On Transformer Indicator Diagrams

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XXXIV. *On Transformer Indicator Diagrams.* By THOMAS R. LYLE, *M.A., Sc.D., Professor of Natural Philosophy in the University of Melbourne* *.

[Plate X.]

1. THE term "transformer indicator diagram" has been applied by Professor Fleming to any series of periodic curves which gives the forms, relative phase-positions, and magnitudes of the waves of current and E.M.F. on both the primary and secondary sides of a transformer when working. Such diagrams have been obtained by many investigators in different ways, but by none of the methods hitherto used has it been possible to determine directly and independently either the wave of magnetic flux F in the core, or the wave of magnetizing-current turns usually represented by the vector sum $n_1C_1+n_2C_2$.

Both these quantities are of fundamental importance in the theory of the transformer. When they are known for any given load, all the other quantities (currents and E.M.F.s) can be determined for the same load when the primary and secondary turns, resistances, and leakage coefficients are known †.

In addition, since, as will be shown later, the integral

$$\int(n_1C_1+n_2C_2)dF$$

for one cycle is equal to the total iron loss per cycle, the advantage of being able to determine both $n_1C_1+n_2C_2$ and F directly and accurately is apparent.

Theoretically, $n_1C_1+n_2C_2$ can be obtained by the vector addition of n_1C_1 and n_2C_2, but as the latter quantities are, when the transformer carries a load, approximately equal in magnitude and opposite in phase, their vector sum is a small quantity compared with either of them. Hence small errors in the magnitudes of n_1C_1 and n_2C_2 may cause a large percentage error in the magnitude of $n_1C_1+n_2C_2$, while very small errors in the magnitudes and phase-difference of n_1C_1

* Read February 22, 1907.

† See Lyle: "The Alternate Current Transformer," Proc. Roy. Soc. Victoria, vol. xviii. pt. 1.

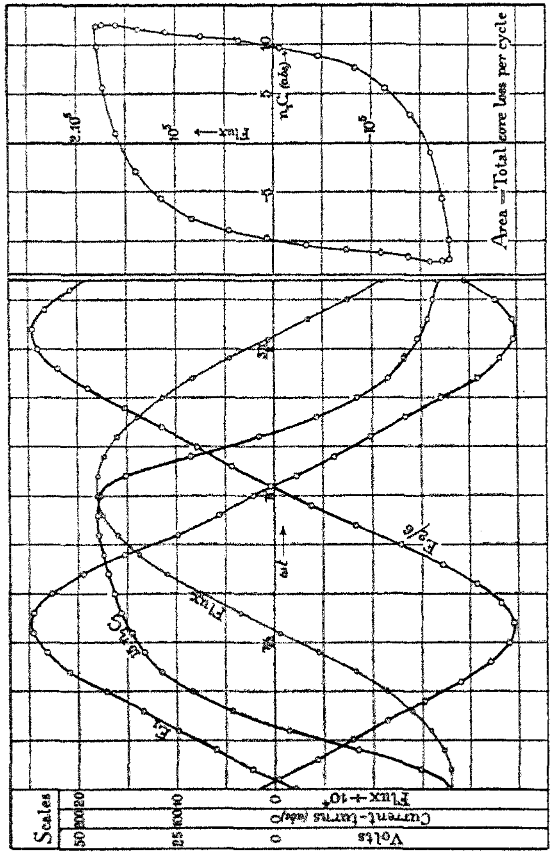


Fig. 2.—Transformer Diagram. No load. Period = ·02015 sec.

