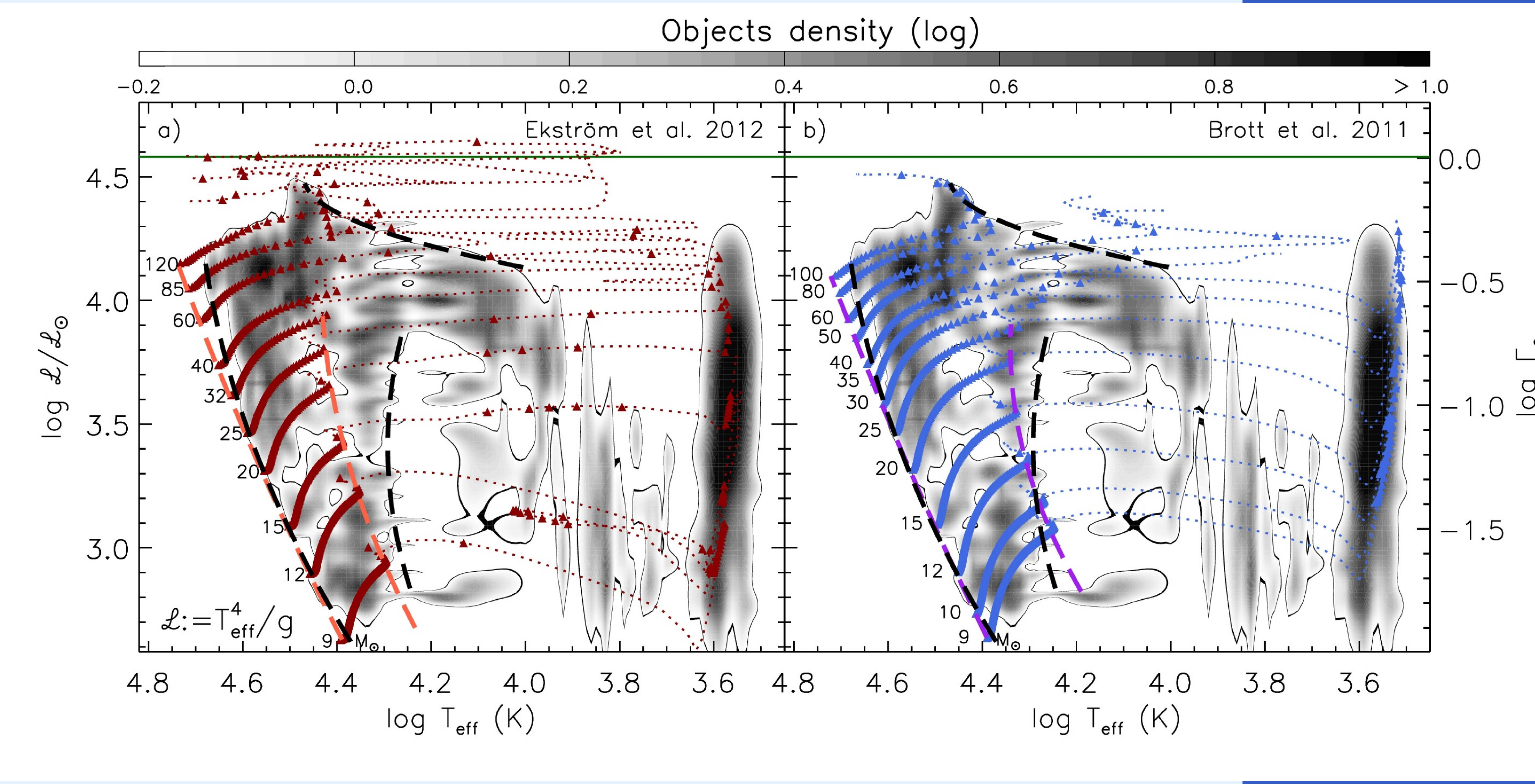
Sanyal et al. 2015

Eddington-limit → Inflation



Sanyal et al. 2015

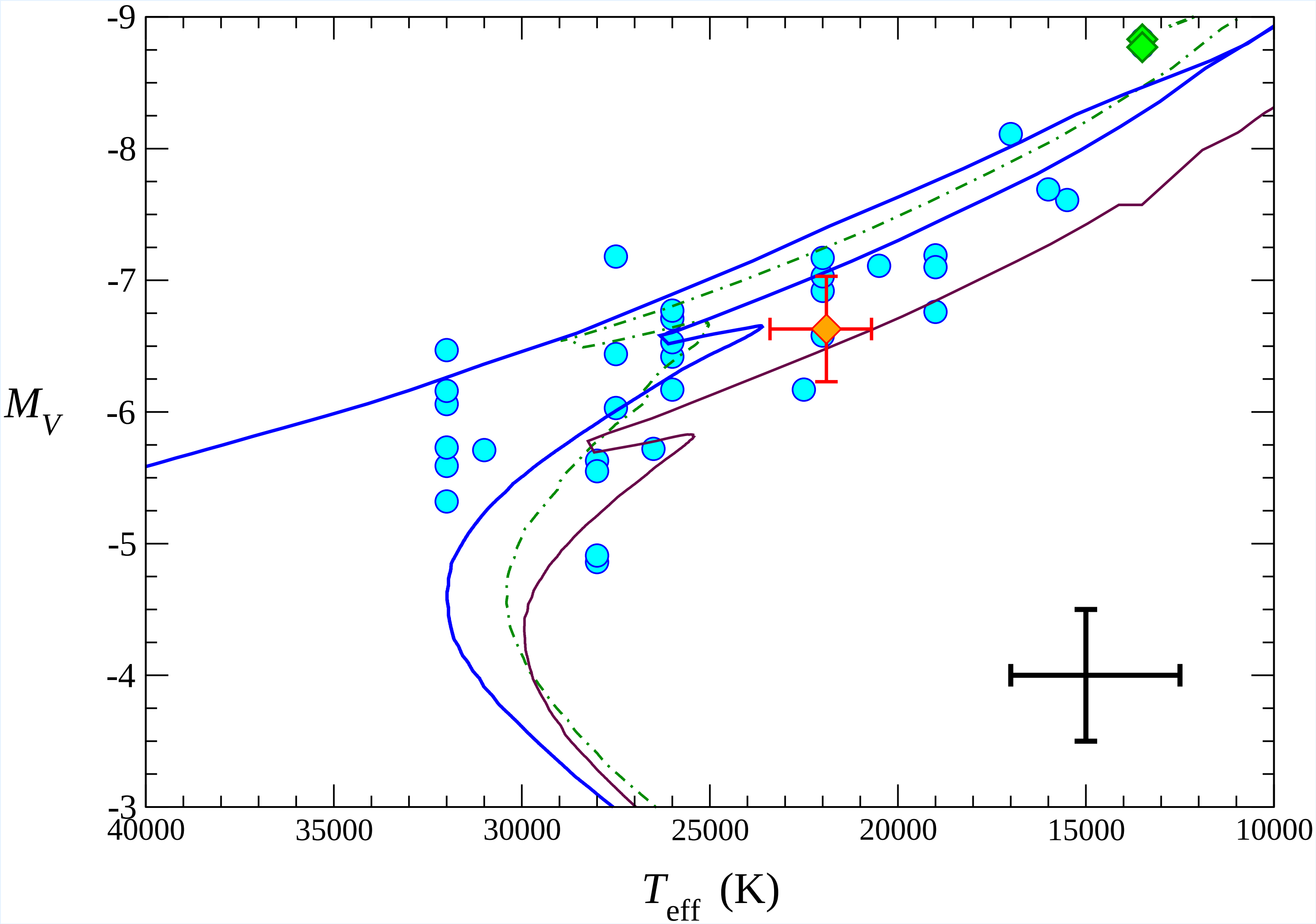
The Galactic sHRD



600 stars: distance- and reddening-independent

Castro et al. 2014

CMD of Westerlund 1



Clark et al. 2014

Start Minilab 2

The Eddington limit and convection

Schwarzschild criterion: $\nabla_{\rm rad} > \nabla_{\rm ad}$

$$\nabla_{\rm rad} = \frac{\Gamma(r)}{4(1-\beta)}$$

$$\nabla_{\rm ad} = \frac{8-6\beta}{32-24\beta-3\beta^2}$$

$$\Rightarrow \nabla_{\rm rad} > \nabla_{\rm ad} \Leftrightarrow \Gamma(r) > (1-\beta)\frac{32-24\beta}{32-24\beta-3\beta^2}$$

