

known skill of the author, who has added to our libraries a most useful and interesting work. Both he and the Wireless Press, which has produced the book, may be cordially congratulated on the result of their labours.

Fermat's Last Theorem.

Three Lectures on Fermat's Last Theorem.

By L. J. Mordell. Pp. vii + 31. (Cambridge: At the University Press, 1921.) 4s. net.

THE "last theorem of Fermat" states that if x, y, z, p denote positive integers, the equation $x^p + y^p = z^p$ is impossible if p exceeds 2: thus no cube can be the sum of two cubes, and so on. If the theorem is true when p is 4, or an odd prime, it is true for all other integral values of p . For three centuries this theorem has baffled the efforts of all who have attacked it, although it has attracted the attention of all first-rate arithmeticians, and a great number of amateurs. For $p=3, 4, 5, 7$ comparatively simple proofs have been discovered; but so far none of these has led to a complete generalisation.

The first great advance in the theory was made by Kummer, in connection with his researches on cyclotomic integers. He showed that if the theorem is false for any particular odd prime p , then p must not be a factor of the numerator of any one of the first $\frac{1}{2}(p-3)$ numbers of Bernoulli. This very recondite test rules out all values of p below 100 except 37, 59, 67. By additional criteria Kummer was able to prove the theorem for these exceptional primes, and hence for all values of p from 3 to 100 inclusive.

Not many years ago (1907) a prize of 100,000 marks was set aside for the first who succeeded in giving a complete proof or disproof of the theorem. Quite recently, new criteria, independent of Kummer's, have been discovered, which have to be satisfied by odd primes p for which the theorem is false, and the simplest of these is the condition $2^{p-1} \equiv 1 \pmod{p^2}$, discovered by Wilferich in 1909. Other tests of a more or less similar kind have been accumulated, and the net result is that any value of p for which the theorem is false must exceed 7000. Gauss's tables of quadratic forms warn us not to draw any conclusions from this result; in fact if N is any assigned integer, however large, a proof that the theorem is true unless $p > N$ gives us no information about the truth or falsity of the theorem in general.

Mr. Mordell's lectures give a clear and interesting account of the history and present state of this subject. Lecture I. gives a statement of the

theorem, and a summary of the work done by Kummer's predecessors; Lecture II. is on Kummer's researches, and more recent investigations of similar type; and Lecture III. gives an account of various results obtained by Libri, Sophie Germain, and others. Full references are given to the original papers, so that a reader within reach of a good reference library can make himself acquainted with details of all that has been done hitherto.

A perplexing circumstance, often alluded to, is the fact that, in a private note, Fermat distinctly asserted that he had proved the theorem. Now Fermat was never convicted of a false assertion, and only once of a wrong conjecture; on the other hand it is extremely improbable that Fermat's proof, if he had one, was in any way analogous to the work of Kummer and his successors. It is not, perhaps, unreasonable to hope that a proof may be found, some day, derived from Diophantine analysis proper, combined with a process of induction, and possibly with some application of analytical geometry, or theory of equations, or both. A really gifted youth, approaching the problem without knowledge of modern analysis, might throw a quite new and unexpected light upon it.

Mr. Mordell's pamphlet ought to do much to stimulate our rising mathematicians, and we hope that it will have a large circulation.

G. B. M.

Chemistry of Coke-oven and By-product Works.

Coke-oven and By-product Works Chemistry.

By T. Biddulph-Smith. Pp. x + 180 + 7 plates. (London: Charles Griffin and Co., Ltd., 1921.) 21s.

THE author states in the preface that his object in compiling this book is to furnish a concise manual covering, so far as space will allow, the general work required for the chemical control of coke-oven and by-product works. As regards the variety of subjects treated, he has doubtless achieved his object, but it is to be regretted that the apparent exigencies of space have caused the manual to become so concise in certain sections as to detract appreciably from the value of the work as a whole.

