

SECOND REPORT OF COMMITTEE OF MATHEMATICS SECTION OF CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS ON REAL APPLIED PROBLEMS IN ALGEBRA AND GEOMETRY.

Summary of First Report.

This committee, created at the annual meeting in November, 1908, presented a preliminary report at the meeting in Chicago, November, 1909. The preliminary report, based upon a year's investigation by the Committee, set forth the theory underlying the real problem movement in secondary mathematics, gave an account of the preliminary investigation regarding the various fields in which direct applications of elementary algebra and geometry are encountered, included a collection of problems made during the year and printed from month to month in *SCHOOL SCIENCE AND MATHEMATICS*, and suggested a list of books and magazines on practical subjects which are sources of applied problems in algebra and geometry, and which may be consulted in search for problem material.

In the discussion of the theory underlying the real problem movement, it was pointed out that the movement aims to reform the teaching of elementary algebra and geometry by teaching the subjects more in relation to their practical uses. The subjects are to be looked at by the student, not as mere pure sciences to be studied solely for the purposes of mental discipline, but rather as scientific instruments in doing the world's work. Mental development is not to be minimized at all, but the functional or application side of the subjects is to be given its due emphasis. The report defended this emphasis, upon psychological grounds. It showed the value of the direct applications of algebra and geometry in practical life and in the motivation or development of interest in the subjects. And it showed the necessity of their uses in the school in assisting the individual's knowledge of the subjects to function, thus supplying the final step in the educative process.

By the problems collected and printed in *SCHOOL SCIENCE AND MATHEMATICS*, it was demonstrated conclusively that elementary algebra and geometry do have numerous direct applications in a great variety of fields. To collect a fund of these problems for school use is only a matter of time and coöperative effort on the part of a large number of people.

Scope of Investigation.

As pointed out in the first report, the investigation has two phases; viz., (1) the determination of the extent to which algebra and geometry do have direct practical application in the world about us, and the collection and editing of a fund of such problems of application; and (2) the determination of the adaptability of the problems collected for use in secondary schools, it being assumed that many of the problems that might be collected are too technical or too difficult to be comprehended by the average high school pupil.

The work of the committee the first year was devoted entirely to the first phase of the investigation. During the past year the work of collecting and editing problems has been continued. From three to four pages of applied problems, collected and contributed by teachers in different places and by members of the committee, have been printed each month in *SCHOOL SCIENCE AND MATHEMATICS*, as was done the year before.

During the past year the attempt has been made to have the problems that have been collected and printed tried out in a large number of good schools in different parts of the country, with a view to determining in a scientific way their adaptability to the comprehension and interests of boys and girls in secondary schools.

Problems Printed.

The new problems printed in *SCHOOL SCIENCE AND MATHEMATICS* since last November are submitted as an integral part of this report.

An examination of the problems printed in *SCHOOL SCIENCE AND MATHEMATICS* by the committee in the last two years reveals the fact that they occur in a great variety of fields of application, as follows: Agriculture, athletics, architecture, brick and masonry construction, bridge building, various phases of business, carpentry, civil engineering, design, dietetics, drafting, embroidery, forestry, games, geography, use of instruments for measuring and construction, different phases of manufacturing, machinery, different phases of practical mechanics, mining, practical mensuration (unclassified), navigation, pattern making, plumbing, railroad work, science, sheet metal work, and steel construction. The fields in which the greatest number of problems have been collected are civil engineering, carpentry, architecture,

machinery and various phases of applied mechanics, design, sheet metal work, science, agriculture, and the use of instruments for measuring and construction. This is probably largely accidental. An exhaustive search probably would reveal other fields as rich in the applications of algebra and geometry. It must be conceded that the work as yet done along the line of collecting real applied problems is a mere scratching on the surface, and can serve only as an index to the great possibilities in the real problem movement if it is carried on thoroughly by the great body of teachers and text-book writers for a number of years.

