Order in the particle zoo

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Abstract

The standard model classifies particles into elementary leptons and hadrons composed of quarks. In this article the existence of an alternate ordering principle will be demonstrated giving particle energies to be quantized as a function of the fine-structure constant, α. The quantization can be derived using an appropriate wave function that acts as a probability amplitude on the electric field. The value of α can be approximated numerically by the gamma functions of the integrals for calculating particle energy. Limiting cases of the quantized energy series are the Planck energy constituting a source term and the energy of the Higgs boson as maximum energy. The series expansion of the energy equation provides quantitative terms for strong, Coulomb and gravitational interaction.
The model can be expressed “ab initio” without use of free parameters.

1 Introduction

Particle zoo is the informal though fairly common nickname to describe what was formerly known as "elementary particles". The standard model of physics [1] divides these particles into leptons, considered to be fundamental "elementary particles" and the hadrons, composed of quarks.
Well hidden in the data of particle energies lies another ordering principle, based on a description of particles as electromagnetic objects. In the model presented here, the particles are interpreted as some kind of standing electromagnetic wave localized due to the effects of the strong force and may be visualized as a rotating electromagnetic field with the E-vector pointing towards the origin [1]. Neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of opposite polarity. To obtain quantifiable results, the electromagnetic field will be modified with an appropriate exponential function, $\Psi(r, \vartheta, \varphi)$, serving as probability amplitude of the field. The two integrals needed to calculate energy in point charge and photon representation exhibit the following two relations:
1) Their product - resulting from energy conservation - is characterized by containing the product of the two gamma functions $\Gamma_{+1/3}\,|\Gamma_{-1/3}| \approx \alpha^{-1}/(4\pi)$,
2) their ratio features a quantization of energy states with powers of $1/3^n$ over some base α_0, a relation that can be found in the particle data with $\alpha_0 = \alpha$ as:

$$W_n/W_e \approx 3/2\left(y_l^m\right)^{-1/3}\Pi_{k=1}^{n}\alpha\hat{}(-3/3^k) \qquad n = \{1;2;3..\} \quad [2] \qquad (1)$$

with W_e = energy of electron, W_n = energy of particle n and y_l^m representing the angular part of $\Psi(r, \vartheta, \varphi)$. For spherical symmetry $y_0^0 = 1$ holds, corresponding particles are e, μ, η, p/n, Λ, Σ and Δ [3].
Apart from calculating energies the model may be used to describe other particle properties. In particular the expansion of the incomplete gamma function appearing in the integrals for calculating particle energy gives quantitative terms for Coulomb and strong interaction. Allowing for excitation into virtual particle states, the strong interaction term may exert a long range effect which can be identified as gravitational interaction.
The model is essentially a static, rest frame approximation. To focus on the more fundamental relationships the discussion of minor aspects of the model parameters is exiled to an appendix, related topics to be marked as [A]. Typical accuracy of the calculations presented is ~ 0.001 (e.g. due to approximation of Γ-functions) which would be in the order of magnitude of possible QED corrections.

2 Results

2.1 Basic calculations

The model is essentially based on a single assumption:
Particles can be described by using an appropriate exponential wave function, Ψ(r), that acts as a probability amplitude on an electromagnetic field.

1 with B-field and propagation velocity perpendicular to it
2 All indices chosen to attribute 0 to the electron; Factor 3/2 is supposed to represent an anomaly of the electron, see [A2,3];
3 The relation of the e, μ, π masses with α was noted in 1952 by Y.Nambu [2]. M.MacGregor calculated particle and constituent quark mass as *multiples* of α and related parameters [3]. This article is a revised + shortened version of [4].

An appropriate form of Ψ can be deduced from three boundary conditions:

1.) To be able to apply Ψ to a point charge $\Psi(r = 0) = 0$ is required, this may be considered by a term such as:

$$\Psi(r) \sim \exp\left(\frac{-\beta/2}{r^{y}}\right) \tag{2}$$

2.) To ensure integrability an integration limit is needed. This may be achieved by Ψ(r) being of type

$$\Psi_n(r) = \exp\left(-\left(\frac{\beta_n/2}{r^x} + \left[\left(\frac{\beta_n/2}{r^x}\right)^2 - 4\frac{\beta_n/2}{\sigma r^x}\right]^{0.5}\right)/2\right) \tag{3}$$

with index n referring to radial quantum number [4].

3.) Ψ should be applicable regardless of the expression chosen to describe the electromagnetic object. In particular requiring a point charge and a photon representation of a localized electromagnetic field (particle) to have the same energy, the exponent of r is required to be x=3 (see (14)), giving finally:

$$\Psi_n(r) = \exp\left(-\left(\frac{\beta_n/2}{r^3} + \left[\left(\frac{\beta_n/2}{r^3}\right)^2 - 4\frac{\beta_n/2}{\sigma r^3}\right]^{0.5}\right)/2\right) \tag{4}$$

Up to the limit of the real solution of (4), $r = r_n$, with

$$r_n = (\sigma\, \beta_n/8)^{1/3}\,, \tag{5}$$

in all integrals over Ψ(r) given below equ. (6) may be used as approximation for (4)

$$\Psi_n(r<r_n) \approx \exp\left(\frac{-\beta_n/2}{r^3}\right) \tag{6}$$

Phase will be neglected on this approximation level, properties of particles will be calculated by the integrals over $\Psi(r)^2$ (hence factor 2 in (2)ff) times some function of r which can be given by:

$$\int_0^{r_n} \Psi(r)^2 r^{-(m+1)} dr \approx \int_0^{r_n} \exp(-\beta/r_n^3) r^{-(m+1)} dr = \Gamma(m/3, \beta/r_n^3)\ \frac{\beta^{-m/3}}{3} = \int_{\beta/r_n^3}^{\infty} t^{\frac{m}{3}-1} e^{-t} dt\ \frac{\beta^{-m/3}}{3} \tag{7}$$

with m = {..-1;0;1;..}. The term $\Gamma(m/3, \beta/r_l^3))$ denotes the upper incomplete gamma function, given by the Euler integral of the second kind with β/r_n^3 as lower integration limit. For $m \geq 1$ the complete gamma function $\Gamma_{m/3}$ is a sufficient approximation, for $m \leq 0$ the integrals have to be integrated numerically.