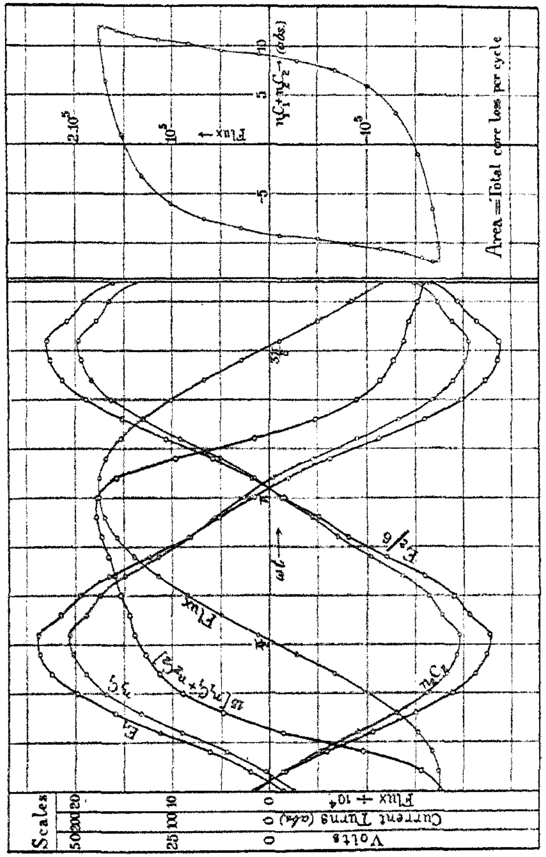


Fig. 3.—Transformer Diagram. Non-inductive load. Period = ·02053 sec.

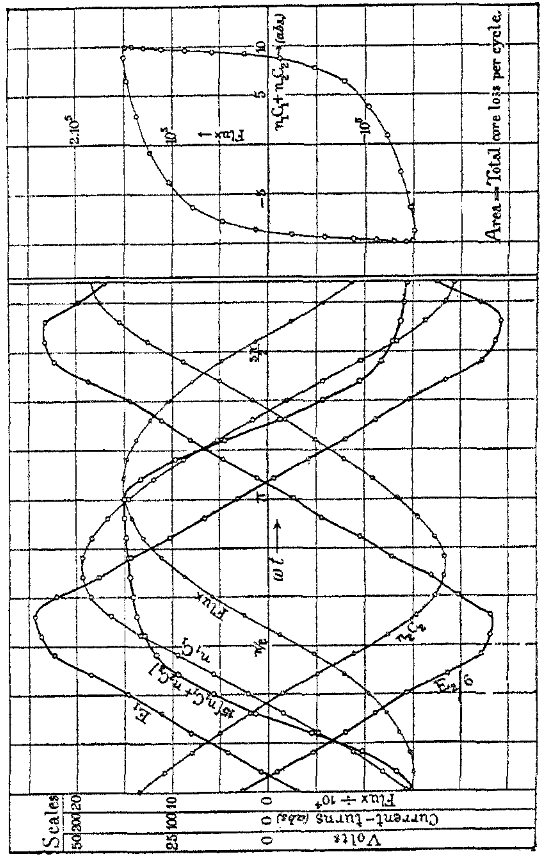


Fig. 4.—Transformer Diagram. Inductive load; power factor = ·73. Period = ·01835 sec.

and n_2C_2 may render the phase of $n_1C_1+n_2C_2$ calculated from them utterly unreliable.

By means of the wave-tracer * designed by the author not only can the E.M.F. and current waves be accurately determined, but also the wave of magnetic flux pulsating in the core of the transformer, and in addition, as will be shown in the sequel, we can obtain by its means the magnetizing-current wave, $n_1C_1+n_2C_2$, with the same accuracy as any of the other quantities.

Incidentally will be given new methods of comparing mutual inductances, and of measuring both mutual and self inductances in terms of a resistance and a time.

2. In the paper just quoted I have shown that if a periodic current C flows in the primary of a pair of coils whose mutual inductance is M, and if the secondary be joined through a suitably arranged commutator (running synchronously with the generator of the periodic current), which commutes twice per period, to a large resistance r and thence to a galvanometer, there will be a steady deflexion γ in the latter which is connected with the instantaneous value C of the periodic current at the instant of commutation by the relation

$$MC=\frac{\lambda rT}{4}\gamma,$$

where λ is the reducing factor of the galvanometer and T the period.

By arranging so that the commutating brushes can be rotated on a divided circle round the drum of the commutator, commutation can be effected at any desired instant of the period, and the corresponding galvanometer reading when multiplied by the factor given in the above equation gives the ordinate of the current wave at that instant.

