LETTERS TO THE EDITOR.

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Non-Euclidean Geometry.

MR. FRANKLAND (NATURE, September 7) has raised the old problem of Bertrand's proof of the parallel-axiom by a consideration of infinite areas. This is perhaps the most subtle and the most specious of all the attempted proofs, and this character it owes to the fact that a process of reasoning which is sound for finite magnitudes is extended to a field which is beyond our powers of comprehension—the field of infinity. The fallacy which underlies Bertrand's proof becomes more apparent in Legendre's simpler device ("Éléments de Géométrie," 12 éd., Note ii.). A straight line divides a plane in which it lies into two congruent parts—this, of course, has no real meaning, since we are dealing with infinite areas, but such is the argument—and two rays from a point enclose an (infinite) area which is less than half the whole plane. Hence, if two intersecting lines are both parallel to the same straight line, the area of half the plane can be enclosed within an area which is less than half the plane.

This is the same sort of paradox as the well-known one by which the part is made to appear equal to, or even greater than, the whole. The even numbers 2, 4, 6, . . . form a part of the aggregate of integral numbers 1, 2, 3, . . ., but a (1, 1) correspondence can be established between them, viz. to $2n$ in the part corresponds n in the whole aggregate, and to n in the whole corresponds $2n$ in the part. Hence the part is equal to the whole. And, again, a (2, 1) correspondence can be established between the part and the whole, viz. to $4n$ in the part corresponds n in the whole, while the numbers of the form $4n+2$ have no correspondent. Thus the part is greater than the whole.

Mr. Frankland's comparison of the areas of a circle and a regular inscribed polygon is not quite fair to the polygon. The area of a regular N-gon, as its radius tends to infinity, tends to a finite limit, $\pi k^2(N-2)$, which, of course, tends to infinity as N is increased. The area of a circle is $4\pi k^2 \sinh^2 r/2k$, which also tends to infinity as r is increased. The first he calls a linear infinity, and the second an exponential infinity, and certainly e^x/x^n tends to infinity with x, if n is any finite number. But what is the relation between r and N? If we take the expression for the area of a regular N-gon inscribed in a circle of radius r, and then let N increase, we get a limit $4\pi k^2 \sinh^2 r/2k$, which is the expression for the area of the circle. Again, if in the regular N-gon with infinite radius we inscribe a circle, its area is $2\pi k^2(\operatorname{cosec}\frac{\pi}{N}-1)$ and this always bears a finite ratio to the area of the N-gon; it is thus an infinity of the same order, if N is increased indefinitely, and the N-gon, the inscribed circle, and the circumscribed circle all tend to the same geometrical limit—the absolute.

The fact that the cuspidal edge of the surface of rotation of the tractrix forms a line of discontinuity in this representation, and that none of the types of surfaces of constant negative curvature exactly images the hyperbolic plane in the properties belonging to analysis situs, appears to be no objection to hyperbolic geometry. An exactly similar difficulty occurs in the representation of elliptic geometry, since there is no continuous surface of constant positive curvature on which two geodesics have but one point of intersection. Geometry has become entirely a matter of postulation; but, at the same time, it is of interest to observe that the non-Euclidean geometries are capable of being truly represented, even within a restricted field, in Euclidean space.

D. M. Y. SOMMERVILLE.

The University, St. Andrews, September 30.

Elements of Comet 1911*f*.

FROM M. Quénisset's observation of September 23, and my own of September 26 and 30, I obtain the following approximate elements:—

$$\begin{aligned} T &= 1911 \text{ November } 12{\cdot}67 \\ \omega &= 123^\circ\ 13{\cdot}4' \\ \Omega &= 35^\circ\ 36{\cdot}6' \\ i &= 102^\circ\ 19{\cdot}8' \\ \log q &= \bar{1}{\cdot}89116. \end{aligned}$$

The comet is now receding from the earth and approaching the sun, and there is no reason to expect much increase in brilliancy. The only point of interest is that when at the descending node on December 16 it will be about half a million miles outside the earth's orbit. The difference of the heliocentric longitudes of the earth and comet will, however, be 132°, so that no near approach is possible.

J. B. DALE.

Craigness, New Malden, Surrey, October 3.

Rainfall in the Summer of 1911 and of 1912.

HAS Mr. MacDowall the courage to apply his own experience, to which he refers in NATURE of September 28, to "supply long-range forecasts of months, seasons, &c."? Will he publish in advance a forecast for the winter 1911–12 or for the spring and summer of 1912, such as he considers could have been done for the summer of 1911? Or is it only after the event that he can discover what points in the past have to be considered and in what grouping they have to be compared in order to yield an *a posteriori* "forecast"?

HUGH ROBERT MILL.

62 Camden Square, London, N.W., October 2.

Miniature Rainbows.

WHEN returning one day in August of last year from the Farne Islands to Berwick in a pleasure steamer, I was standing in the bow of the boat, and was much struck by the display of a permanent rainbow in the spray that was thrown up. The rainbow was inverted, the result, presumably, of my position above it. The sea was very rough, and thus the spray was constant.

EDWARD A. MARTIN.

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THE STONE AGES OF SOUTH AFRICA.[1]

THE papers in this volume are a very full and important addition to the work already published by Mr. J. P. Johnson; but it is doubtful whether they bring us any nearer to a solution of one of the most interesting questions connected with archæological or palæontological discoveries in South or Central Africa —namely, the approximate age to which the existence of man can be traced back in South Africa, East Africa, the Congo basin, West Africa, and the Sudan. Though Dr. Péringuey would seem, from one or two phrases, to lean to the theory of a very ancient date for the human colonisation of tropical Africa, he has to admit repeatedly that so far no cogent evidence has been produced in the shape of geological features associated with the finds of human remains or implements to indicate, as positively as is the case in Europe and Asia, the period in the earth's history with which such remains are to be associated.

As our knowledge advances towards perfection, as we become better and better able to read that new Bible, the book of the Earth itself, we may have to revise our estimate of the ages of the hitherto discovered prehistoric, palæolithic, and eolithic human remains in Europe and Asia. Still, there can be little

[1] Annals of the South African Museum: vol. viii., part 1, containing the Stone Ages of South Africa, &c. By Dr. L. Péringuey, with further contributions by Mr. A. L. Du Toit and Dr. F. C. Shrubsall. (London: Printed for the Trustees of the South African Museum by West, Newman and Co.) Price 40s.