Coefficient β_n contains a particle specific factor and is proportional to particle energy W as $\beta_n \sim W_n^{-3}$ (11), for particle n it may be given as partial product of a value for a reference particle, β_{ref}, carrying the dimensional term, β_{dim}, times particle specific dimensionless coefficients, α_n, of succeeding particles representing the ratio of β_n and β_{n+1}:

$$\beta_n = \beta_{ref} \Pi_{k=1}^{n} \alpha_k = 2\sigma \beta_{dim} \alpha_{ref} \Pi_{k=1}^{n} \alpha_k \tag{8}$$

Index n will indicate solutions of (4) i.e. spherical symmetric solutions. For the angular terms of Ψ(r, ϑ, φ), to be indicated by index l, only rudimentary results exist, their contribution has to be incorporated in parameter σ which according to (4) is related to the solution for a bound state and r_n. Coefficients r_n and σ determine the integration limit of the integrals over Ψ and thus are a crucial factor in particular for the (semi-classical) calculation of angular momentum J. Coefficients r_n and σ will have the following values (see [A1,2]):

$$r_n \approx 1.5133\ \alpha^{-1}\, |\Gamma_{-1/3}|\, \beta_n^{1/3}/3 \tag{9}$$

$$\sigma = 8\, r_n^3/\beta_n = 8(1.5133\ \alpha^{-1}\, |\Gamma_{-1/3}|/3)^3 = 1.772E+8\ [-] \tag{10}$$

However, factor 1.5133 is also part of a minor term depending on the radial quantum number, n (see [A2,3]). Thus in the following β_n may be split into $\sigma/1.5133^3 = 5.112E+7$ [-] and n-dependent terms containing factor 1.5133^3.

Particle energy is expected to be equally divided into electric and magnetic part, $W_n = 2W_{n,el} = 2W_{n,mag}$. To calculate energy, the integral over the electrical field E(r) of a point charge is used as a first approximation.

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4 Note: β_n will be defined as product of σ and a particle specific term, α(n), in (8) times factor 2 due to Ψ appearing as Ψ^2 in (7), $\beta_n = 2\sigma\alpha(n)\beta_{dim}$. Parameter β is kept for brevity. The associated differential equation is given in 2.6.

Using (7) for m = 1 gives ($b_0 = e^2/(4\pi\varepsilon)$ used as abbreviation, e=elementary charge, ε=electric constant):

$$W_{pc,n} = 2\varepsilon_0 \int_0^\infty E(r)^2 \Psi_n(r)^2 d^3r = 2b_0 \int_0^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr = 2\, b_0\, \Gamma_{1/3}\, \beta_n^{-1/3}/3 \qquad (11)$$

Using equation (7) for m = -1 to calculate the Compton wavelength, λ_C, in the expression for the energy of a photon, hc_0/λ_C, gives the following expression for λ_C :

$$\lambda_{C,n} \approx \int_0^{\lambda_{C,n}} \Psi_n(r)^2 dr = \int_{\beta/\lambda_{C,n}^3}^{\infty} t^{-4/3} e^{-t} dt \;\; \beta_n^{1/3}/3 \approx 36\,\pi^2\, |\Gamma_{-1/3}|\, \beta_n^{1/3}/3 \qquad (12)$$

to be used in:

$$W_{Phot,n} = hc_0/\lambda_{C,n} = \frac{hc_0}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} = \frac{3hc_0}{36\pi^2 |\Gamma_{-1/3}|\, \beta_n^{1/3}} \qquad (13)$$

The energy of a particle has to be the same in both photon and point charge description. Equating (11) with (13) and rearranging to emphasize the relationship of α with the gamma functions ($\Gamma_{1/3} = 2.679$; $|\Gamma_{-1/3}| = 4.062$) gives (note: h => ħ):

$$\frac{4\pi\Gamma_{1/3}|\Gamma_{-1/3}|}{0.998} = \frac{9hc_0}{18\pi b_0} = \frac{\hbar c_0}{b_0} = \alpha^{-1} \qquad (14)$$

i.e. the value of α is determined by the Γ-functions appearing in the integral related to the spherical symmetry of the point charge, originating from a 3D integral, and the 1D integral for calculating (photon wave -) length, $3\int^\infty \exp(-x/r^3)\, r^{-2}\, dr \;\; 3\int^\infty \exp(-x/r^3)\, dr \approx 4\pi\, \Gamma_{+1/3}\, |\Gamma_{-1/3}|$, turning α into an essentially geometric coefficient.