$\Rightarrow \Gamma(r) \to 1 \Longrightarrow$ convection

Joss et al. 1973, Langer 1997

Eddington factors

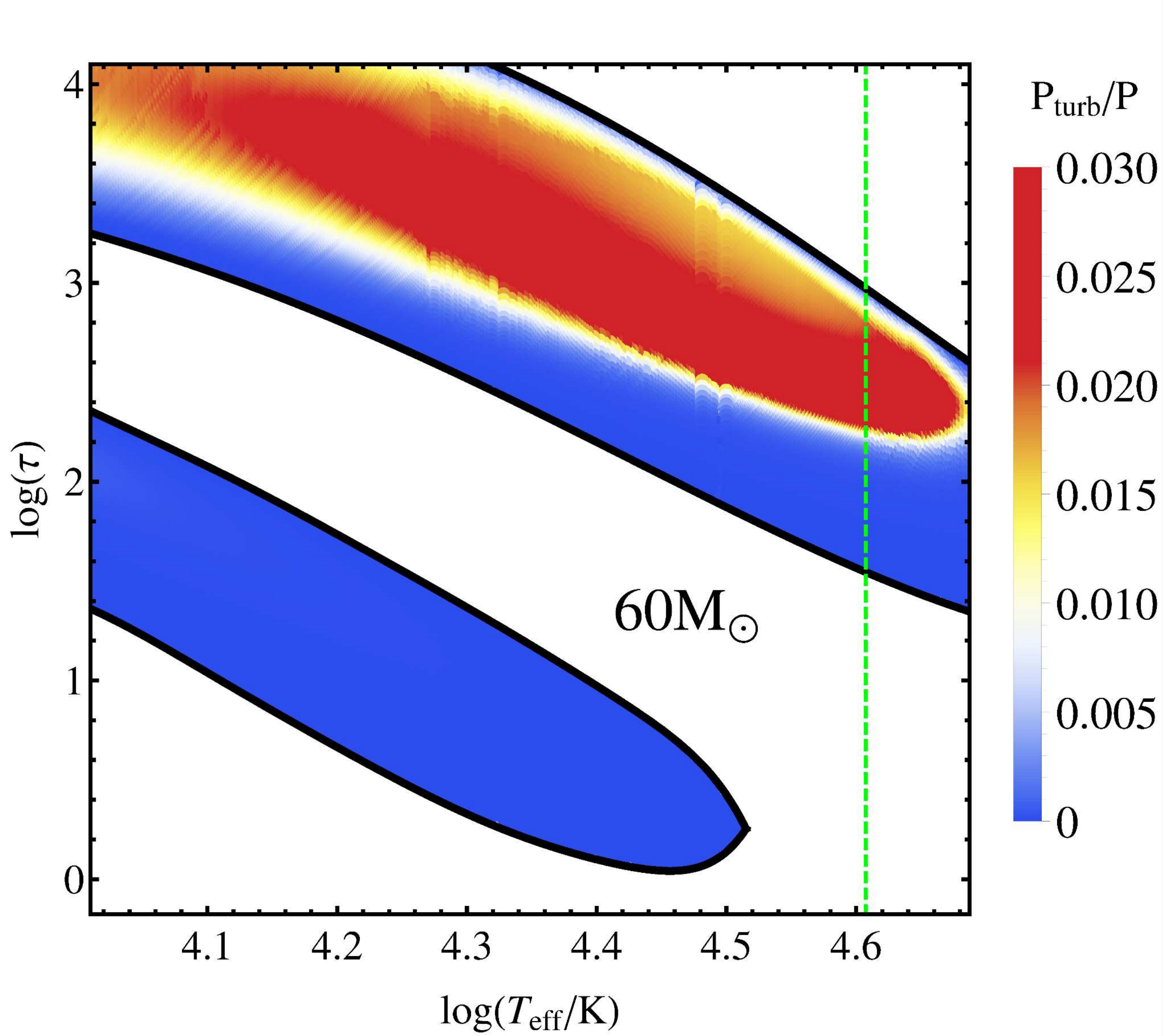
- surface:
  $\rho \downarrow \quad \Rightarrow \kappa \downarrow$
  $\Rightarrow L < L_{\rm Edd}$
- deep interior:
  $\Gamma \to 1 \quad \Rightarrow$ adiabatic convection
  $L_{\rm rad} = L - L_{\rm conv} < L_{\rm Edd}$
- subsurface: convection inefficient but $\kappa \uparrow$
  $\Rightarrow$ neither convection nor radiation can transport energy
  $\Rightarrow L \to L_{\rm Edd}$

