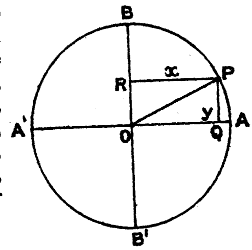
In finding the average kinetic energy of Q from A to A' and back, we have really to find the average value of QP^2, and multiply the result by $\frac{1}{2}m\omega^2$.

Suppose the particle starts from A. Divide the whole period in which the particle goes from A to A' and back into a large number $4n+4$ equal intervals of time; and let $P_1, P_2, \ldots P_n, B$ be the positions of the point P at the end of the 1^{st}, 2^{nd}, ... n^{th}, $(n+1)^{th}$ intervals respectively. Then, since P describes its circle uniformly, the arcs AP_1, P_1P_2, ... $P_{n-1}P_n$, P_nB are all equal. Hence, if x_r, y_r are the perpendiculars from P_r to BB' and AA', the average value of QP^2 during the first quarter of the period is



$$\frac{y_1^2+y_2^2+\ldots+y_n^2+a^2}{n+1}=\frac{x_n^2+x_{n-1}^2+\ldots+x_1^2+a^2}{n+1}$$

$$=\frac{x_1^2+y_1^2+x_2^2+y_2^2+\ldots+x_n^2+y_n^2+2a^2}{2(n+1)}$$

$$=\frac{(n+2)a^2}{2(n+1)}=\frac{a^2}{2}, \text{ when } n \text{ is made indefinitely great.}$$

Hence the average value of the kinetic energy $\frac{1}{2}m\omega^2 . QP^2$ during the first quarter of the period is $\frac{1}{4}m\omega^2a^2$, or half the maximum kinetic energy; and it is obviously the same for the whole period.

Corollary. For vibrations of a similar nature in the same medium, the period is the same for all particles, and therefore ω is constant.

Hence the average energies of the particles for such vibrations vary as the squares of their amplitudes. J. H. HERSCHKOWITZ.

41. *Extension of Euclid I. 47 to n-sided regular polygons.*

The following is a short summary of an article published in the *Indian Journal of Education* for December, 1895, showing how Euclid I. 47 can be extended to n-sided regular polygons without going beyond the first book.

Take equal lengths Aa, Bb, ... Ll, on the sides AB, BC, ... LA, of a given regular n-sided polygon; and upon them as bases describe outwards the isosceles triangles $AA'a$, $BB'b$, ... $LL'l$, having their base angles all equal to π/n. Then the regular polygon $A'B'$ ... L' is equal to the original one AB ... L together with the n triangles described.

This is proved by showing that $A'a$ is equal and parallel to BB'; so that, if $A'B'$ cuts AB in a', the triangles $a'aA'$, $a'BB'$ are equal in all respects. Similarly, if $B'C'$ cuts BC in b', the triangles $b'bB'$, $b'CC'$ are equal, and so on. Now, if in the figure made up of $AB \ldots L$ and the n triangles $AA'a$, $BB'b$, ..., the triangles $a'BB'$, $b'CC'$, ... be replaced by $a'aA'$, $b'bB'$, ..., the new figure thus formed is $A'B' \ldots L'$, which proves the theorem.

Take a point O such that $A'O$ is bisected at right angles by AB. Then AOA' is an isosceles triangle, whose vertical angle OAA' is $2\pi/n$, equal to the triangle $AA'a$. Hence the n equal triangles $AA'a$, $BB'b$, ... are together equal to n times the triangle AOA', or to the n-sided regular polygon described on OA'. Also, AO is equal and parallel to $A'a$, and therefore equal and parallel to BB'; hence OB' is equal and parallel to AB, and at right angles to OA'. Hence we see that, in the right-angled triangle $A'OB'$, the n-sided regular polygon $A'B' \ldots L'$ described on $A'B'$ is equal to that described on OB' (the polygon $AB \ldots L$) together with that described on OA' (the n triangles $AA'a$, ... $LL'l$). Thus the extension of Euclid I. 47 to n-sided regular polygons is proved. V. RAMASWAMI AIYAR.