2.2 Quantization with powers of $1/3^n$ over α

Inserting (8) in the product of the point charge and photon expression of energy, W_n^2, gives:

$$W_n^2 = 2b_0 hc_0 \frac{\int^{r_{l,n}} \Psi_n(r)^2 r^{-2} dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr} \sim \frac{1}{\beta_n^{2/3}} \sim \frac{\alpha_0^{1/3}\alpha_1^{1/3}.....\alpha_n^{1/3}}{\alpha_0\alpha_1....\alpha_n} \qquad n = \{0;1;2;..\} \qquad (15)$$

The last expression of (15) is obtained by expanding the product $\Pi_n^{-2/3}$ included in $\beta_n^{-2/3}$ of (8) with $\Pi_n^{1/3}$ From this term it is obvious that a relation $\alpha_{n+1} = \alpha_n^{1/3}$ such as given by equation (1) yields the only non-trivial solution for W_n^2 where all intermediate particle coefficients cancel out and W_n becomes a function of coefficient α_0 only. By comparison with experimental data α_0 may be identified as $\alpha_0 = \alpha_e \approx \alpha^9$ and the α-product can in general be given by:

$$\frac{\alpha^3\alpha^1....\alpha\wedge(9/3^n)\alpha\wedge(3/3^n)}{\alpha^9\alpha^3\alpha^1....\alpha\wedge(9/3^n)} = \alpha\wedge(3/3^n)/\alpha^9 \qquad n = \{0;1;2;..\} \qquad (16)$$

The corresponding term for particle energies will be (using (14)):

$$W_n = \left(\frac{4\pi b_0^2}{\alpha} \frac{\int^{r_n} \Psi_n(r)^2 r^{-2} dr}{\int^{\lambda_{C,n}} \Psi_n(r)^2 dr}\right)^{0.5} = \left(\frac{(2b_0)^2\Gamma_{1/3}^2}{9[\alpha 4\pi\; |\Gamma_{-1/3}|\,\Gamma_{1/3}]\,\beta_n^{2/3}}\right)^{0.5} = \qquad n = \{0;1;2;..\} \qquad (17)$$

$$= 2b_0 \frac{\Gamma_{1/3}}{3\beta_n^{1/3}} = 2b_0 \frac{\Gamma_{1/3}}{3(2\sigma\beta_{dim})^{1/3}} \alpha\wedge(1.5/3^n)/\alpha^{4.5} \approx W_e\, \frac{3}{2}\, \Pi_{k=1}^{n+1} \alpha\wedge(-3/3^k)$$

yielding equation (1) for spherical symmetry. In the last term of equ. (17) the additional factor ≈ 3/2 has to be inserted *ad hoc* to represent the anomaly due to the energy ratio of e, μ, $W_\mu / W_e = 1.5088\ \alpha^{-1}$, see [A2]. The quantization condition given by (17) is not exclusive but considered to relate to rest mass of particles, not excluding the existence of particles with any other mass.

Equation (9) will be given accordingly as ($\alpha_e = (1.5133)^3\, \alpha^9$):

$$\beta_n \approx \beta_e \left(\frac{2}{3}\right)^3 \Pi_{n+1} = \frac{2\sigma}{1.5133^3} \beta_{dim} \alpha_e \left(\frac{2}{3}\right)^3 \Pi_{k=1}^{n+1} \alpha \wedge (9/3^k) \approx 2\sigma \beta_{dim} \left(\frac{2}{3}\right)^3 \Pi_{k=0}^{n} \alpha \wedge (9/3^k) \qquad n=\{0;1;2..\} \qquad (18)$$

with factor $(2/3)^3$ representing factor 3/2 in (1) and Π_{n+1} representing the dimensionless term in β_n relating particle n to the electron. A fit of W_e will give β_{dim} = 1.77E-33 [m^3]. Extending the model to energies below the electron with a coefficient of α^3 in (1) gives a state of energy ~ 0.2eV which is roughly in a range expected for a neutrino [5].

2.3 Non-spherical symmetric states

Up to here only spherical symmetry, y_0^0, and Ψ(r) have been considered [5]. The ratio of the volume integrals attributed to spherical harmonic Y_1^0 and Y_0^0 gives a factor of 1/3. Assuming Y_1^0 to be a sufficient approximation for the next angular term and $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3}$ (V = volume) to be applicable for non-spherically symmetric states as well, will give $W_1^0/W_0^0 = 3^{1/3} = 1.44 = (y_1^0)^{-1/3}$. A change in angular momentum is expected for this transition which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2. Such angular terms have to be attributed to the parameter σ, see 2.4. Results for particles assigned to y_0^0, y_1^0 are presented in table 1.