An examination of these problems shows also that they illustrate a large part of the subject matter of elementary algebra and geometry. In algebra they are distributed as follows: linear equations in one unknown number, 25; systems of linear equations, 6; quadratic equations, 4; formulæ solved as equations, 10; formulæ used for computation, 11; advance topics, 8. In geometry they are distributed by books as follows: Book I, 17; Book II, 29; Book III, 30; Book IV, 18; Book V, 29; Book VII, 27; Book VIII, 11.

Adaptability of Problems.

At the request of the Mathematics Section when the first report was submitted, the problems printed in *SCHOOL SCIENCE AND MATHEMATICS* up to November, 1909, were rearranged and classified according to the topics of algebra or the books of geometry which they illustrated, and reprinted in pamphlet form for class room use. Nearly three thousand copies of this problem pamphlet have been requested by teachers since the middle of the last school year. A few copies still are available.

During the year a circular was sent out to a large number of schools where this problem pamphlet was being used, asking those schools to be prepared to make a report to the committee at the end of the school year regarding the adaptability of such problems as were tried in one or more classes to the interests and capacities of the students. It was requested that the following points be covered in the reports:

1. In what grades were the problems used?
2. What problems tried do you think too difficult for the classes in which used? Are they more difficult than the corresponding problems of the text-book?
3. Were the problems of algebra more or less difficult than those of geometry? What types?
4. Do boys or girls find the problems more difficult?
5. How much explanation by the teacher of the meanings of the problems was necessary?

6. Do pupils show more interest in these problems than in the abstract or artificial ones?

7. In what kinds of problems, in what fields of application, do pupils show most interest?

8. Do boys or girls show the greater interest in these problems?

9. Encourage pupils to bring in real problems of local application in the manual training work, etc. What success along this line?

10. Other points, not here covered, that you have observed in the use of the problems. Write the committee any general suggestions that you have to make as the result of your experience, or on general grounds.

Unfortunately, the problem pamphlet did not get distributed to teachers in time for use in classes the first half of the school year. Consequently, not many schools were able to make an extensive use of the problems. In many cases, local conditions prevented a thorough trial of the problems in the class room. As a result, the reports from many schools were apologetic in nature, the teachers expressing the hope of being able to use the problems in a more systematic way during the coming school year. Instead of fifty, as we had hoped, only fifteen schools, representing the experiences of about thirty-five teachers, were in a position to make comprehensive reports, covering practically all of the above points. Several other schools sent incomplete statements, or general expressions of commendation or criticism of the problems, based upon their slight use of them. Hence the expectations of the committee of being able to report at this meeting, in a thorough scientific fashion, regarding the adaptability of the problems collected, have been only partly realized.

The summaries of the reports of the fifteen schools giving the problems the most careful trial are here submitted, in the belief that in as far as they go, they will be found to furnish an approximately accurate estimate of the value of the problems printed by the committee during the first year of the investigation.

In some schools only the geometry problems were used. In some cases the problems were used largely in reviews. But in most cases they were taken along with the regular work of the classes.

In the following summary the numbers correspond to the numbers of the questions or topics in the circular sent to the teachers:

1. A few problems were used in first year classes, and some in fourth year classes; but in the great majority of cases they were used in second and third year classes.

2. Problems 58, 60, 63, 66, 70, and 71 only were reported

as too difficult for the classes in which they were tried by one or more schools. It must be remembered that some problems were tried in only a few schools. Some teachers omitted purposely the problems they judged would be found too difficult for their classes. Consequently, this list of the problems that are too difficult for secondary schools probably should be somewhat extended.

One-third of the schools found the problems, on the average, not more difficult than the corresponding problems of text-books. Two schools thought that the pupils found them easier, because of the greater interest they aroused. The rest of the schools judged that their pupils found the applied problems, on the average, slightly more difficult than those in the text-books.