Turbulent pressure

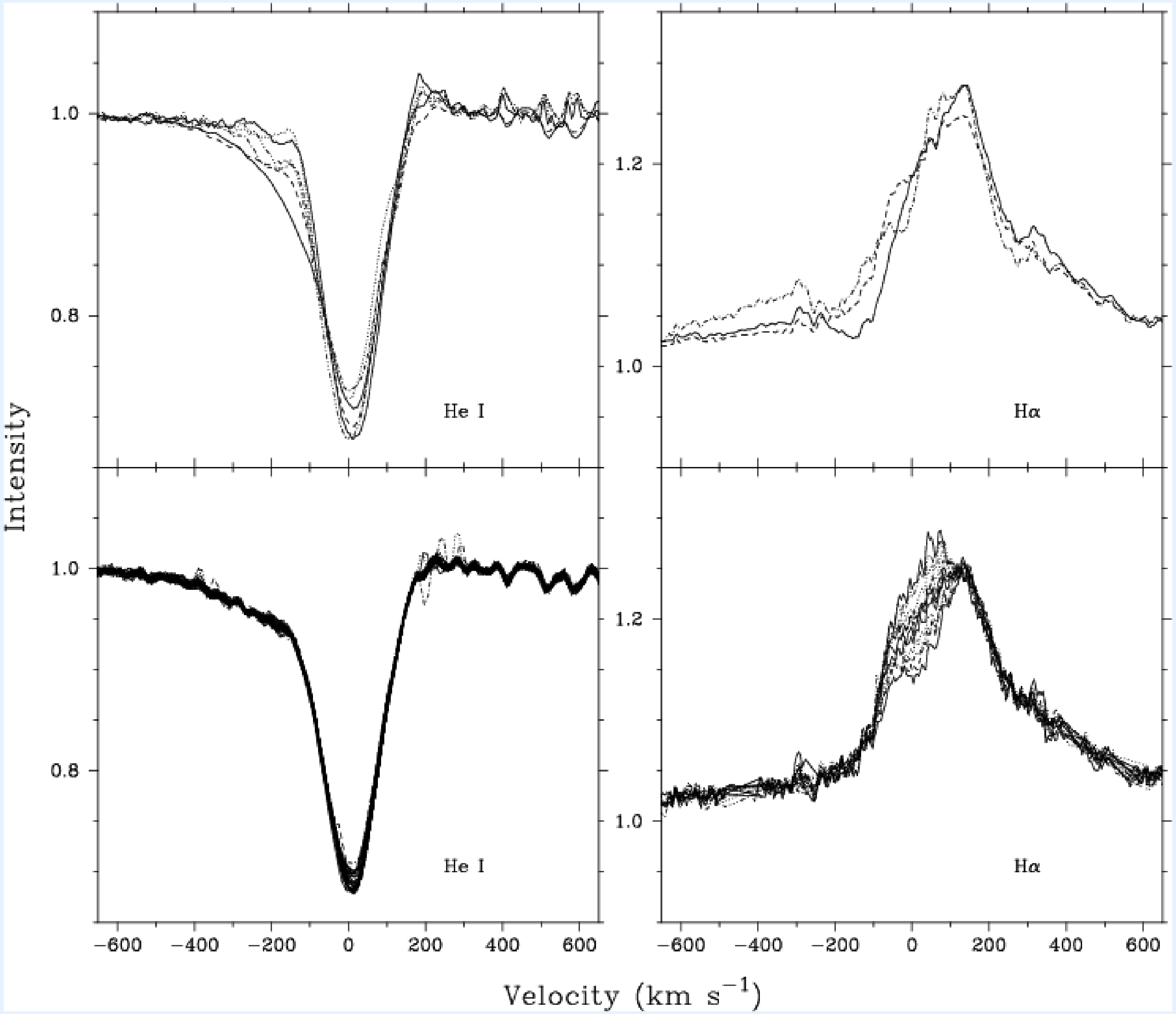
subsurface density $\downarrow \Rightarrow \tau_{\rm therm} < \tau_{\rm dyn}$

- $F_{\rm conv} = c_{\rm P} \rho v_{\rm conv} \Delta T$
- inefficient convection $\Rightarrow \Delta T \rightarrow 0$
- $\Rightarrow v_{\rm conv} \uparrow\uparrow \rightarrow v_{\rm sound}$
- $P_{\rm turb} = \frac{1}{3} \rho v_{\rm conv}^2 = \frac{1}{3} \left(\frac{v_{\rm conv}}{v_{\rm sound}} \right)^2 P_{\rm gas} \Uparrow$

