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SOME REMARKS ON UTILITY

THE enquiry that follows was suggested by some subtle and interesting arguments which Professor Patten has published in various places. It is not, however, confined to a consideration of his views, but rather seeks to take up and examine a few related points that bear directly or indirectly upon "Consumers' rent."

The first question clearly is as to the definition and measurement of utility. Here we may notice a broad distinction between writers who do, and those who do not, treat the term as equivalent to pleasure. Jevons and Dr. Patten¹ appear to hold that this identification can and should be made, but Professor Fisher refuses to make it and, following Professor Marshall, defines utility with reference to desire.

It was natural for Jevons, as an exponent of Benthamite psychology, to take the former view,² while to Professor Patten, as a utilitarian philosopher, an identification which afforded a theoretical basis for the summation of happiness along the lines of consumers' rent must have been attractive. Clearly, however, in an article of this kind it is inadvisable to adopt any definition which implies a particular answer to a disputed question, and therefore, since utility is at all events primarily a measure of desire (as for example when the utility curve is made the basis of a money demand curve), it seems best to define it exclusively as such.

The question of the unit of utility then becomes simple. What has been said should have made it clear that Professor Edgeworth's "just perceivable increment of pleasure,"³ is unsuit-

¹ *Theory of Prosperity*, p. 13.

² It is interesting to note that the same bias led Jevons to ignore the element of sacrifice of opportunities for pleasure in "disutility," and to identify this with pain.—an error which Marshall has quietly, and Patten elaborately, corrected.

³ *Mathematical Psychics*, p. 99.

able for our present purpose; and that we must fall back on the desire for some definite object as our unit. The quantity and kind of commodity taken must clearly be something arbitrary, just as the yard is an arbitrary unit of length. It does not, therefore, seem possible to improve upon Professor Fisher's method of taking the desire of an arbitrarily chosen individual A for B a small increment of a given commodity under given circumstances of total consumption, subjective state, and so on, as the unit of utility, and calling it a "util." This unit will be upon precisely the same footing as the units of length and weight.¹

Having thus got our unit, we have next to ascertain upon what the number of utils that any quantity of a given commodity has for any specified individual depends, or in other words, of what variables its utility to that individual is a function. The well-known law of diminishing utility tells us that U , the total utility of commodity A to individual I , is a function of Q_1 , the quantity of A in the hands of I ,² and that, generally speaking, $\frac{d^2U}{dQ_1^2}$ is negative. A second well-known law tells us that U also varies with Q_2 the quantity of the commodities that may be used as complements, and with Q_3 , the quantity of the commodities that may be used as substitutes for A , that are in the hands of I . Thus $U = F(Q_1, Q_2, Q_3)$.

It should further be observed that U varies with Q_4 , the quantity of things in general over which I has command. It is, of course, a platitude that an alteration in the amount of a person's wealth will alter the price measure that he applies to a given utility. But the point here is that a sudden increase of wealth not merely changes the price that will be given, but the actual intensity of the desire for any commodity; and that whereas, after an increase in his wealth I will generally give more money for a "util," a given increment of A will have for him fewer utils. The reason for this is that the new commodities in general over which he gets command become rival claimants for the time and attention he had formerly given to A .³ Thus I

¹ Cp. *Mathematical Investigations in the theory of value and prices*, p. 18.

² Of course, if we choose, we can take account of time, and speak of the rate of utility per rate of commodity consumed per week. For our purpose, no difference would be made by doing this, and we refrain merely to keep the discussion from becoming too cumbrous.

³ Of course, in this argument, it is assumed that habits are not changed. We are considering a momentary and not an historical utility function.

will desire less intensely to listen to a barrel organ if he can afford to go to the theatre, and will pick fewer blackberries from the hedgerow when he fares sumptuously every day. The point is that all commodities are to a greater or less extent substitutes for one another, owing to the fact that our time and capacities are limited. Consequently just as I 's desire for an n^{th} increment of tea varies with the quantity of tea, coffee, and sugar that he has, so also, though, of course, to a much smaller degree, it varies with the amount of things in general at his command. $U = F(Q_1, Q_2, Q_3, Q_4)$.