It is evident that a small portion of the problems printed must be discarded as too difficult for use at any place in the high school course. A more thorough trial with classes would be necessary to determine just what individual problems should be eliminated on the ground that they are too difficult. This sifting process, as applied to all of the real problems now collected or that will be collected in the future, as this movement progresses, will come as the results of the extended experiences of teachers in dealing with such problems.

3. No important conclusion can be reached in answer to question 3. Some schools used only the geometry problems, and hence had no basis for comparison. Two schools reported the algebra problems the less difficult, but the majority found no difference in the difficultness of the problems in algebra and geometry.

4. It was the experience of six schools that girls found the problems more difficult than boys did. The other schools could detect no difference in the capacities of girls and boys in solving the problems.

As suggested by two or three teachers, the majority of the problems touched more nearly upon the experiences of boys than those of girls. They had more meaning to boys than to girls. This fact would probably account for the experiences of those teachers who reported that the problems were found more difficult by girls than by boys.

5. The amount of explanation of the meanings of the problems on the part of the teachers was reported by schools as follows: "Considerable. Might have been greatly reduced after considerable experience with such problems;" "Little;" "Little,

if any more than book problems;" "Too technical problems were avoided;" "Little or none on those used;" "In some cases;" "Little;" "None;" "Considerable at times;" "Little with those tried;" "Very little;" "Considerable for second year students;" "Some explanation with a few problems;" "Very little."

While it must be remembered that some teachers purposely avoided using those problems that in their judgment were too technical for their pupils, it is safe to conclude from these reports that the great majority of the problems that have been printed are not too technical or too far removed from the experiences of the average boy or girl to be used in ordinary secondary schools. In some cases real problems that may be found expressed in language too technical to be understood by the average boy or girl may easily be made intelligible by rewording them in nontechnical language, or when possible, illuminating them by pictures of the things involved. This does not detract at all from the reality of the problems. In fact, the most real of problems encountered in life are those found involved in situations, and not formulated in language at all.

Just which ones of the problems printed by the committee refer to things too far removed from the experiences of the average boy or girl, or which ones need to be reworded in nontechnical language, is a matter to be determined by the extensive experiences of teachers in the future.

6. All schools but two testified to the greatly increased interest of the pupils in these applied problems over the problems of the text-books.

The report from Portland, Oregon, says: "While I have had this work only twice before, I have found such a vast difference in the interest displayed this term that I am heartily in favor of such problems."

The report from Central High School, Toledo, Ohio, says: "At the end of the semester, I asked for written opinions about the work. Most pupils thought the concrete problems were easier than the abstract ones of the text. A few girls thought them difficult but nevertheless more interesting."

The report from Gary, Ind., says: "Pupils show very much more interest in these problems than in the abstract or artificial ones."

The report from Oakland, Cal., says: "In reply to the question, 'Do pupils show more interest in these problems than in the abstract or artificial ones?' Yes. Are enthusiastic."

Of the schools that tried some of the problems and made incomplete reports, not included among the fifteen summarized here, all commended the problems tried as possessing greater interest to the pupils than the problems in the text-books.

At one school the parents of some of the pupils, who were mechanics or carpenters, became interested in the problems.

We have found, therefore, that this one of the main arguments in favor of the use of real applied problems of algebra and geometry in the school room, which has been advanced on theoretical grounds, is being substantially supported by the experiences of those who have made actual use of the problems in teaching. This is a matter of the greatest interest to those who are following the real problem movement.

7. No reliable record as to what kinds of problems pupils are most interested in has been obtained. One report says that there was greater interest in manual training problems, and problems connected with business. Another says that the greatest interest was shown in problems in forestry, architecture, and physics. Still another found the greatest interest in those problems of geometry involving constructions, and in the problems in architecture and designing. Others, not differentiating as to special fields of application, reported the general conclusion, which one would deduce on theoretical grounds, that those problems proved of greatest interest that touched in some way the pupil's immediate life. This suggests, of course, the desirability of finding problems of local application in each school community and problems actually encountered in the work of different individual pupils in and out of school, a matter discussed at another point in this report.