Turbulent pressure: 60 $M_\odot$ model

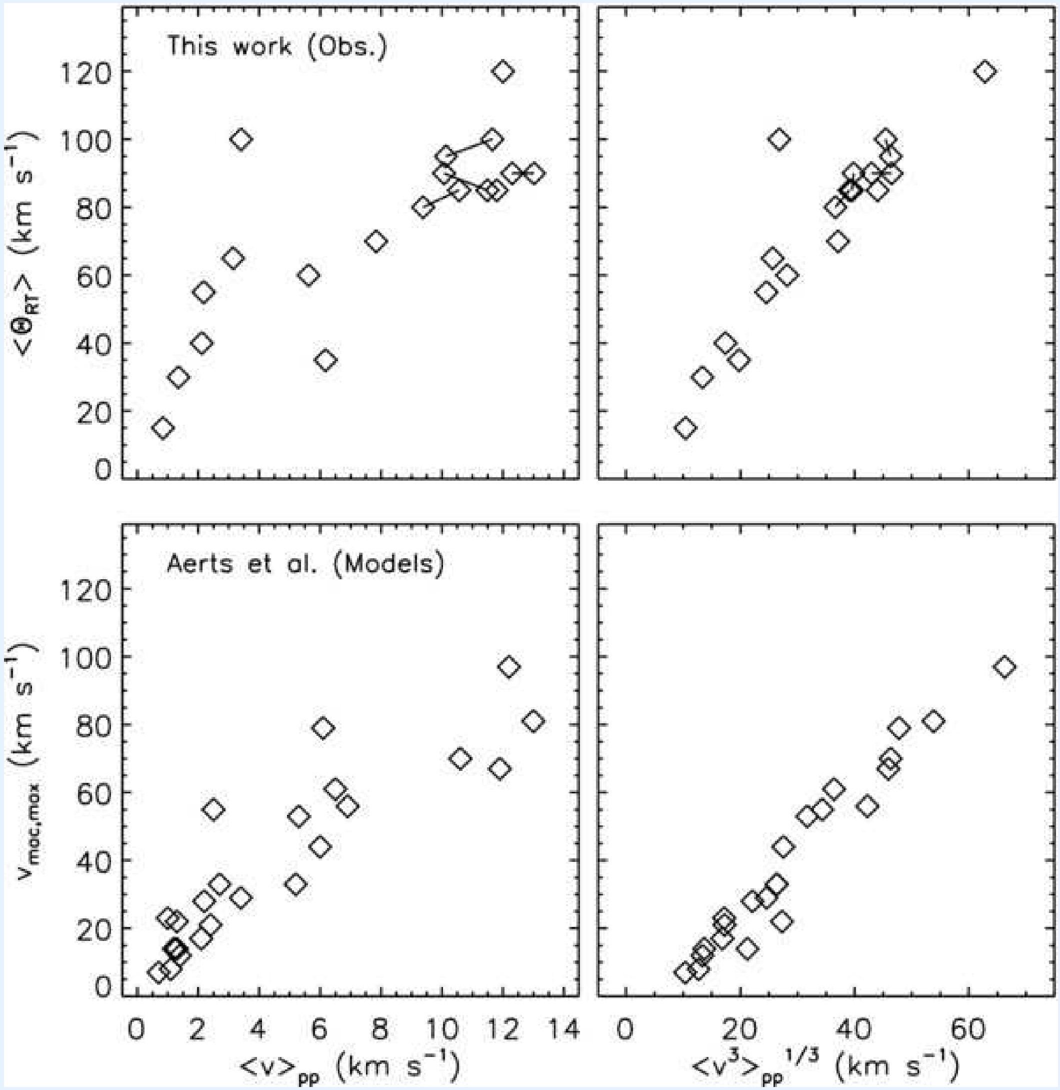


Line profile variability in α Cam



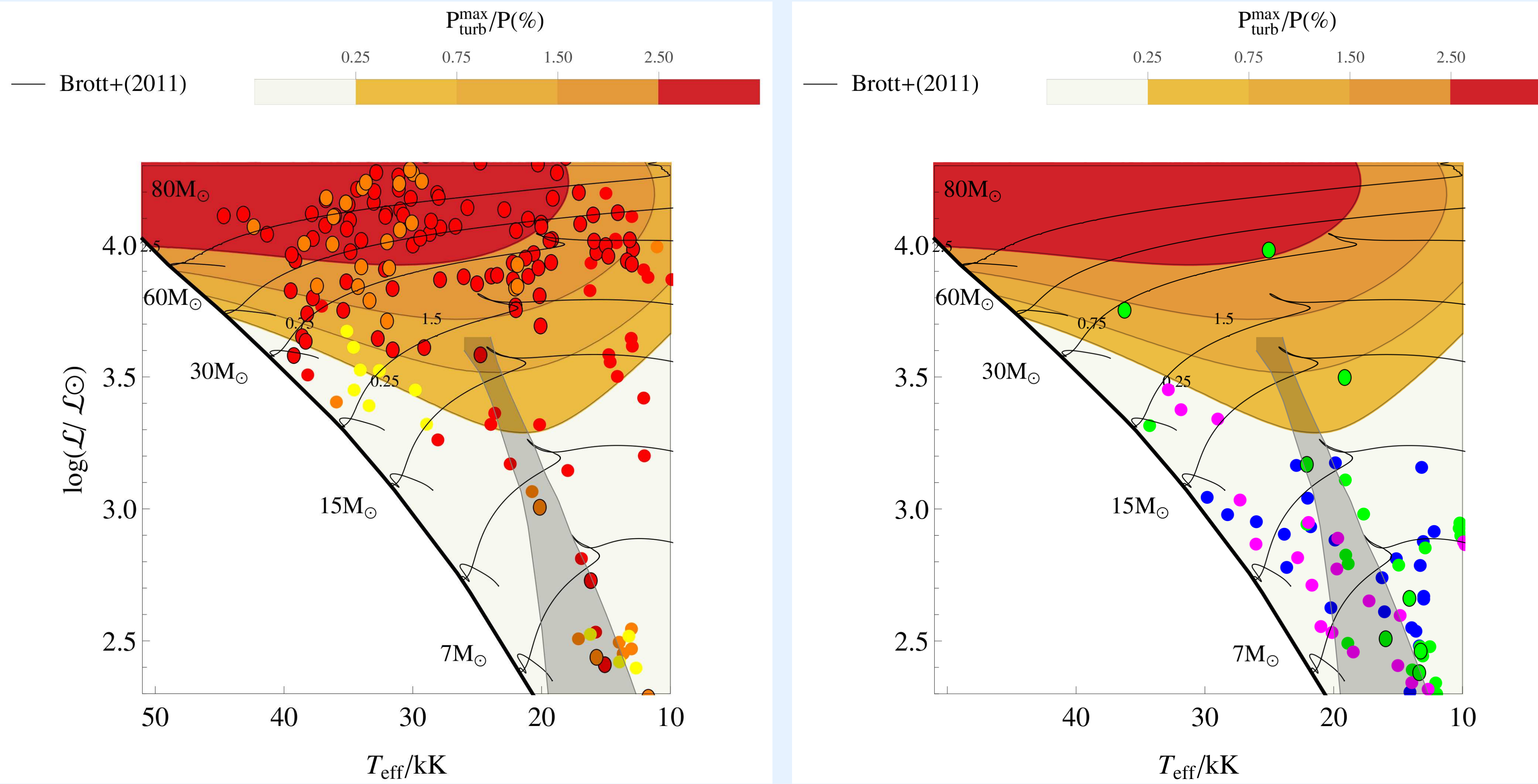
Prinja et al. 2006

Line profile variability and macroturbulence



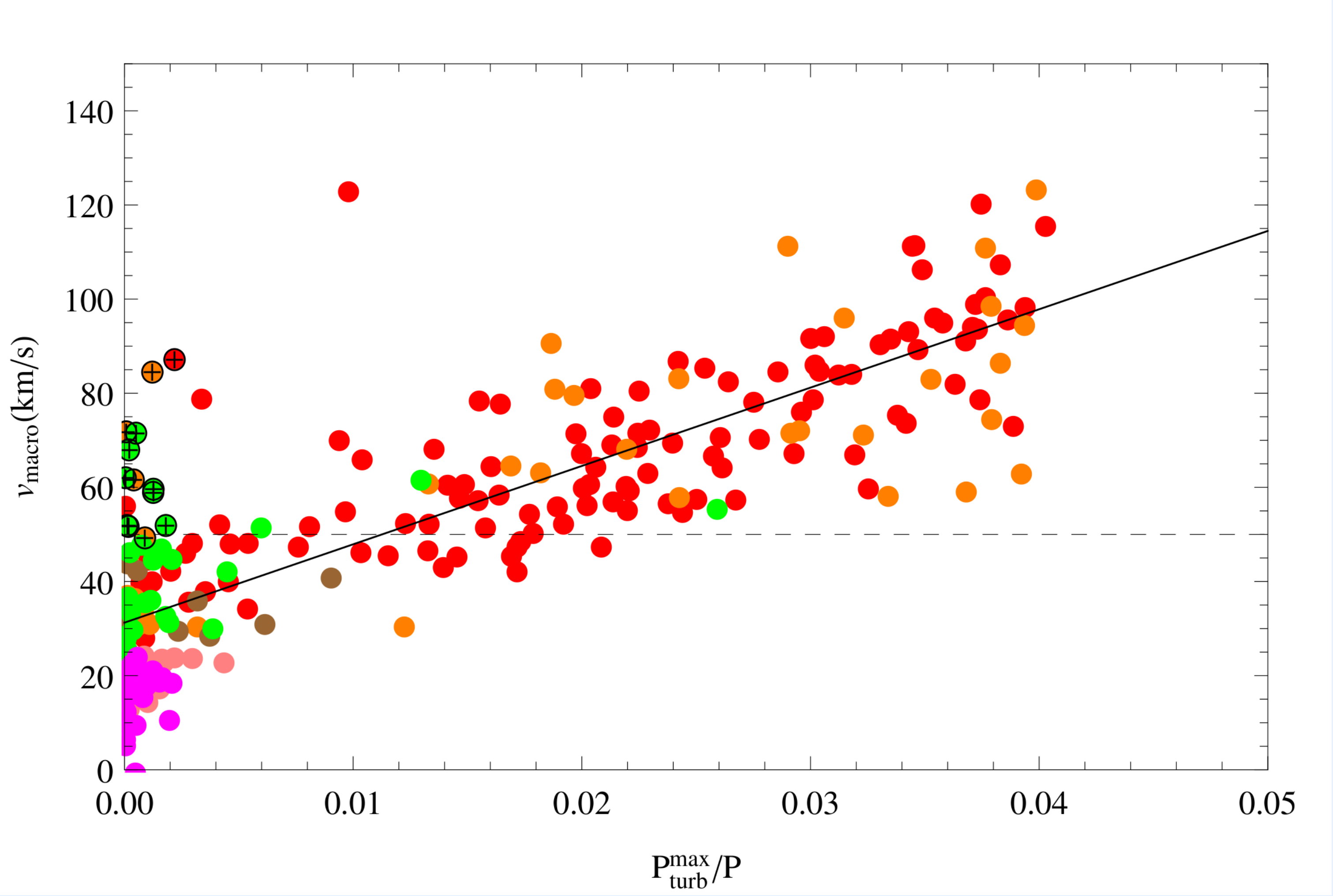
Simon-Diaz et al. 2010

Turbulent pressure and macroturbulence



Grassitelli et al. 2015

Turbulent pressure vs. macroturbulence