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient (energy) equ (1)	α-coefficient in ß equ (18), (28), (38)f	W_{calc}/W_{Lit}	J	r_n [fm]
Planck	(-1,∞)	1.0 E+21*	$(2/3\ \alpha^{-3})^3\ 3/2\ \alpha^{-1}\ 2$ source term, relative to e ! *	$((3/2)^3\ \alpha^9)^3 / (3\ \alpha^{-1})^3$ source term, relative to e ! *	0.9994 rel. to e !*	-	-
e^{+-}	**0, 0**	**0.51**	$\mathbf{2/3\ \alpha^{-3}}$	$(3/2)^3\ \boldsymbol{\alpha^9}$	**1.0001****	**1/2**	**1412**
μ^{+-}	**1, 0**	**105.66**	$\boldsymbol{\alpha^{-3}\alpha^{-1}}$	$\boldsymbol{\alpha^9\alpha^3}$	**1.0000**	**1/2**	**6.83**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}\ 1.44$	$\alpha^9\alpha^3/3$	1.0918	0	4.74
K		495				0	
η^0	**2, 0**	**547.86**	$\boldsymbol{\alpha^{-3}\alpha^{-1}\alpha^{-1/3}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1}$	**0.9933**	**0**	**1.32**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})\ 1.44$	$\alpha^9\alpha^3\alpha^1/3$	1.0124	1	0.92
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})\ 1.44$	$\alpha^9\alpha^3\alpha^1/3$	1.0028	1	0.92
K*		894				1	
p^{+-}	**3, 0**	**938.27**	$\boldsymbol{\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1\alpha^{1/3}}$	**1.0016**	**1/2**	**0.76**
n	**3, 0**	**939.57**	$\boldsymbol{\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1\alpha^{1/3}}$	**1.0003**	**1/2**	**0.76**
η'		958				0	
Φ^0		1019				1	
Λ^0	**4, 0**	**1115.68**	$\boldsymbol{\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}}$	**1.0106**	**1/2**	**0.63**
Σ^0	**5, 0**	**1192.62**	$\boldsymbol{\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}}$	$\boldsymbol{\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}}$	**1.0046**	**1/2**	**0.61**
Δ	**∞, 0**	**1232.00**	$\boldsymbol{\alpha^{-9/2}}$	$\boldsymbol{\alpha^{27/2}}$	**1.0025**	**3/2**	**0.59**
Ξ		1318				1/2	
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})\ 1.44$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}/3$	0.9796	3/2	0.53
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})\ 1.44$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}/3$	0.9724	3/2	0.45
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})1.44$	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}/3$	1.0046	3/2	0.43
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})$	$\alpha^{27/2}/3$	1.0026	1/2	0.4
Higgs	∞,∞ ***	1.25 E+5	$(\alpha^{-9/2})\ 3/2\ \alpha^{-1}/2$	$(\alpha^{27/2})/(3/4\ \alpha^{-1})^3$	1.0192	0	0.006

Table 1: Particle energies for $\mathbf{y_0^0}$ **(bold)**, y_1^0 [6]; col. 2: radial, angular quantum number; col. 3: energy values of [6] except*, see 2.7; col. 4: α-coefficient according to (1), including (2/3) α^{-3} of electron; col.6: W_{calc} calculated using the slightly more precise [A(38)] in place of (18); ** Using [A (42)]; *** see 2.4;

2.4 Upper limit of energy

According to (4) a bound state requires approximately σ > 1. Higher angular terms will reduce the value of σ. The variable part in σ is given by the term $(1.5133\ \alpha^{-1})^3$ thus the maximum angular contribution to W_{max}

5 y_l^m defined as $\quad y_l^m = \int\int \Psi(\varphi,\vartheta)^2 \sin(\vartheta) d\varphi\, d\vartheta \;/\; 4\pi$

6 up to Σ'0 all resonance states given in [6] as **** included; Exponents of -9/2, 27/2 for Δ and tau are equal to the limit of the partial products in (1) and (18); r_n calculated with (5); 1.5133 rounded to 3/2;

should be:

$$\Delta W_{max,\ angular} = 1.5133\ \alpha^{-1} \tag{19}$$

Considering that according to equation [A2,(37)] factor 1.5133 will approach 3/2 for n -> ∞, the maximum energy will be $W_{max} = W_e\ 1.5{*}1.5133\ \alpha^{-2.5}$ = 4.05E-8 [J].

In the simple visualization sketched in the introduction the "rotating E-vector" might be interpreted to cover the whole angular range in the case of spherical symmetric states while a p-like state of an Y_1^0-analogue might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the angular limit case, l -> ∞, a state of minimal angular extension representing the original vector, however, extending in both directions from the origin and featuring parity p= -1. Considering only „half" such a state, extending in one direction only and having p= +1, would notably feature an energy of 1.01 W_{Higgs}, suggesting the Higgs boson as possible high energy end of (1).

2.5 Expansion of the incomplete gamma function $\Gamma(1/3,\beta_n/r^3)$

The series expansion of $\Gamma(1/3,\beta_n/r^3)$ in the equation for calculating particle energy (11) gives [7]:

$$\Gamma(1/3,\ \beta_n/(r^3)) \approx \Gamma_{1/3} - 3\left(\frac{\beta_n}{r^3}\right)^{1/3} + \frac{3}{4}\left(\frac{\beta_n}{r^3}\right)^{4/3} = \Gamma_{1/3} - 3\frac{\beta_n^{1/3}}{r} + \frac{3}{4}\frac{\beta_n^{4/3}}{r^4} \tag{20}$$

and for $W_n(r)$:

$$W_n(r) \approx W_n - \frac{2}{2} b_0 \frac{3\beta_n^{1/3}}{3\beta_n^{1/3} r} + 2 b_0 \frac{3}{4}\frac{\beta_n^{4/3}}{3\beta_n^{1/3} r^4} = W_n - \frac{b_0}{r} + b_0\frac{\beta_n}{2r^4} \quad {}^{7} \tag{21}$$

The 2nd term in (21) drops the particle specific factor β_n and gives the electrostatic energy of two elementary charges at distance r. The 3rd term is an appropriate choice for the 0th order term of the differential equation below. It is supposed to be responsible for the localized character of an electromagnetic object and thus may be identified with the "strong force" of the standard model.