Take now a triad (see fig. 1, T) of coils of which p_1 and p_2 are to serve as primaries and the remaining one s placed between p_1 and p_2 to serve as common secondary. Let s be connected as before through commutator and resistance r to the galvanometer, and let M_1 be the mutual inductance of p_1 and s, and M_2 that of p_2 and s. Then, when a current C_1 circulates in p_1 and none in p_2, the galvanometer deflexion γ_1

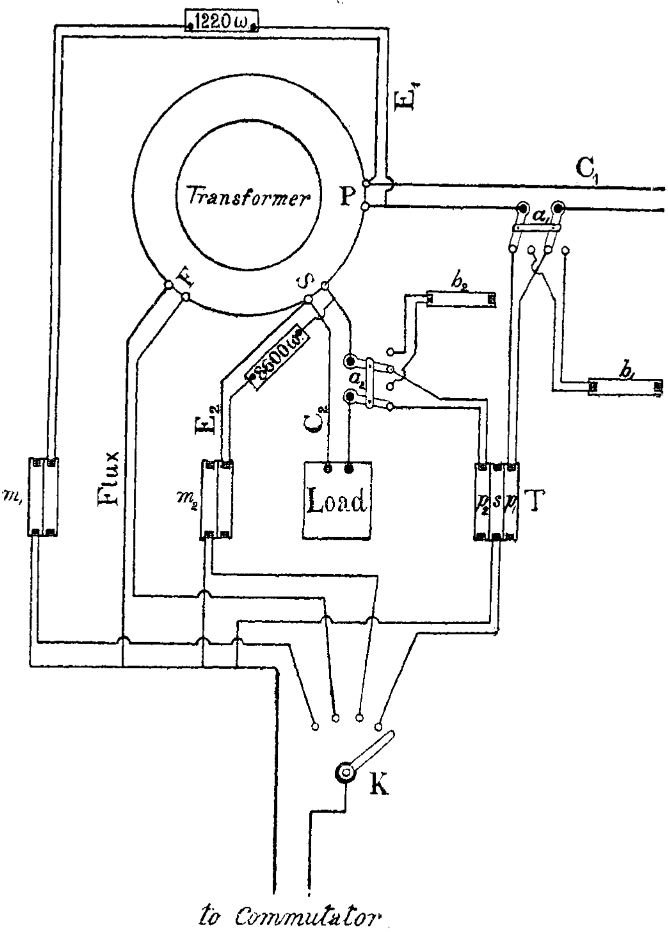
* Lyle: "Wave-Tracer and Analyser," Phil. Mag. Nov. 1903.

produced is connected with the ordinate of C_1 corresponding to the instant in the period at which commutation takes place by the equation

$$M_1C_1 = \frac{\lambda r T}{4} \cdot \gamma_1,$$

and when a current C_2, equiperiodic with C_1, circulates in

Fig. 1.



p_2 and none in p_1, a galvanometer deflexion γ_2 is produced which is similarly connected with the corresponding ordinate of C_2 by the equation

$$M_2C_2 = \frac{\lambda r T}{4}\gamma_2.$$

When both C_1 and C_2 are flowing in their respective primaries at the same time, the galvanometer deflexion γ will be the sum of γ_1 and γ_2 produced by C_1 and C_2 separately; hence

$$M_1C_1 + M_2C_2 = \frac{\lambda r T}{4}\gamma.$$

If now n_1 and n_2 are numbers such that

$$\frac{M_1}{n_1} = \frac{M_2}{n_2} = M,$$

then
$$n_1C_1 + n_2C_2 = \frac{\lambda r T}{4M}\gamma;$$

so that if γ be the galvanometer deflexion for a given position of the commutating brushes when C_1 is flowing in p_1 and C_2, correctly directed, in p_2, the ordinate of the wave which is the vector sum of n_1C_1 and n_2C_2 at the instant of commutation is the product of γ by $\lambda r T/4M$.

The same principle can obviously be extended to the determination of the vector sum of any number of equi-periodic currents each affected by an independent numerical multiplier.

In the paper already quoted it is also shown that if a coil of v turns of wire is looped on a magnetic circuit in which a periodic magnetic flux of period T is pulsating, and if the ends of the coil are connected through the synchronous commutator that changes twice per period to a high resistance r, and thence to a galvanometer as before, the ordinate F of the flux-wave and the corresponding galvanometer deflexion β for any position of the commutating brushes are connected by the relation

$$F = \frac{\lambda r T}{4v}\beta.$$

Hence any desired set of ordinates of a flux-wave can be obtained.