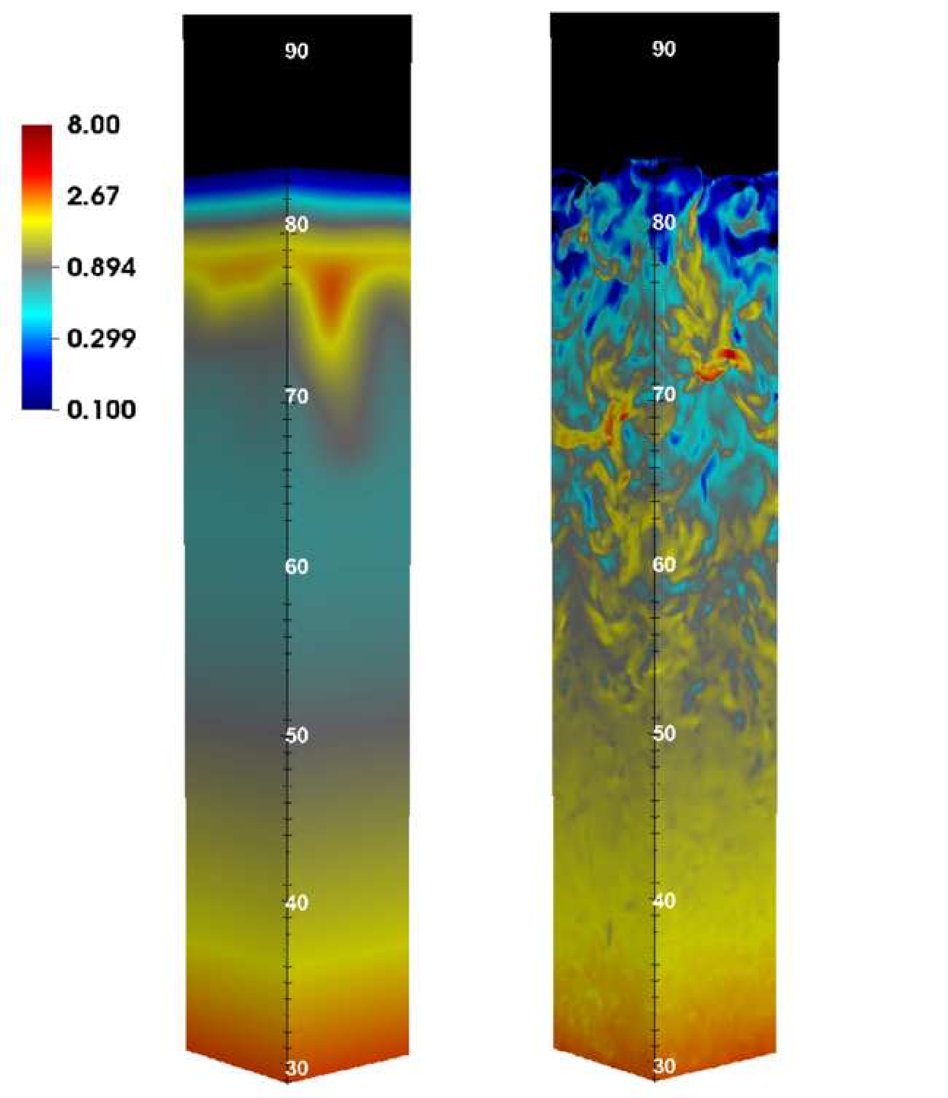
Grassitelli et al. 2015

Massive stars at the Eddington limit

κ_{Fe}-peak in subsurface layers:

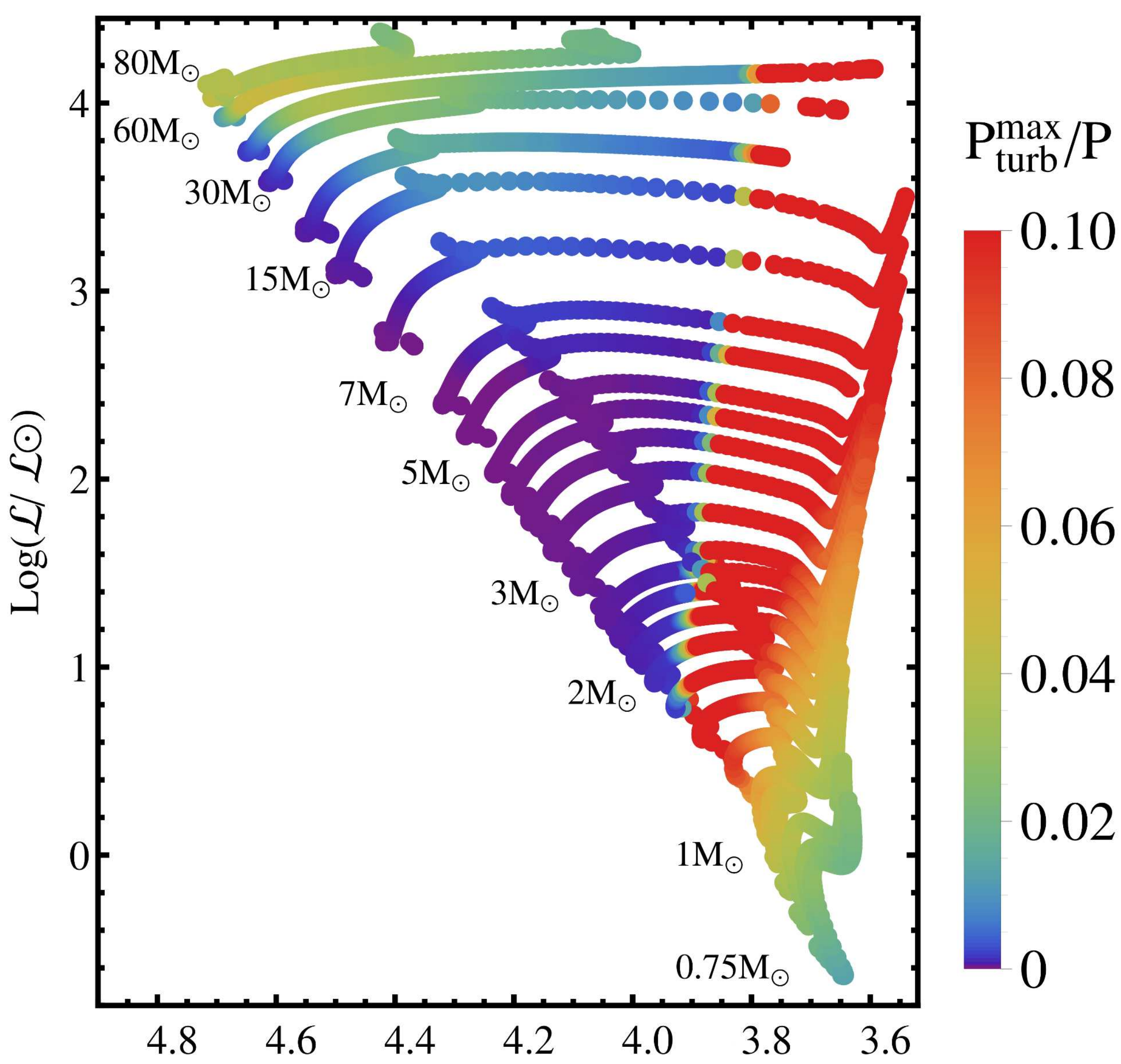
- $\rightarrow$ inflation
- $\rightarrow$ turbulent pressure fluctuations
- $\rightarrow$ high order non-radial pulsations
- $\rightarrow$ macroturbulence
- $\rightarrow$ line profile variability