2.6 Differential equation

The approximation $\Psi(r<r_n)$ of equation (6) provides a solution to a differential equation of type

$$-\frac{r}{6}\frac{d^2\Psi(r)}{dr^2} + \frac{\beta_n/2}{2r^3}\frac{d\Psi(r)}{dr} - \frac{\beta_n/2}{r^4}\Psi(r) = 0 \tag{22}$$

However, the discriminant form of $\Psi(r)$ of equ. (4) would be provided by a slightly different equation

$$-r\frac{d^2\Psi(r)}{dr^2} + \frac{\beta_n/2}{r^3}\frac{d\Psi(r)}{dr} - \frac{\beta_n/2}{\sigma\ r^4}\Psi(r) = 0 \tag{23}$$

Equation (23) may be turned into a more conventional expression containing terms used in quantum mechanics by using a quantum mechanical operator for kinetic energy, $T = (\hbar c_0)^2\, r/b_0$, i.e. $c_0^2 r/b_0$ representing 2/(2m) [8], and the 3rd term in (21) for potential energy, V:

$$V(r) = b_0\ \beta_0/(2\ r^4) = b_0\ [2\sigma\ \alpha_0'\ \beta_{dim}]\ /(r^4) = b_0\ [2\sigma/1.5133^3\ \alpha_0''\ \beta_{dim}]\ /(r^4) \tag{24}$$

resulting in:

$$-\frac{(\hbar c_0)^2 r}{b_0}\frac{d^2\Psi(r)}{dr^2} + \alpha^{-2} r V(r)\frac{d\Psi(r)}{dr} - \frac{\alpha^{-2}V(r)}{\sigma}\Psi(r) = 0 \tag{25}$$

Introduction of $(\hbar c_0)^2/b_0$ as well as b_0 in V(r) corresponds to an expansion by $(\hbar c_0)^2\alpha^{-2}/b_0^2$, requiring factor α^{-2} to be included in (25) to reproduce (23).

2.7 Planck energy

By defining the Planck energy W_{Pl} as [9]

$$W_{Pl} = c_0^2\ (b_0/G)^{0.5} = c_0^2\ (\alpha h c_0/G) = 1.67\text{E+8 [J]} \tag{26}$$

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7 Signs not adapted to conventional definition. The 2nd term may be divided by two since it represents only an electrostatic contribution, to be complemented by another, magnetic term.

8 Assuming $W_{n,kin} = W_n/2$

9 This conforms to the assumption that gravitational interaction is a higher order, nonlinear effect of electromagnetic interaction and as such should be of less or equal strength compared to the latter.

gravitational attraction F_G between two particles m and n in the classical limit can be expressed as:

$$F_{G,m,n} = \frac{b_0 W_n W_m}{W_{Pl}^2} \frac{1}{r^2} \tag{27}$$

Equation (27) is a restatement of Newton's law with no additional insight unless an expression for W_{Pl} independent of G would be at hand. Expanding relationship (18) to higher powers of α i.e. $\alpha_e^3 = (1.5133^3\,\alpha^9)^3$ provides just that. The relationship is quantitative if using [10]

$$\frac{W_e}{0.9994\,W_{Pl}} = \left(1.5133^3\,\alpha^9\right)\left(1.5133^{-1}\,\alpha\right)/2 = 1.5133^2\,\alpha^{10}/2 = \left(\frac{F_{G,e}}{F_{C,e}}\right)^{0.5} = \alpha_0 = 4.903\text{ E-22} \tag{28}$$

i.e. the ratio of the electrostatic part of W_e, $W_{e,elst} = W_e/2$ to the electrostatically defined W_{Pl}, given as α_e^3 times the angular limit factor according to (19), 1.5133 α^{-1} [11].

Using [A(36)] to express factor 1.5133 gives (F_G, F_C = gravitational, Coulomb forces):

$$\left(\frac{W_e}{W_{Pl}}\right)^2 = \left(\frac{F_{G,e}}{F_{C,e}}\right)_{calc} = \left[\frac{(4\pi)^2\,\Gamma_{-1/3}^4\,\alpha^{12}}{2}\right]^2 = 1.0008^2\left(\frac{F_{G,e}}{F_{C,e}}\right)_{exp} = \frac{G W_e^2}{c_0^4 b_0} \quad ^{12} \tag{29}$$

3 Discussion

3.1 Standard model of particle physics

The standard model of particle physics is very much elaborated in particular with regard to hadron properties and the reliability of the model presented here will depend crucially on reproducing the symmetry properties as represented by the various quarks. A major discrepancy to the SM arises from the capability of this model to calculate both lepton and hadron properties. The standard model distinguishes quite rigidly between both types, postulating that a set of physical objects characterized by an almost identical set of experimental observables is based on completely different physical principles. The distinctive observable for both particle groups is assumed to be the strong force which is postulated to be zero for leptons, which per se is not verifiable beyond experimental accuracy. The three generation model, attributing a neutrino to each charged lepton, serves as supporting argument. However, the total number of neutrinos is not beyond doubt [5], [8] and neutrino oscillation obscures the earlier assumption of clearly distinct particles. Last not least, a distinctive interaction of neutrinos with the charged leptons might simply be due to a very weak strong interaction of the particles involved, not requiring any assumption beyond that.

According to this model weak strong interaction for leptons is expected in scattering events since the effect of strong interaction between particles is considered to be due to wave function overlap [9] depending on

1) comparable size and energy of wave functions,

2) sufficient net overlap: If regions with same and opposite sign balance to give zero net overlap, no interaction occurs. From condition 1) it is obvious that the wave functions of neutrino and electron will not show effective interaction with hadrons [13]. In the case of the tauon the second rule is crucial. In this model the tauon is at the end of the partial product series for y_1^0 and should exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign prohibiting net overlap and effective interaction with hadrons of higher symmetry, such as the proton.

In the standard model mass of elementary particles is generated by the “Higgs mechanism”. In this model the Higgs boson is a candidate for the highest energy state indicating some relationship between the Higgs mechanism and the generation of standing electromagnetic waves of well defined rest energy. A fundamental break of symmetry associated with the creation of a „localized“ photon in this model is the generation of +/- charge due to the persistent orientation of the E-vector towards the origin [14].