3. From what precedes it is obvious that in order to obtain practically the vector sum of n_1C_1 and n_2C_2, where n_1 and n_2 are numbers and C_1 and C_2 currents of equal period, it is necessary to have a triad p_1, p_2, and s of coils such that the

mutual inductance of p_1 and s is to that of p_2 and s as n_1 is to n_2. These can be wound in three grooves turned out of a circular disk of seasoned wood that has been well baked and then soaked in melted paraffin. A rough approximation to the required ratio of M_1 to M_2 can be obtained by attention to the relative numbers of turns, and a preliminary adjustment can most quickly be made by the wave-tracer as follows:—Send a periodic current of suitable value through the common secondary s. Join together one end from each of p_1 and p_2 and connect the junction to one fixed brush of the commutator. Bring the other ends of two terminals of a two-way key, from whose moving tongue a connexion is made, to the other fixed brush. Join the two movable brushes of the commutator through a suitable resistance r to the galvanometer. Thus, when a current C is flowing in s either p_1 or p_2 can be put in circuit with the galvanometer, and the deflexions γ_1 and γ_2 due to p_1 and p_2 respectively for every position of the moving brushes are in the ratio of M_1 to M_2, since

$$C=\frac{\lambda r T}{4M_1}\gamma_1=\frac{\lambda r T}{4M_2}\gamma_2.$$

Obviously in making the comparison it is desirable to place the moving brushes so that the deflexions obtained are near the maximum ordinate of the current wave.

Otherwise we might put a resistance r_1 in the galvanometer circuit when p_1 is in, and r_2 when p_2 is in, so as to obtain equal deflexions; then we would have that

$$\frac{M_1}{M_2}=\frac{r_1}{r_2}.$$

Hence the coils s and p_1 may be finished and their terminals permanently fixed, and, using the latter (equal deflexion) method, a resistance r_1 found to give a large deflexion when p_1 is in circuit and the brushes placed so as to give approximately the maximum ordinate of the current wave. Now substitute r_2 obtained by the above relation, allowing for galvanometer or other appreciable resistance that may be in circuit, and switch p_2 on to the commutator and galvanometer. An assistant can increase or reduce the turns (or fractions) until the same deflexion is obtained as was given by p_1 with r_1 in circuit. Spare wire should be left in case the further treatment of the coils causes any change in their M ratio.

As it is necessary, if high accuracy with the wave-tracer be desired, that the M coils used should have high insulation, the triad should now be thoroughly dried in an oven to drive off moisture from the cotton or silk covering and placed in an iron dish that can be fitted with an air-tight lid that has an exhausting tube through it. A layer of solid paraffin is in the bottom of the dish, and on this the coil is placed and weighted with a piece of metal. The lid is luted on with melted paraffin, the air pumped out, and the dish then heated so as to melt the paraffin it contains. The coil now sinks in the melted paraffin, and as the air has been removed it becomes thoroughly impregnated. We can thus obtain coils of high insulation and permanent mutual inductance if, in addition, proper attention is paid to the insulation of the terminals.

A final and careful adjustment of the ratio of M_1 to M_2 should now be made with the wave-tracer.

The absolute values of M_1 and M_2 must also be known, and they can easily be obtained by either of the methods just described in terms of a known standard of mutual inductance. An alternating current is sent through the primary of the standard, and either p_1 or p_2 of the triad (the one whose M is nearest in value to that of the standard) and the secondary of the standard and s of the triad arranged so that either can be switched on to the commutator and galvanometer. Then for any position of the commutating brushes the two deflexions are proportional to the mutual inductances when the resistance in circuit is fixed, or if equal deflexions be obtained the inductances will be proportional to the resistances. Obviously it will be desirable to make a number of independent comparisons by varying the position of the commutating brushes about the place at which the maximum ordinate of the current is obtained.

The absolute value of a mutual inductance can be determined by the wave-tracer in terms of a resistance and a time, as follows. An alternating current is sent through a Kelvin balance, and at the same time a number (usually 30) of equispaced ordinates (γ) of it embracing one full wave is obtained by the wave-tracer using the mutual inductance M to be measured; as

$$C_t = \frac{\lambda r T}{4M} \gamma$$

the square root of the mean squares of C_t, which is the Kelvin balance reading (B say), is $\lambda r T/4M$ times the square root of mean squares ($\bar{\gamma}$ say) of the galvanometer readings, hence

$$M = \frac{\lambda r T}{4} \frac{\bar{\gamma}}{B};$$

T is given by the chronograph attached to the wave-tracer, and the ratio of λ, the reducing factor of the galvanometer, to B can be obtained as follows. Send a continuous current $=B$ through the balance and through a standard resistance ρ (usually ·1 ohm). From the terminals of ρ lead a shunt circuit through a resistance R to the galvanometer and let the deflexion of the latter be d.