But this is not all. The utility of A to I is a function not only of the quantity of different commodities that he possesses but also of the quantity that other people possess. Thus Mr. Henry Cunynghame¹ points out that the utility of diamonds varies inversely, and that of top-hats directly, with the extent to which they are respectively in common use. In fact, my desire for the first class of commodities is compounded of my desire for the article and my desire for the uncommon, while my desire for the second class is similarly compounded of a desire for (or, in the case of top-hats, possibly an aversion to) the article and a desire to be "in the swim." It should, however, be noticed that this involves a consideration of the distribution of the commodity as well as of the mere quantity of it, to which alone Mr. Cunynghame refers. If a million extra diamonds or top-hats were miraculously created and entailed, without the power to lease, upon a single individual for ever, the effect on my utility curve for these two articles would be of a degree of smallness that even in this study of small quantities it would be pedantic not to neglect. Similarly if the million extra top-hats went to the normally top-hatted class and merely allowed them to have three each instead of two, my utility curve would not be appreciably affected, while it might be if the miracle resulted in the so-called lower classes adopting this head-gear. Again, if there were a second miracle destroying two-thirds of all existing top-hats, the curve would not be affected if one were left to all accustomed to wear them, but it would be affected if the destructive force were concentrated upon those belonging to people in my immediate neighbourhood, just as it is at present affected by the fact that in Cambridge a beneficent fashion has already antici-

¹ ECONOMIC JOURNAL, vol. ii., "Some Improvements in Simple Geometric Methods of treating Exchange Value, Monopoly, and Rent."

pated the work that in London it still needs a miracle to perform.¹

It is difficult properly to express this dependence of U upon the way in which A is distributed in mathematical form. The expression $F(q, n)$, where q means the total quantity of A and n the number of persons among whom it is divided, is inadequate, first, because it ignores the difference that would be made by a transference of increments of A from persons in the same class or neighbourhood to others, and *vice versa*, and, secondly, because it treats the influence of an even distribution among the n persons as the same as that of an uneven one. Now this will not do, for the utility of a third carriage to me will not be the same if each of my hundred neighbours has two carriages as it will be if ninety-nine of them have one each and the hundredth has one hundred and one. To be accurate therefore we should have to make U a function of many variables, representing the quantity of A in the hands of each other person and also of the proximity of each in place or station to I . This complex expression may be abbreviated into the form $F\{K(ab)\}$, a_1 being the quantity possessed by the first neighbour and b_1 his "distance," a_2 being the quantity possessed by the second neighbour and b_2 his distance, and so on, K being a symbol akin to, though not identical with, the ordinary Σ . Thus we get, so far, $U = F\{Q_1, Q_2, Q_3, Q_4, K(ab)\}$.

It is not necessary to labour the matter further, for it is at once clear that for complete accuracy we should have to take account also of variations induced in U by changes in the quantity of commodities other than A , whether substitutes,

¹ The desire for the uncommon or to be "in the swim" generally resolves itself on analysis into one for distinction. This desire operates to impel me towards commodities that are consumed by the classes with whom I wish to be associated, and away from those that are common among the classes from whom I wish to separate myself. It is this that makes some decorations more valuable in my sight than others, because the possession of them would raise me to a loftier social pinnacle. A man of good family might quite possibly consider himself insulted by the offer of the badge of some order that is usually bestowed upon persons whom he deems his inferiors.

Furthermore, the distinctions which the possession of a particular thing confers upon me, and therefore my desire for it, varies with changes which, for any reason, take place in the attitude assumed towards it by distinguished persons. Thus when royalty limps, court ladies desire to limp, and "Caracalla caused yellow-amber to bear a great price throughout the Roman Empire, by ordering it to be imported for his ornaments because it was the colour of his mistress' hair." (Montanari, *Della Moneta*, p. 59, quoted by Dr. Sewell.) Similarly, an experienced housekeeper once observed to the writer that when a family eats little meat, the servants come to think that it is not so much of a luxury after all and eat less themselves.

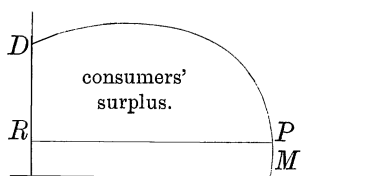
complements or things in general, in the possession of his neighbours in time, place or station. U would therefore finally emerge as a function of innumerable variables, and the employment of a common expression $U=F(Q_1)$ demands justification, or rather an explicit statement of the assumptions made in using it.