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10 Note the difference of α-coefficients in β- and energy expressions. The energy of the hypothetical Planck state is $W_{Planck} \sim (\alpha_0^3\,\alpha_0)^{-1/3} 1.5133\alpha^{-1}$, while for the electron $W_e \sim (\alpha_0)^{-1/3}$ holds

11 Planck energy is included in table 1 for reference, it is not expected to represent a rest mass particle.

12 Using [A, (42)] for calculating W_e would give G as: $G_{calc} = \frac{c_0^4}{\varepsilon_c}\frac{2^{10}\pi^{11/3}}{9}\frac{\Gamma_{-1/3}^{14}}{\Gamma_{1/3}^2}\alpha^{30} \approx \frac{c_0^4}{\varepsilon_c}\frac{2}{3}\frac{1}{4\pi}\alpha^{24} \approx 1.001\,G_{exp}$

13 As for energy density ~ W_m/W_n^4 : e/p ~ E-13, μ/p ~ 6E-4; μ/π ~ 1/3, i.e. in case of μ/π some measurable effect might be expected; different symmetry may play an additional role.

14 Applies equivalently for partial charge in case of neutral particles.

3.2 Gravitation

Within this model particles might interact via direct contact in place of boson-mediated interaction. The particles are not expected to exhibit a rigid radius. Within the limits of charge and energy conservation a superposition of many states might be conceivable, extending the particle in space with radius r_{VS} appropriate for energy of each virtual particle state (VS) [15], providing a source of energy at a distance r_{VS} from the primary particle and in turn contributing to the stress-energy tensor responsible for curvature of space-time that manifests itself in gravitational attraction. The appearance of the angular limit term of (19) in (28)ff is supposed to reflect the instantaneous character of such an interaction. The value of the ratio of F_G and F_C indicates that the probability for the generation of such states is associated wth the strong interaction term, see 3.3.

Such an interpretation implies that at least part of the energy in the "vacuum" is supplied by virtual particle states. They are distinguished by the product $W_n r_n$ being constant, giving a non constant spatial contribution to vacuum energy.

The model might fit a modified Kaluza-Klein (KK) theory in non-compactified form [10], where charge is related to motion in the 5^{th} dimension and the 5^{th} coordinate might be related to rest energy. In this model energy of a VS is intrinsically connected to distance itself, corroborating such an extension of space-time. An alternate view on gravitational attraction might be to consider the wave function of a particle contributing an additional factor to lower total Ψ^2 values on site of a second particle thereby reducing particle energy and resulting in an attractive force, relating Ψ^2 directly to the stress-energy tensor. Components of Ψ might provide appropriate candidates for the scalar field, $\mathbf{\Phi}$, of KK, see [A4].

3.3 Summary

In the following an attempt to summarize the results in a coherent manner is tried with some additional aspects emphasized.

The reference value, W_e, will be replaced with a more general condition:

a) the dimensionless parameter α_0 of V(r), according to (24) will be given by the energy ratio of a ground state (≙ state of point charge) and some maximum allowed energy state as dimensionless parameter, i.e. $\alpha_0 = W_{GS}/W_{max}$.

b) For the dimension part [m³] an expression in appropriate natural units might be a suitable choice, β^*_{dim}. As first guess it will be assumed that β^*_{dim} may be approximated by $\beta^*_{dim} \approx r_{GS}^3$.

This gives the following results:

I According to (11), (17) $\alpha_{GS} \approx \alpha_0$ and $\alpha_{max} \approx \alpha_{GS}^3$ have to hold.

II Using (5) and β^*_{dim} requires $\alpha_0 = \alpha_{GS}$ to be roughly in the order of $\alpha^{7.5}$ [16], providing an approximate absolute energy of the ground state without need for any reference.

III In the following $\alpha_0 = W_e/W_{Pl}$ according to (28) will be used, with $\beta_0 = \sigma/1.5133^3\,\alpha_0\,\beta^*_{dim}$ [17] and β^*_{dim} given by [A3 (43)], $\beta^*_{dim} \approx (e_c/\varepsilon_c)^3$ as suitable choice for a natural unit system[18]. To get absolute values for energy, either [A3 (43)] may be used, giving $W_n = f(e_c, \varepsilon_c)$ or W_{Pl}, giving $W_n = f(G, e, \varepsilon)$, i.e. in both cases no free parameter will be required for the model.

IV A long range effect of the strong interaction term may be exerted via virtual particle states (i.e. angular limit state, $\sigma \to (2/3\,|\Gamma_{-1/3}|)^3$). Using the 3^{rd} term of (21), equ. (24) and β^*_{dim} representing the cube of the natural unit for length, $\beta^*_{dim} = R_1^3 \approx (3/\Gamma_{1/3}\,\alpha^{-1} r_e)^3/2 \approx (\alpha^{-1} r_e)^3$ [19], for any VS at $r = \alpha^{-1}\, r_{VS} = \Pi_{VS}^{1/3}\,\alpha^{-1} r_e \approx R_{VS}$ equ. (30) will hold:

$$W_{VS}(r) \approx b_0\,\frac{\beta_{VS}/2}{(\alpha^{-1}\,r_{VS})^4} \approx \frac{b_0}{2}\,\frac{(2/3|\Gamma_{-1/3}|)^3\alpha_0\Pi_{VS}(3/\Gamma_{1/3}\alpha^{-1}\,r_e)^3}{1.5133^3(\alpha^{-1}\,r_{VS})^3(\alpha^{-1}\,r_{VS})\;2} \approx 2b_0\,\frac{\alpha_0\Pi_{VS}(\alpha^{-1}\,r_e)^3}{(\Pi_{VS}^{1/3}\alpha^{-1}\,r_e)^3(\alpha^{-1}\,r_{VS})} = 2b_0\,\frac{\alpha_0}{R_{VS}} \approx \frac{b_0}{R_{VS}}\left(\frac{F_{G,e}}{F_{C,e}}\right)^{0.5} \tag{30}$$

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15 Not virtual in a Heisenberg sense, energy is provided by the primary particle; energy is not restricted to the set of (1).