Then

$$\lambda d = \rho \frac{B}{R},$$

and hence

$$M = \rho T \frac{r\bar{\gamma}}{4Rd}.$$

4. The transformer, some of whose indicator diagrams will be given, was a small experimental one of the ring type of about one-half kilowatt capacity. It was used as a step-up one of ratio 1 to 6 transforming from about 40 volts (virtual) to 240 at about 50 periods per second.

Its details are as follows:—

Core:—144 annular laminæ annealed and paper insulated.

Laminæ:—Internal diameter .... 15·25 cm.
External diameter ...... 27·75 cm.
Thickness .... ·047 cm.

Primary coil:—No. of turns $(n_1) = 100$.
Resistance (warm) $= 0·0676$ ohm.

Secondary coil:—No. of turns $(n_2) = 600$.
Resistance (warm) $= 2·034$ ohms.

Before either coil was wound a single turn of well-insulated wire was looped on the core to serve as a search-coil for the determination, by means of the wave-tracer, of the magnetic flux pulsating in the core. The secondary coil was wound next the core.

The primary current was drawn from the alternating side of a small rotary converter that was supplied with direct current from storage-cells. The spindle of the commutator and that of the converter were in line and rapidly connected so that perfectly synchronous commutation was obtained. A chronograph took a continuous record of the period, recording once every 200 alternations.

The arrangement of the transformer and the mutual inductances by means of which the different waves were determined is shown in fig. 1. The primary current C_1 from the converter enters by the two leads marked C_1. In one of these leads is placed a two-pole switch a_1, by which C_1 can be sent through the primary p_1 of the triad T, or deflected through an equal compensating coil b_1, leaving p_1 completely disconnected from the live circuit in case the triad is being used for the determination of C_2.

E_1 is determined by obtaining the trace of the current wave it sends through a non-inductive resistance of 1220 ohms. The mutual inductance m_1 of ·00061 henry is used for obtaining this trace. E_2 is similarly determined, the non-inductive resistance in circuit being 8660 ohms and the mutual inductance m_2 ·003535 henry.

The secondary current C_2 may, by means of the switch a_2, be directed through the primary p_2 of the triad T or through the equal compensating coil b_2. When $n_1C_1+n_2C_2$ is being determined, both C_1 and C_2 flow through their respective primaries p_1, p_2 of the triad in the proper relative directions, the common secondary s being joined as shown to the commutator and thence through a resistance to the galvanometer. When C_1 alone is being determined, C_2 is deflected through its compensating coil b_2; similarly when C_2 alone is being determined, C_1 is deflected through b_1. The mutual inductance of p_1 and s was ·0000485 henry, while that of p_2 and s was ·000291 henry, which bear the same ratio to each other as n_1 to n_2, that is as 1 to 6.

The points marked F in fig. 1 represent the terminals of the single loop of insulated wire wound round and close to the transformer core. When the flux-wave is being obtained these are connected direct to the commutator, and thence through a suitable resistance to the galvanometer.

By means of the key K either the flux leads or those of any of the secondaries of the different mutual inductances can be connected with the commutator so as to obtain the corresponding wave-trace.

A glance at fig. 1 will show that the true primary current is less than the measured C_1 by the small current by which E_1 is determined. To correct for this (if correction is necessary) we subtract from each galvanometer C_1 reading an easily determinable fraction of the equi-phase E_1 galvanometer-reading. A similar correction has to be applied to the C_2 readings; but in this case the corrections have to be added. The galvanometer-readings for $n_1C_1+n_2C_2$ have also to be similarly corrected, and in this case the corrections are of great importance as the C_1 correction is affected by the factor n_1 and the C_2 correction by the factor n_2, and both corrections are approximately in the same phase.

5. For the present paper three separate sets of connected transformer quantities were determined, and they will be given below by their harmonic expressions and also represented by curves or *indicator-diagrams*.