Now this expression $U=F(Q_1)$ is not used by authoritative writers absolutely, but always with the condition "other things being equal." In other words the remaining variables upon which U depends are temporarily treated as constants. This is, of course, a perfectly legitimate proceeding, but it is important to distinguish between those influences which are thus provisionally impounded, and those which may be ruled out of court altogether as of negligible importance. In practically all cases it would be agreed that the effect of changes in the quantity (or rate) of other commodities, which are neither complements nor substitutes of A , possessed by other people, is thus negligible. But this cannot generally be said of any of the elements in the expression $U=F\{Q_1, Q_2, Q_3, Q_4, K(ab)\}$. The last element might be thought a trivial one, but, though it may be admitted that commodities like diamonds, whose attraction lies partly in their character as advertisements of wealth, are exceptional, the same cannot be said of that class of which top-hats have been taken as typical. The notion that our position requires us to live in such and such a style, and to do such and such a thing, is a widely prevalent one, and what our "position requires" is, of course, simply what other people in like position as to class, place or time are accustomed to do. Therefore it should be clearly understood that the variables, which, for convenience, we have designated by the expression $K(ab)$ are merely impounded and not dismissed as negligible when we write $U=F(Q_1)$.¹

With these explanations we seem entitled to write down the expression, and to draw, for any individual at any moment, a utility curve, which, according to common agreement, will be negatively inclined after a certain point in its course. Here, however, we are met by an argument which Professor Patten directs against the doctrine of consumers' rent. The first assumption of that doctrine is that, for an individual at any moment the successive increments of any commodity will, *other*

¹ The same remark probably holds good, in some cases, of the expressions $K(cb)$, $K(db)$, where $c_1, c_2, \&c$, are complements of A , and $d_1, d_2, \&c$, substitutes for A in the hands of persons at "distances" $b_1, b_2, \&c$.

things being equal, afford increments of utility, the magnitude of each of which is independent of the total quantity of the commodity that the individual may choose to take. That is to say, the utility of my first pound of tea will be a_1 , of my second a_2 , of my third a_3 , etc., whether I consume three pounds or fifty. Thus if I can obtain fifty pounds at a cost per unit equal to a_{50} , I obtain on my first pound a consumers' surplus of $a_1 - a_{50}$, on my second one of $a_2 - a_{50}$, and so on, this being represented graphically thus,



Against this Professor Patten argues that when we get more of any commodity we cannot express the utility of previous units by the same vertical line as before.¹ And he gives his reasons elsewhere² as follows: "It is quite different to ask what a man would give for a single roll of bread in a desert or for two rolls, etc., than [from what] it is to ask what utility³ will he get from the first roll at a meal when he eats four rolls, from the second roll and so on. . . Professor Marshall's consumers' surplus would be much larger than mine because he estimates the surplus, not from a given situation of the consumer, but from a series of situations representing different stages of supply." This argument is made plausible by its illustration, but a moment's reflection will show that its conclusion is fallacious. Of course, it is perfectly true that the utility of the first loaf in a desert is greater than that of the first loaf at a meal at which I eat four, but the reason is, not as Professor Patten implies, that I am eating a greater number of loaves, but that I have at my command meat and other things not available in a desert, which serve as substitutes for loaves. In fact the condition "*ceteris paribus*" upon which Professor Marshall's whole argument explicitly depends, is, in this illustration, violated. What Professor Patten has shown is that when I can eat a great many other things I am likely to desire all possible

¹ *Theory of Prosperity* (106).

² *Cost and Utility*, *Annals of the American Academy*, p. 31.

³ I have substituted the word "utility" for pleasure, in order to be able to deal with one controversial point at a time.

increments of bread less than I did in a desert, or, in other words, that my whole utility curve is lowered. What he thinks he has shown is that the utility of my first loaf is less if I have four than if I had one, or in other words, that, as my expected consumption of bread increases, the utility curve for the earlier increments ought to be redrawn lower down. This Professor Patten certainly has not shown, nor can any one who reads Professor Marshall's discussion in the *Principles* fail to see that, so far as the utility curve of a single individual is concerned, the criticism fails.¹

When we pass from the utility curve of a single individual to that of a "market" we have to face the problem of how to treat the variables $K(ab)$. Except upon the assumption that their influence is of negligible importance we cannot obtain a curve for the whole market by compounding those of the different individuals in it, for we have only been able to draw the curve for I_1 to represent the utility of successive increments to him, *when the consumption of I_2, I_3 and so on stands at some definite point*. In fact we only become entitled to draw this curve by keeping $K(ab)$ strictly "in the pound," and it is only possible to combine it with a similar curve for I_2 by taking them out of the pound. Thus, except upon the assumption that the influence of $K(ab)$ is negligible, we commit the same fallacy when we combine individual utility curves into a market utility curve, as we do when we add together the consumers' rent from coffee, estimated upon the assumption of a given consumption of tea, and the consumers' rent from tea, estimated upon a similar assumption with regard to coffee.