Inflation: 3D

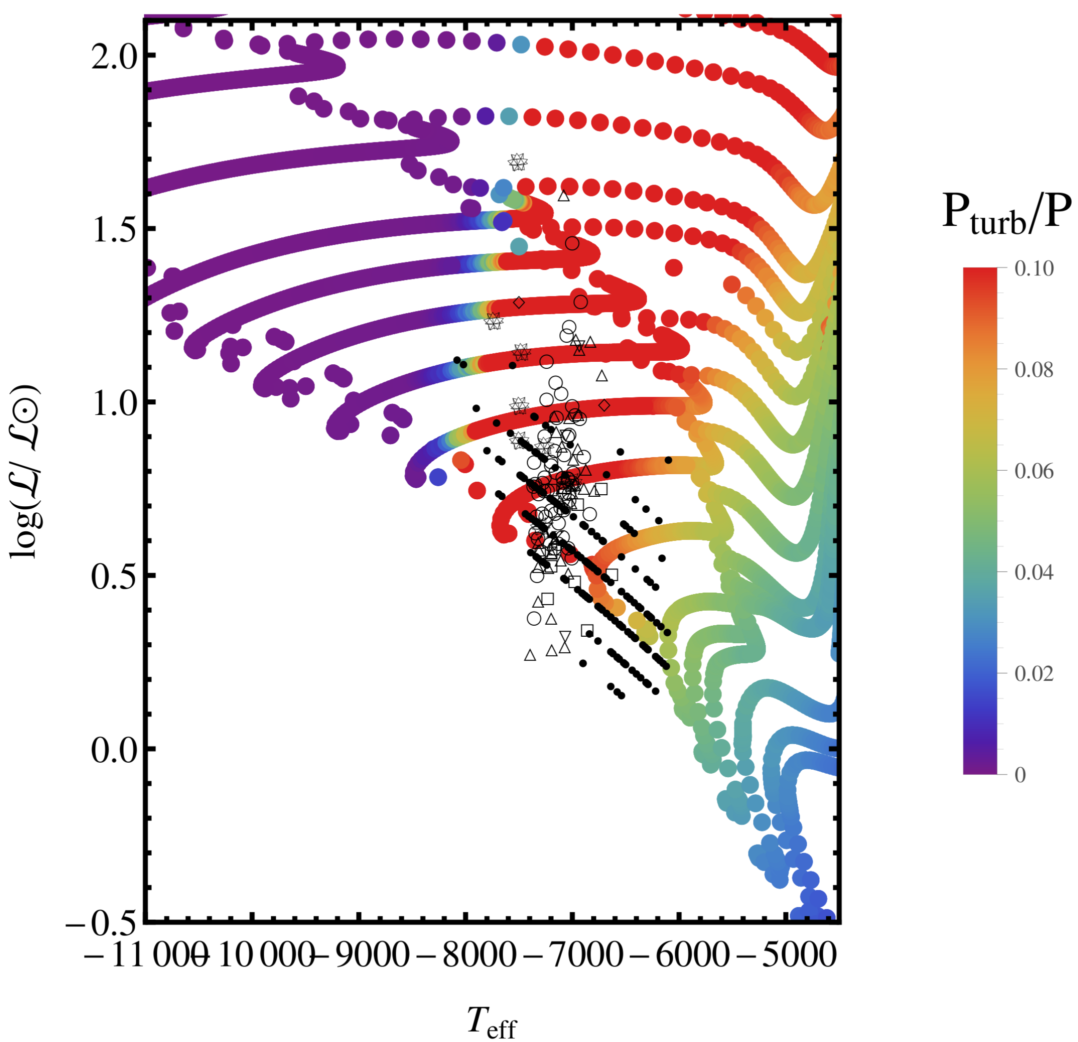


Jiang et al. 2015

Turbulent pressure in cool stars



Turbulent pressure: γ Doradus pulsators



Grassitelli et al. 2015

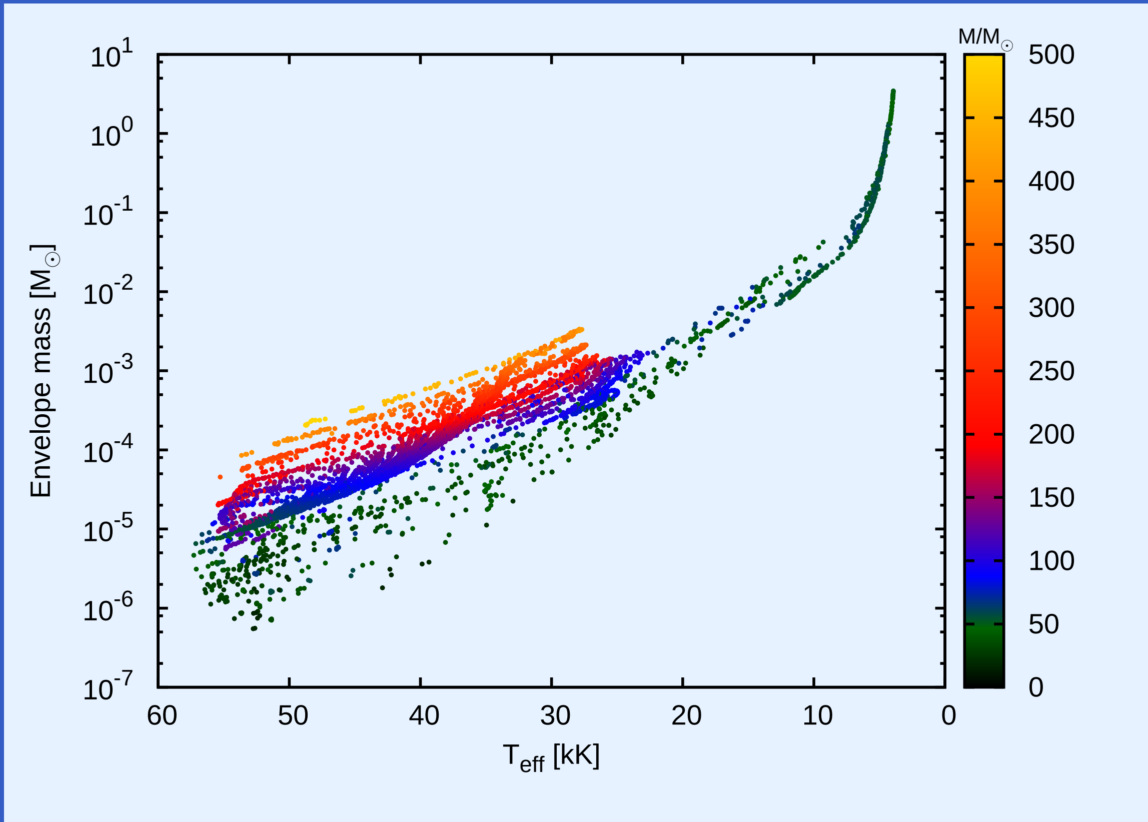
Low order pulsations

- pulsation instability strip on the blue side of $P_{\rm turb}$-strip
- flux blocking layer is closer to the surface
- $\Rightarrow$ κ-mechanism $\rightarrow$ unstable inflation

S Dor Variables & LBVs

inflated envelopes: stable?

- pulsations (hard to observe) (Glatzel+ 1999; Grassitelli et al. 2015)
- high $\dot{M}$ $\rightarrow$ env. loss $\rightarrow$ re-growth ... à la S Dor (Gräfener et al. 2013, Sanyal et al. 2015)
- cool SGs: $M_{\rm inf} > 1\,{\rm M}_\odot$ $\rightarrow$ $\kappa\uparrow$ & $L_{\rm conv}\downarrow$ $\rightarrow$ LBV giant eruption? (under investigation!)