8. Four schools did not give answers to question 8: "Do boys or girls show the greater interest in these problems?"

Seven schools reported that boys showed the greater interest; three reported that girls and boys showed equal interest; and one reported that girls showed the greater interest, which was in a single class where there were only a few boys.

The greater interest thus shown by boys can be accounted for by the fact that the problems enter more extensively into the experiences of boys than of girls. The need for more problems that relate to the experiences of girls is evident. The experience of the committee has been that such problems are more difficult to find than those that relate to the experiences of boys.

9. Of the fifteen schools reporting, five have attained some degree of success in having pupils bring in, or propose, problems of local interest that are either suggested by the problems printed by the committee in the problem pamphlet used by the class, or encountered by some pupil in his work in other lines in or out of school. This is very gratifying.

Miss Sarah Ruby, Portland, Ore., says: "Problem 42 suggested various forms of trefoils and quadrifoils which they had observed in church windows and elsewhere. I found them so interested in these and various designs of parquet floors that I asked them to keep a notebook for them. When their notebooks were handed in I found as many as seventy designs in several books. At the time of the equinox, some of the classes who were studying physical geography suggested the problem of finding the latitude of Portland by means of the shortest shadow of an object. Following this, I had them look up Eratosthenes and gave them a few facts from which they worked out Eratosthenes's method of measuring the circumference of the earth. When studying the theorems of equal triangles, I told them that Thales was thought to have measured the distance of a vessel at sea by means of one of them, and left them to tell how it might be done. They at once suggested their use for timber cruising, * * * and one boy, who had observed the method used at the aerial navigation meet for finding the height of the airship, suggested that application. * * * Some of the problems proposed, such as * * * finding the width of Mt. Hood at the snow line (a girl asked about this), we have left for the work in similar triangles."

Mr. S. F. Beard, Bradford, O., says: "I have encouraged pupils to bring to the school room simple machines, and then with due study and measurement, to lay off their designs on paper. In other cases sketches have been made by the pupils, and these have been studied in class. The materials for this work have been found in the home, on the farm, in the carpenter's shop or blacksmith's shop, when bridges have been built, now and then from the railroad, or from whatever source may be at hand. For me there have been many surprises at the sources from which pupils themselves draw many problems. * * * No. 38 was caught by a girl, and readily solved, with an accurate drawing. A catalogue from a manufacturer of manual training tools was used freely in making original real applied problems of this kind. * * * No. 42 awakened quite

a field in trefoils and quadrifoils. The pupils enjoyed finding these figures in windows, gables, and even originated many. *

* * In addition, designs from wall paper, oilcloth, parquetry, linoleums, steel ceilings, etc., were passed in review. * * * This was indeed a fertile field for us."

Miss Marie Gugle, Toledo, O., says: "Just a few pupils have used their geometry in their outside experiences. A summary of these is as follows:

"(1) Used much in manual training department, especially in pattern making.

"(2) In athletics, in laying out baseball diamond and measuring distances with short tapes and from several angles.

"(3) Several have helped their fathers who are carpenters or other kinds of mechanics.

"(4) In making pennants and in lettering. One boy does quite an extensive business in making and selling pennants; and he, perhaps, more than any other student, has made use of his geometry."

Some of the real problems of geometry encountered by this boy in the manufacture of pennants were printed in *SCHOOL SCIENCE AND MATHEMATICS* in October, 1910.

Miss Lida C. Martin, Decatur, Ill., had one boy bring in several problems encountered at a local bridge factory. One of these was printed in *SCHOOL SCIENCE AND MATHEMATICS* in October, 1910.

Problems of General and Local Interest.