Therefore we must draw our market utility curve differently. Take successive increments of A in the order of the magnitude of the utility they yield to no matter whom.² A 's desire for one increment of tea, that being the only increment, is a_1 , B 's desire for an increment, A having one, is a_2 , A 's desire for a second, A having one and B one, is a_3 , C 's desire for one, A having two and B one, a_4 , and so on.

Now this curve, it is clear, has certain peculiarities. In drawing the vertical lines to represent the utility of successive increments of commodity, we have taken account of the element

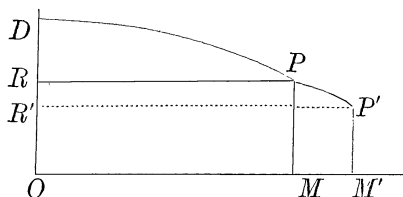
¹ Of course, Professor Marshall's demand curves in this connection are *not* taken from a series of situations. He carefully impounds the element of "time," just as we have so far done throughout this whole discussion.

² Note the assumption that the "util" can serve as a unit of measurement for the desires of different individuals.

$K(ab)$. We have done this because we found that otherwise we should involve ourselves in a self-contradiction if we constructed a market curve at all, and by this means we continue to get a valid one for marginal utilities. But now the question arises, is this curve, like that for a single individual, a true "particular utilities" curve as well as a true "marginal utilities curve"? Except upon the assumption that the influence of the element $K(ab)$ is negligibly small, the answer would appear to be in the negative. For though the thick straight line MP represents the utility of the OM^{th} unit to the person who has it, under certain conditions, it does not do so under all conditions. It does so, in fact, only when there are OM units of commodity A in the neighbourhood, and when these are distributed in a particular way. When the total consumption of the market rises to OM' units, this condition is violated, and MP no longer represents the utility of the OM^{th} unit. Thus when the area DPR is treated as a measure of consumers' surplus obtained from a consumption of OM increments of commodity in a market, the same kind of fallacy is committed as when an ordinary marginal supply price curve is treated as a "particular expenses curve" and made to yield a graphic representation of producers' surplus. The marginal utility curve can be used for a particular utilities curve only when $K(ab)$ exercises a negligible influence.

The practical question that arises, of course, is as to the extent and frequency with which this assumption can legitimately be made. Are we safe in treating the marginal utilities curve as an accurate approximation to a particular utilities curve? The answer seems to be that it is only for considerable changes of market consumption that the required assumption is illegitimate. It is highly improbable that a *slight* change in the consumption of anything would have an appreciable influence upon the utility of earlier increments, because a considerable change of consumption is necessary to make us aware that any change in "commonness" has taken place. Consequently when the consumption of a commodity increases, say ten per cent., passing from OM to OM' , we may fairly regard the marginal utility lines between PM and $P'M'$ as being approximately equivalent to particular utility lines. Therefore the area $PMM'P'$ represents the total utility derived from the last MM' units of commodity. And further, though the area $DPMO$ cannot be regarded as equivalent to the total utility of the previous OM units, yet it may be said that the total utility of these units is approximately the same with a consumption of OM as with one of

OM' . Therefore the change of total utility brought about by the increase of consumption = $(X + \text{area } PMM'P') - X = \text{area } PMM'P'$; and the change of consumers' surplus =
 $(X + PMM'P' - P'M'OR')$
 $- (X - PMOR) = PMM'P' + PMOR - P'M'OR' = PP'R'R.$



Thus, though we cannot get from our figures a legitimate representation of the total consumers' rent afforded by any commodity, we can get such a representation of the changes of consumers' rent due to small fluctuations of consumption. And, curiously enough, the area over which consumers' rent curves are theoretically inefficient is precisely the same as that over which they are practically inefficient also, for it is only with regard to *small* price¹ fluctuations that statistics of consumption are generally obtainable. It is very fortunate that in many problems, like those with regard to taxation which Professor Marshall has suggested, it is with these small changes only that we are concerned.² The engine which he has devised, though limited in range, can therefore often serve us.