Sanyal et al. 2015

Start Maxilab

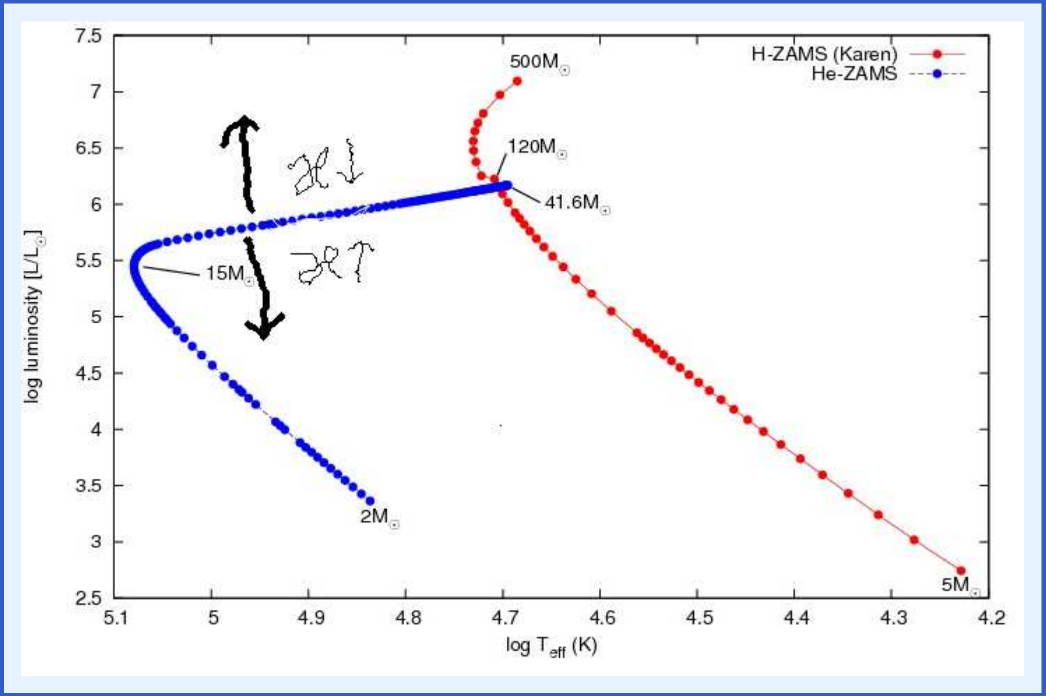
Summary

- consider only electron scattering: the Eddington limit is not reached by stars
- in any case: stars don't reach the Eddington limit at their surface
- stars of all masses can reach the Eddington limit in their sub-surface layers
- massive stars: inflation, turbulent pressure, macroturbulence, ...
- low mass stars: turbulent pressure, macroturbulence, non-radial pulsations, ...
- pulsations: unstable inflation
- stars close to the Eddington limit: dynamical timescale variations

Wolf-Rayet stars

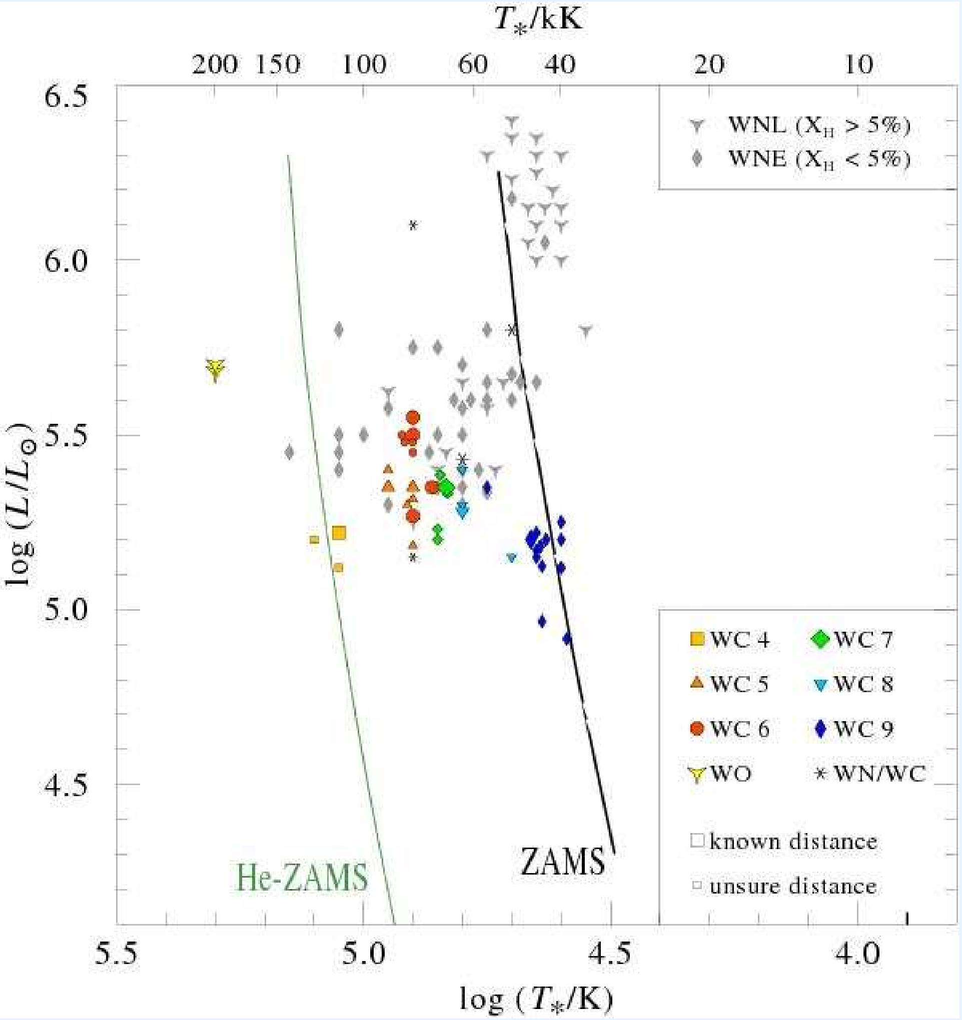
inflation limit:

- $\kappa \downarrow \Rightarrow$ luminosity-limit $\uparrow$
  e.g., ULXB: "porosity"
  (Shaviv 2010)
- $\kappa \uparrow \Rightarrow$ luminosity-limit $\downarrow$
  (Gräfener et al. 2013)
- similarly: metallicity
- not: convective efficiency



