



41. Extension of Euclid I. 47 to n-Sided Regular Polygons

Author(s): V. Ramaswami Aiyar

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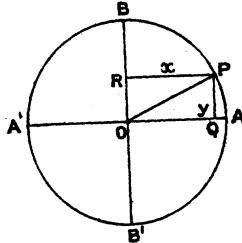


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In finding the average kinetic energy of Q from A to A' and back, we have really to find the average value of QP^2 , and multiply the result by $\frac{1}{2}m\omega^2$.

Suppose the particle starts from A . Divide the whole period in which the particle goes from A to A' and back into a large number $4n+4$ equal intervals of time; and let P_1, P_2, \dots, P_n, B be the positions of the point P at the end of the 1st, 2nd, ..., n^{th} , $(n+1)^{\text{th}}$ intervals respectively. Then, since P describes its circle uniformly, the arcs $AP_1, P_1P_2, \dots, P_{n-1}P_n, P_nB$ are all equal. Hence, if x_r, y_r are the perpendiculars from P_r to BB' and AA' , the average value of QP^2 during the first quarter of the period is



$$\begin{aligned} \frac{y_1^2 + y_2^2 + \dots + y_n^2 + a^2}{n+1} &= \frac{x_n^2 + x_{n-1}^2 + \dots + x_1^2 + a^2}{n+1} \\ &= \frac{x_1^2 + y_1^2 + x_2^2 + y_2^2 + \dots + x_n^2 + y_n^2 + 2a^2}{2(n+1)} \\ &= \frac{(n+2)a^2}{2(n+1)} = \frac{a^2}{2}, \text{ when } n \text{ is made indefinitely great.} \end{aligned}$$

Hence the average value of the kinetic energy $\frac{1}{2}m\omega^2 \cdot QP^2$ during the first quarter of the period is $\frac{1}{2}m\omega^2 a^2$, or half the maximum kinetic energy; and it is obviously the same for the whole period.

Corollary. For vibrations of a similar nature in the same medium, the period is the same for all particles, and therefore ω is constant.

Hence the average energies of the particles for such vibrations vary as the squares of their amplitudes.

J. H. HERSCHKOWITZ.

41. Extension of Euclid I. 47 to n -sided regular polygons.

The following is a short summary of an article published in the *Indian Journal of Education* for December, 1895, showing how Euclid I. 47 can be extended to n -sided regular polygons without going beyond the first book.

Take equal lengths Aa, Bb, \dots, Ll , on the sides AB, BC, \dots, LA , of a given regular n -sided polygon; and upon them as bases describe outwards the isosceles triangles $AA'a, BB'b, \dots, LL'l$, having their base angles all equal to π/n . Then the regular polygon $A'B' \dots L'$ is equal to the original one $AB \dots L$ together with the n triangles described.

This is proved by showing that $A'a$ is equal and parallel to BB' ; so that, if $A'B'$ cuts AB in a' , the triangles $a'Aa', a'BB'$ are equal in all respects. Similarly, if $B'C'$ cuts BC in b' , the triangles $b'Bb', b'CC'$ are equal, and so on. Now, if in the figure made up of $AB \dots L$ and the n triangles $AA'a, BB'b, \dots$, the triangles $a'BB', b'CC', \dots$ be replaced by $a'Aa', b'Bb', \dots$, the new figure thus formed is $A'B' \dots L'$, which proves the theorem.

Take a point O such that AO is bisected at right angles by AB . Then AOA' is an isosceles triangle, whose vertical angle AOA' is $2\pi/n$, equal to the triangle $AA'a$. Hence the n equal triangles $AA'a, BB'b, \dots$ are together equal to n times the triangle AOA' , or to the n -sided regular polygon described on OA' . Also, AO is equal and parallel to $A'a$, and therefore equal and parallel to BB' ; hence OB' is equal and parallel to AB , and at right angles to OA' . Hence we see that, in the right-angled triangle $A'OB'$, the n -sided regular polygon $A'B' \dots L'$ described on $A'B'$ is equal to that described on OB' (the polygon $AB \dots L$) together with that described on OA' (the n triangles $AA'a, \dots, LL'l$). Thus the extension of Euclid I. 47 to n -sided regular polygons is proved.

V. RAMASWAMI AIYAR.