It does not seem possible, at this stage, to avoid the psychological controversy which, by the adoption of Professor Fisher's definition of utility, we have hitherto left aside. Within the limits in which it can properly be used what precisely does our consumers' rent area signify? In practical problems, of course, it is a price area and not a utility area that is presented to us, but it would be useless to repeat here what Professor Marshall has said with reference to this difficulty and to the circumstances under which it may, for some purposes, become comparatively unimportant. Let us therefore assume the establishment of a utility area, and ask precisely what it means. Clearly, from our definition, the area measures the intensity of the desires which

¹ Note the transition, for purposes of illustration, from utility to price.

² This correspondence between the practical and the theoretical range of the doctrine of consumers' rent is, indeed, "very fortunate." The good fortune is due, however, not to some pre-established harmony, but to the wonderful skill which Professor Marshall has brought to bear upon the construction of a suitable technical apparatus.

OM successive increments of the given commodity go to satisfy. In a sense, therefore, since it measures the desires which are satisfied, and which, but for the commodity, would have remained unsatisfied, it may be said to measure the satisfaction of desire. But are we entitled to identify the satisfaction of desires of given intensity with a given intensity or quantity of "satisfaction"? Unless satisfaction be taken as equivalent to pleasure this second phrase seems to have no clear meaning, and therefore, when it is adopted, there is a great temptation to slide unawares into statements which assume that the pleasure derived or expected from the satisfaction of a given desire bears some definite relation to, and can be expressed in terms of, the intensity of the desire that is satisfied. But this assumption is not one that can be taken as obviously true, and therefore not one that should be allowed to remain concealed in a phrase.¹ We must proceed to examine it on its merits.

To those who adopt the position of psychological hedonism there is, indeed, no difficulty. If it is impossible for any one ultimately to desire anything except his own pleasure as such, it is self-evident that the intensity of our desire for any means of pleasure will vary with the amount of pleasure that that means is expected to yield. We may, of course, calculate wrongly, and the pleasure actually experienced may be different from that which we expected, but the intensity of our previous desire must always be commensurate with the pleasure that, whether rightly or wrongly, we did expect.

Psychological hedonism is not, however, a position that at the present time has many adherents. It has been attacked not only by idealists like Green, but with still more fatal effect by the greatest of modern Utilitarians, the late Professor Sidgwick. After these attacks the present writer at all events can only regard it as an untenable and exploded doctrine. We do not desire only pleasure, but numerous other things as well. When one is doing a problem, one does not desire the pleasure of finding the solution, but just to find the solution, and, when one is playing a game, it is not the pleasure of winning that one wants, but just to win. Indeed if it were not so, it would be

¹ Professor Marshall evades the difficulty by defining pleasure with reference to desire: "Those pleasures are equal for our purposes the desires for which are equally strong incentives to action, for persons who are *prima facie* similar and similarly situated." (*Principles of Economics*, p. 76.) But the late Professor Sidgwick writes, "I do not judge pleasures to be greater or less exactly in proportion as they exercise more or less influence in stimulating the will to actions tending to sustain or produce them." (*Methods of Ethics*, p. 126.)

hard to see how the attainment of our solution or our victory could afford us any pleasure whatever, for we would *ex hypothesi* be attaining something other than that which we desired.¹

As soon, however, as it is granted that we may desire other things besides pleasure, we are no longer entitled to say that the intensity of our desires and the pleasure we expect from their satisfaction are *necessarily* proportionate to one another. The satisfaction of all desires, whether they be directed towards pleasure or towards anything else, is, of course, generally pleasurable, but when the desires are "disinterested," there seems no reason for assuming a constant relation between their intensity and the quantity of pleasure, that, as a matter of fact, does, or, if it is thought about at all, is expected to, accompany their satisfaction.

If this be granted, how far does the admission affect the practical usefulness of "consumers' rent" as a technical apparatus? Clearly it would invalidate any attempt to use it for the summation of total happiness—an attempt which has already been shown to be hopeless for other reasons,—but we have no ground for supposing that it would interfere with more modest applications of it. For we may fairly expect that most material commodities, and especially those of wide consumption, that are required, as articles of food and clothing are, for direct personal use, will be wanted as a means to pleasure, and will consequently be desired with varying intensity in proportion to the pleasure they are expected to yield. For practical purposes therefore, no new difficulty is introduced by the change of opinion that has recently taken place with reference to pleasure and desire.

A. C. PIGOU.

¹ Cp. Sidgwick, *Methods of Ethics*, Bk. I. Ch. IV. ; T. H. Green, *Prolegomena to Ethics*, Bk. III. Ch. I.