16 $r_{GS}^3 \approx \sigma^2\alpha_0\beta^*_{dim} \approx \sigma^2\alpha_0\,r_{GS}^3 \;\Rightarrow\; \alpha_0 \approx \alpha^{7.5}$; using $\beta^*_{dim} \approx (e_c/\varepsilon_c)^3$ would result in $\alpha_0 \approx \alpha^{10}$.

17 W_e will be given as: $W_{e,calc} = 2b_0\,\Gamma_{+1/3}/3\,(\sigma/1.5133^3\,\alpha_0\,\beta^*_{dim})^{-1/3} = 8.16E-14$; note that according to [A (40)] α_0 does include factor 2 of β while α_0' of (24) does not.

18 With e_c, ε_c referring to a unit system with c_0 split symmetrically into electric and magnetic constant

19 Relationship with r_e is phenomenological only; Capital R denotes distance in natural units, R_1 = 1[length].

The crucial factor that turns the r^{-4} dependence of the strong interaction term into r^{-1} of gravitational interaction is the proportionality of β_n to the cube of any characteristic particle length, r_n, $r_{m,n}$, $\lambda_{C,n}$ etc. which is valid for each particle state subject to the relations of this model.
The composition of the stress-energy tensor from virtual states should be based on a much more complex mechanism requiring consideration of all possible virtual states at a particular point. However, equ. (30) indicates that the result of an appropriate average of such states may be subsumed in such a simple term. Equ. (30) is a representation of the gravitational potential of the electron, terms for other particles may be obtained by inserting values according to (1) in (30) which might be interpreted as the intensity/frequency of emergence of virtual states being proportional to the energy of the primary particle.

Conclusion

Using an exponential function $\Psi(r,\vartheta,\varphi)$ as probability amplitude to modify the electric field $E(r,\vartheta,\varphi)$ gives the following results:
- the fine-structure constant, α, is defined by the product of the Γ-functions in the integrals over $\Psi(r)$ related to spherical and 1D symmetry, $4\pi\ \Gamma_{+1/3}\ |\Gamma_{-1/3}| \approx \alpha^{-1}$,
- a quantization of energy levels given by a partial product of terms α^(-1/3^n)
- the limits of the partial product are given by the Planck energy as source term and the energy of the Higgs boson as the upper limit,
- qualitative explanations for particle properties e.g. the lepton character of the tauon,
- a function for particle energy including terms for rest energy, electromagnetic interaction and a 3rd term which at short range yields effects associated with strong interaction,
- the relationship with Planck energy yields the gravitational constant in electromagnetic terms, consistent with a mechanism of the strong interaction term representing virtual particle states.

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Appendix

[A1] Angular momentum

A simple relation with angular momentum J for spherical symmetric states will be given by applying a semi-classical approach using

$$J = r_2 x\, p(r_1) = r_2 W_n(r_1)/c_0 \tag{31}$$

with $W_{kin,n} = 1/2\ W_n$, using term $2b_0$ of (11) as constant factor, integrating over a circular path of radius $|r_2| = |r_1|$ and setting the terms of (9), (10) as integration limits. This will give :

$$|J| = \int_0^{r_n}\int_0^{2\pi} J_n(r)\,d\varphi\,dr = 4\pi\frac{b_0}{c_0}\int_0^{r_n}\Psi_n(r)^2 r^{-1}dr \tag{32}$$

From (8) follows for m = 0:

$$\int_0^{r_n} \Psi_n(r)^2 r^{-1}\, dr \;=\; 1/3 \int_{8/\sigma}^{\infty} t^{-1} e^{-t} dt \;\approx\; 5.45 \;\approx\; \alpha^{-1}/8\pi \tag{33}$$

Inserting (33) in (32) would provide:

$$|J| = \; 4\pi \frac{b_0}{c_0} \frac{\alpha^{-1}}{8\pi} \; = 1/2\ [\hbar] \tag{34}$$

The derivation of (34) requires a lot of assumptions. However, the parameter 1.5133 α^{-1} associated with this reasoning is relevant for many relations of this work supporting the significance of 1.5133 α^{-1}, see [A2]. In general the whole complex of angular part of the wave function, wave function phase, angular momentum, magnetic moment etc. of this model is not well understood yet and has to be elaborated in more detail.

[A2] Coefficient ~1.5

The value of 1.51 α^{-1} in r_l, σ originates from the relationship with J through equ. (32) and is obviously close to the ratio W_μ/W_e = 206.8 = 1.5088 α^{-1}. The source of this anomaly is supposed to be the electron rather than the muon, which is a middle term of product (18) and the equations will be arranged accordingly in (18) by factor $(2/3)^3$ representing a general factor of all particles to be canceled by a factor ≈ $(3/2)^3$ in α_e. Several options for the exact value of ~1.51 involved in this model have been considered: 3/2, $|\Gamma_{-1/3}|/\Gamma_{1/3}$ = 1.516, π/2 = 1.571, 1.5088 etc. The value 1.5133 has been chosen due to