It is evident that there are many real applied problems of algebra and geometry that are of such a nature as to appeal to the interests and comprehension of pupils of all localities. They are problems encountered in fields of activity that enter into the experiences of the average person everywhere. A number of problems printed in *SCHOOL SCIENCE AND MATHEMATICS* are of this kind. Such problems as these should in time be worked into our text-books, where they will be available for use in the most economical way in all schools. Just as in elementary school mathematics the successful text-books which are written for use in all parts of the country can contain only those problems that have to do with things relating to the experiences of all children, so a text-book in elementary algebra or geometry written for general use in all localities can contain only such real problems as touch the experiences and interests of all pupils to a greater or less extent. It is hoped that, in the future, writers of text-

books on algebra and geometry for secondary schools will introduce in a careful way more and more of those real applied problems that are of interest to all pupils and that have been tried out in schools. In fact, it can be only through this introduction of real applied problems into the text-books that these problems can ever become available for general use in all schools, and become the integral part that they should become of our courses in elementary algebra and geometry. It must be not only through the activities of individuals and organizations of teachers such as ours, but through the writers of text-books, co-operating with these, that this real problem movement should be carried forward in the future.

But the great majority of applied problems are of interest only to individuals or special groups of individuals. A problem of interest to pupils in one locality, or to pupils looking forward to some one vocation, may be foreign to the experiences of, or without interest to, all other pupils. Hence, the desirability of supplementing, in each school, the printed problems adapted to the use of pupils everywhere, which it is hoped will in time be incorporated into the text-books, by the problems of local interest that have to do with things in the immediate environment of the school. That problems of this type can be found in practically every community there can be little doubt. Illustrations of this are shown in the problems printed in *SCHOOL SCIENCE AND MATHEMATICS*, October, 1910, that were encountered by pupils in Toledo, O., and Decatur, Ill. Others were suggested above, in the reports of teachers in Portland, Ore., and elsewhere. This problem material of local interest must be collected by the individual teacher in each community, aided by the pupils themselves, acting through her inspiration.

Prospect.

There is much evidence of the growing popularity of this real problem movement throughout the country. The committee has received increasing encouragement in the work from many interested and enthusiastic teachers in all parts of the United States. At the meeting of the National Education Association in Boston, July, 1910, a paper was read by Mr. W. E. Breckenridge, Stuyvesant High School, New York City, on applied problems. This paper will be found printed in the *Proceedings of the N. E. A. for 1910*. Teachers will find in it a strong argument for the use of real applied problems, thus furnishing

an index to the interest in the real problem movement now being manifested in the Eastern States. That the movement will prove to lead in time to a permanent reform in the teaching of secondary mathematics we firmly believe. And it will be a matter for gratification if the work of this committee, superficial as it has been, in demonstrating the possibilities in the movement, and in helping to stimulate the interests of teachers in it, through problems collected and printed, through some of the sources of problem material that have been suggested, and through the efforts to get the adaptability of problems tested in the school room, may not prove a total waste.

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FORMULAE FOR RATIONAL RIGHT TRIANGLES.

The following table is given in the February, 1910, number of *Indian Education*.

	A	B	$\sqrt{A^2+B^2}$	Authority
1	$2a(a+1)$	$2a+1$	$2a^2+2a+1$	Pythagoras
2	$4a$	$4a^2-1$	$4a^2+1$	Plato
3	\sqrt{ab}	$(a-b)/2$	$(a+b)/2$	Euclid
4	$2ab/(b^2+1)$	$a(b^2-1)/(b^2+1)$	a	Diophantus
5	$2ab$	a^2-b^2	a^2+b^2	Brahmagupta
6	\sqrt{a}	$\frac{1}{2}(a/b-b)$	$\frac{1}{2}(a/b+b)$	"
7	a	$2ab/(b^2-1)$	$a(b^2+1)/(b^2-1)$	Sridhara
8	a	$\frac{1}{2}(a^2/b-b)$	$\frac{1}{2}(a^2/b+b)$	Bhaskara
9	$2lmn$	$l(m^2-n^2)$	$l(m^2+n^2)$	Gen'l Formula