Sanyal et al. 2015

Galactic WR stars

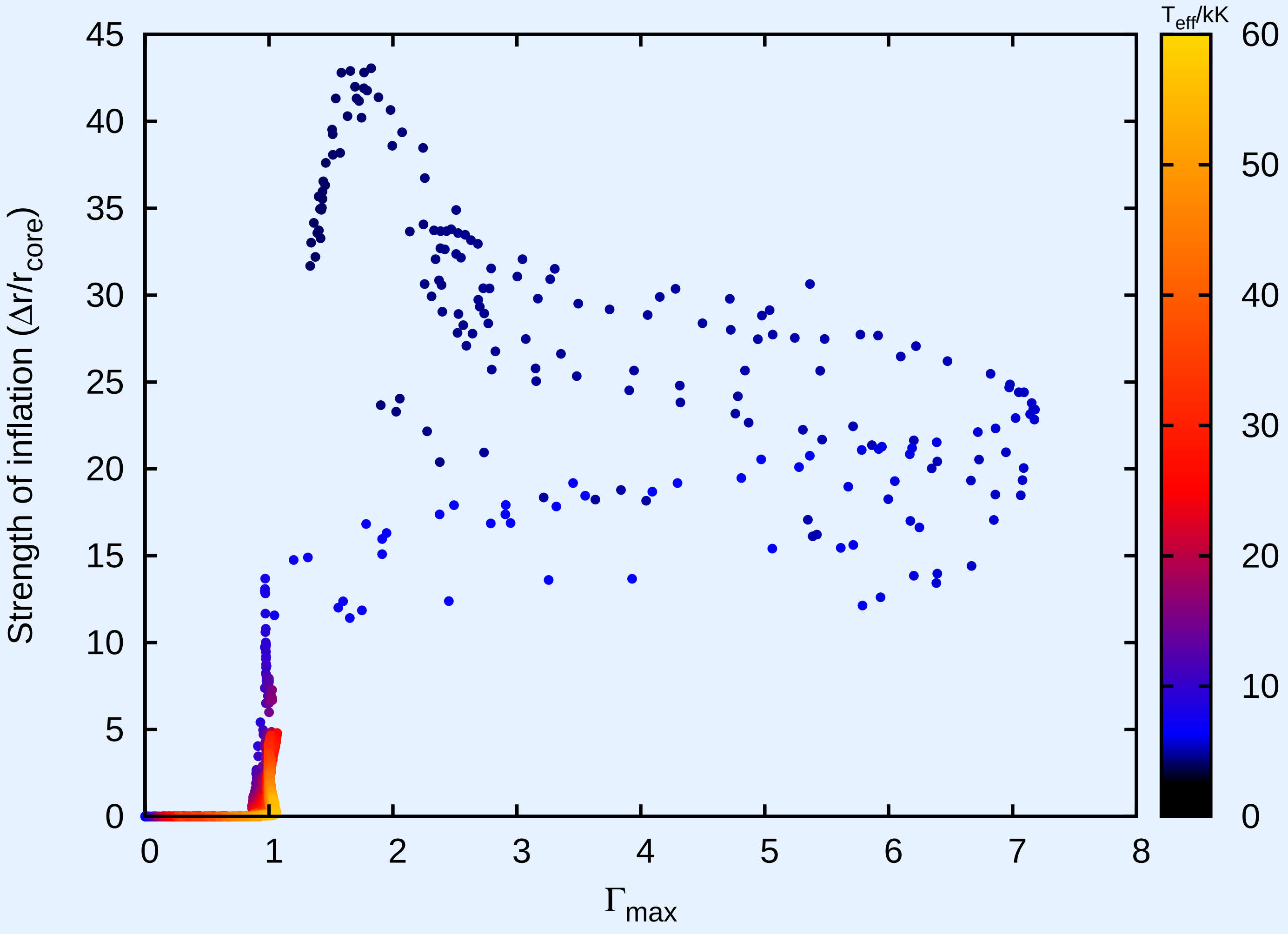


Sander et al. 2012

Inflation for $\log L > 5.3$, and ...

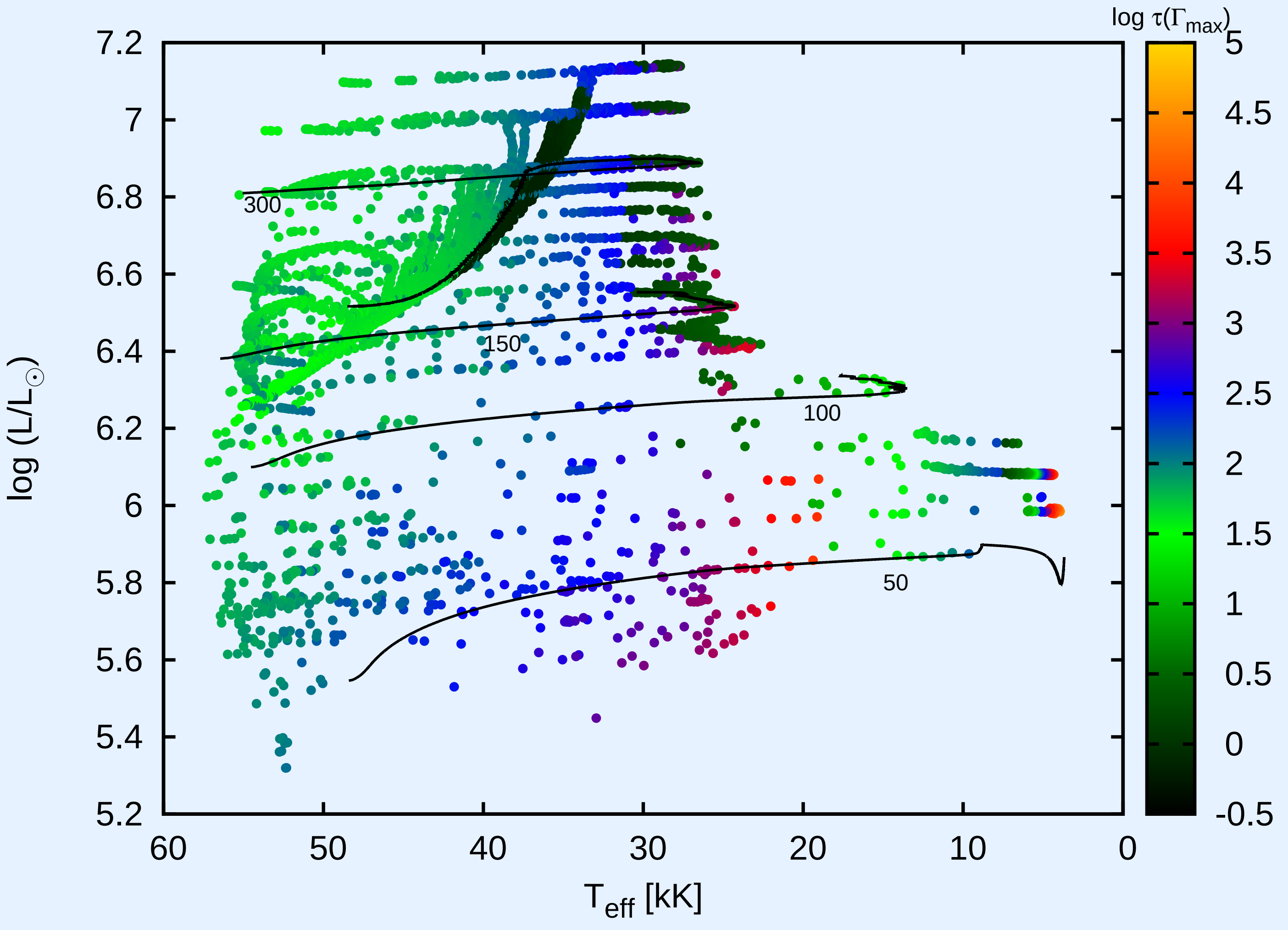
- rotation (Ω-limit): θ-dependant inflation $\rightarrow$ B[e] supergiants?
- binaries: $M \downarrow \Rightarrow \Gamma \uparrow$
- post-RSG, post-AGB: $M \downarrow \Rightarrow \Gamma \uparrow$
- accretion: $L \uparrow$
- “slow” eruptions: (Novae, R Corona Borealis, ...)
- ...

Inflation (Gamma)



Sanyal et al., 2015

Tau (Gamma-max)



Sanyal et al., 2015