1. a possible geometrical interpretation (using(14)) of the term in σ:

$$1.516\ \alpha^{-1}\ |\Gamma_{-1/3}|\ /3 = \ |\Gamma_{-1/3}|\ /\Gamma_{1/3}\ \ 4\pi\ |\Gamma_{-1/3}|\ \Gamma_{1/3}/0.998\ \ |\Gamma_{-1/3}|/3 = \ \frac{1}{0.998}\ \frac{4\pi\left|\Gamma_{-1/3}{}^{3}\right|}{3} \tag{35}$$

giving 1.5133 as

$$1.5133 \ = 1.516 * 0.998 \ = \ 4\pi\,\Gamma_{-1/3}^{2}\ \alpha \tag{36}$$

2. Factor 1.5088 of the ratio W_μ/W_e being subject to a 3rd power relationship of the same kind as the α coefficients:

$$\left(\frac{1.5133}{1.5088}\right)^3 = \left(\frac{1.5133}{1.5}\right) \tag{37}$$

indicating that the radial terms of β_n and the angular components of σ are not correctly separated yet or may not be separable even in the case of spherical symmetric states.

[A3] Particle parameter β

Apart from the particle coefficients α_n, parameter β_n may be analyzed further. To avoid introducing additional parameters one might test an approach giving β as function of b_0 and σ. A suitable expression will be:

$\beta \sim \sigma\, b_0^2/(2\pi)^3$ and since (37) will be used within the particle specific factor, coefficient 1.5133 of σ will be placed there, giving for the general term (i.e. excluding the electron) [20] :

$$\beta_n \;=\; \beta^{\#}_{dim}\ \left(\frac{2}{3}\right)^3 \frac{2}{(2\pi)^3}\ \frac{\sigma}{1.5133^3}\ \ b_0^2\,\Pi_{k=0}^{n}\left[\alpha^3\left(\frac{1.5133}{1.5}\right)\right]\wedge\left(\frac{3}{3^k}\right) \qquad n = \{1,2,...\} \tag{38}$$

for the electron:

$$\beta_e \;=\; \beta^{\#}_{dim}\ \left(\frac{2}{3}\right)^3 \frac{2}{(2\pi)^3}\ \frac{\sigma}{1.5133^3}\ \ b_0^2\ \left[\ \frac{3}{2}\ \alpha^3\left(\frac{1.5133}{1.5}\right)\right]^3 \tag{39}$$

the particle specific factor is given in square brackets.

The terms in (38)f are related to the component of (28), 1.5133^{-1} α/2, by:

$$\left(\frac{2}{3}\right)^3 \frac{2}{(2\pi)^3} \;\approx\; 1.5133^{-1}\ \alpha/2 \tag{40}$$

The dimension parameter $\beta^{\#}_{dim}$ = 9.64 E+25 [m/J²] yields a simple term if a unit system with symmetric splitting of c_0 into constants ε and μ, is used, i.e. in SI units the modification:

$$c_0^2 \; = (\varepsilon_c\ \mu_c)^{-1} \tag{41}$$

with ε_c = (2.998E+8 [m²/Jm])$^{-1}$ = (2.998E+8)$^{-1}$ [J/m]
μ_c = (2.998E+8 [Jm/s²])$^{-1}$ = (2.998E+8)$^{-1}$ [s²/Jm]

i.e. the numerical values for c_0, $1/\varepsilon_c$, $1/\mu_c$ are identical, the units of ε_c, μ_c are expanded by [Jm] for the convenience of this model. From b_0 follows for the square of the elementary charge: e_c^2 = 9,67E-36 [J²]

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20 $(2\pi)^3$ is related to the topic in [A1];

This allows to give $\beta^{\#}_{dim}$ as:

$$\beta^{\#}_{dim} = \frac{1}{e_c \varepsilon_c} \quad = 9.64\text{ E+25 } [\mathrm{m/J^2}] \tag{42}$$

or alternatively β^{*}_{dim} as:

$$\beta *_{dim} = \beta^{\#}_{dim} b_0^2 = \frac{1}{(4\pi)^2} \left(\frac{e_c}{\varepsilon_c}\right)^3 \quad = 5.13\text{ E-30 } [\mathrm{m^3}] \tag{43}$$

making e_c/ε_c a suitable choice as natural unit for length.

[A4] Kaluza-Klein parameters

In Kaluza-Klein theory the electromagnetic field is a source for the scalar field Φ:

$$g^{\alpha\beta} \nabla_\alpha \nabla_\beta \mathbf{\Phi} = 1/4 \mathbf{\Phi^3} F^{\alpha\beta} F_{\alpha\beta} \tag{44}$$

with $F^{\alpha\beta}$ being the electromagnetic tensor.

The basic structure of the Ψ term and its components might provide an ansatz for appropriate relationships, see e.g.:

$$\mathbf{\Phi}(r) \sim \left(\frac{e_c}{\varepsilon_c(r) r}\right)^2 \Psi(r)^2 \quad => \quad \nabla_r \nabla_r \mathbf{\Phi}(r) \approx \left(\frac{e_c}{\varepsilon_c(r) r}\right)^6 \left(\frac{e_c}{\varepsilon_c(r) r^2}\right)^2 \Psi(r)^2 + ... \approx \mathbf{\Phi}(r)^3 \; E(r)^2 \; \Psi(r)^2 + ... \tag{45}$$

Like this model Kaluza-Klein implies a direct connection between electromagnetism and gravitation given among other things via the electromagnetic coupling constant in the metric, κ, which could be given, using note 12, in actual electromagnetic parameters in place of G:

$$\kappa = \left(\frac{4 G 4 \pi \varepsilon_c}{c_0^2}\right)^{0.5} = \left(\frac{8}{3}\right)^{0.5} c_0 \alpha^{12} \tag{46}$$