The most valuable section of the manual is that relating to the coal-tar naphthas. There is no doubt that the author has taken considerable pains to collect together the work of some of our best analytical chemists on methods of evaluating the constituents of coal-tar naphthas—work which

was carried out during the war period when the adequate examination of these products was a matter of such great importance. Although most of the methods dealt with have already been described either in technical journals or in the proceedings of technical societies, chemists will welcome the accumulation of this information within the covers of one volume. Moreover, the admirable summary of "The Constituents of Coal-tar and their Properties," compiled by Dr. Spielman, appears in a revised form as an appendix, and the inclusion of this information may prove useful to the coke-oven chemist by saving reference work.

The rest of the book is disappointing. In describing analytical methods the author has obviously attempted to do more than supply indications of the method recommended by him, but has failed to furnish sufficient detailed information to be of real service to the works chemist.

The chapter dealing with the fractions of coal-tar other than the naphtha fraction is meagre, and the weakness of this section accentuates the fact that chemists have not yet given adequate study to the analytical methods required in the examination of the heavier fractions of coal-tar, which are no less important than the naphtha distillate. It is in this section of the book that a recommendation appears which would have shocked those older and well-established chemists who attempted to teach us our subject, and who, not living in these times of efficiency systems and labour-saving devices, paid due reverence to instruments by which accurate measurements might be made. The recommendation refers to the crystallisation of crude tar acids, the instructions being to cool the liquid, "stirring continually with a Fahrenheit thermometer graduated in tenths of a degree."

The treatment accorded in other sections of the book to the analysis of gases, calorimetry, and the examination of chemical products made and required in the recovery works is all too brief. The analysis of coal-gas, which is acknowledged to be so intricate as to require considerable experience before trustworthy results can be expected, is dealt with in a few pages, whilst the method of procedure recommended is archaic. The estimation of naphthalene is carried out by a method which would be quite unpractical if small quantities of ammonia were present in the gas, though no mention is made of this fact.

Finally, the manual contains the usual collection of tables and conversion factors in the second appendix—so useful to reader, author, and publisher.

E. V. EVANS.

NO. 2723, VOL. 109]

Lichens.

- (1) *Lichens*. By A. L. Smith. (Cambridge Botanical Handbooks.) Pp. xxviii + 464. (Cambridge: At the University Press, 1921.) 55s. net.
- (2) *A Handbook of the British Lichens*. By Annie Lorrain Smith. Pp. vii + 158. (London: The British Museum (Natural History), 1921.) 6s. 6d.

(1) **F**OR many years botanists have been without a guide to the large mass of facts that have been added year by year to our knowledge of lichens. Miss A. Lorrain Smith has therefore done a good work in compiling a very comprehensive handbook on this group of plants. The growth of our manufacturing and even our garden cities proves fatal to all except a few insignificant lichens. They are driven away to those far-off parts of the country where the air is still fresh and pure. This circumstance very possibly, but the absence of any comprehensive handbook on, and guide to, the lichens certainly, is a reason why so little interest is taken in this group. Yet, ecologically, it is one of the most interesting groups. Lichens grow on the outskirts of vegetation, as pioneers of the plant world, preparing the way for moss, fern and flowering plant. They are most intimately in touch with the substratum in its virgin condition. Few ecologists, however, properly consider lichens. Anatomically, the lichen thallus very directly reflects the nature of the substratum. A great deal, however, still remains to be done in this direction. The elaborate and careful work of the late Abbé Hue has, unfortunately, not brought much morphological order into our knowledge of lichen structure.

The whole question of the dependence of one organism, whether animal or plant, on another, or even others, again whether animal or plant, is every day becoming of greater interest. The views of various lichenologists on this matter are placed before us by Miss Lorrain Smith, but the simple word symbiosis is the term most favoured. It does not define the relationship between alga and fungus in too great a detail. Terms like helotism (due, by the way, to Warming and not to Nienburg), parasitism, consortium, endosaprophytism, and others, may cover certain individual cases, but the relationship of alga to fungus certainly varies in different species, or possibly even in different individuals of one species growing under different conditions. There is no doubt that on the whole the lichen-fungus fully controls growth and reproduction of the gonidial alga,

B