These are for the transformer :—

(1) At no load.
(2) At (q.p.) full non-inductive load.
(3) At (q.p.) full inductive load.

The method by which the wave-tracer galvanometer-readings are reduced to absolute measure and the harmonic expression for the periodic quantity deduced from them has been fully explained in former papers *.

In the present experiments, for each of the periodic quantities determined, 30 wave-tracer galvanometer-readings were taken, each differing from the next in order by 12°. The full wave was thus covered. Corresponding deflexions in each half of the wave were added, that is the 1st and 16th, the 2nd and 17th, and so on, and 15 equispaced *ordinates per*

* "Wave-Tracer and Analyser," Phil. Mag. Nov. 1903. "Variation of Magnetic Hysteresis with Frequency," Phil. Mag. Jan. 1905. "Expeditious Practical Method of Harmonic Analysis," Phil. Mag. Jan. 1906.

half-wave obtained. These were subjected to harmonic analysis, and the amplitudes of the different harmonics were affected by their proper factors to reduce them to flux, current, or E.M.F., as the case might be, in absolute measure.

The results obtained are as follows :—

(1) *For the transformer at no load.*

Period = ·02015 sec.; $\omega = 311{\cdot}8$.

$$\frac{E_1}{10^8} = 57{\cdot}71 \sin(\omega t - 6{\cdot}98) - 4{\cdot}34 \sin 3(\omega t - 15{\cdot}7) + 0{\cdot}35 \sin 5(\omega t - 29).$$

$$C_1 = {\cdot}1304 \sin(\omega t - 44{\cdot}23) + {\cdot}0224 \sin 3(\omega t - 33) + {\cdot}0057 \sin 5(\omega t - 27).$$

$$F = 184700 \sin(\omega t - 97{\cdot}11) + 4300 \sin 3(\omega t - 105{\cdot}9) - 300 \sin 5(\omega t - 84).$$

And as E_2 at no load $= -n_2 \frac{dF}{dt}$, we find that

$$\frac{E_2}{10^8} = 345{\cdot}5 \sin(\omega t - 187{\cdot}11) - 24{\cdot}1 \sin 3(\omega t - 195{\cdot}9) + 2{\cdot}8 \sin 5(\omega t - 210)$$

(The different quantities are in absolute units and the phase angles in degrees.)

(2) *For the transformer at (q.p.) full non-inductive load.*

Period = ·02053 sec.; $\omega = 306$.

The load was a manganin non-inductive resistance of 104·7 ohms.

$$\frac{E_1}{10^8} = 56 \sin(\omega t - 2{\cdot}59) + 4{\cdot}18 \sin 3(\omega t - 51{\cdot}9) + {\cdot}78 \sin 5(\omega t - 61{\cdot}2).$$

$$C_1 = 1{\cdot}945 \sin(\omega t - 5{\cdot}71) + {\cdot}145 \sin 3(\omega t - 50{\cdot}1) + {\cdot}034 \sin 5(\omega t - 33{\cdot}7).$$

$$n_1 C_1 + n_2 C_2 = 12{\cdot}6 \sin(\omega t - 45{\cdot}21) + 2{\cdot}67 \sin 3(\omega t - 33{\cdot}4) + {\cdot}75 \sin 5(\omega t - 30)$$

$$F = 178900 \sin(\omega t - 93{\cdot}22) + 3900 \sin 3(\omega t - 83{\cdot}2) - 400 \sin 5(\omega t - 93).$$

$$C_2 = {\cdot}309 \sin(\omega t - 183{\cdot}16) + {\cdot}022 \sin 3(\omega t - 233{\cdot}5) + {\cdot}005 \sin 5(\omega t - 216).$$

And as $E_2 = R_2 C_2 = 104{\cdot}7 \times 10^9 . C_2$, we find that

$$\frac{E_2}{10^8} = 323{\cdot}5 \sin(\omega t - 183{\cdot}16) + 23 \sin 3(\omega t - 233{\cdot}5) + 5{\cdot}2 \sin 5(\omega t - 216).$$

(3) *For the transformer at (q.p.) full inductive load. Power factor* = ·73.

Period = ·01835 sec.; $\omega = 342{\cdot}4$.

The load was a manganin non-inductive resistance in series with a copper wire inductance-coil. The total external resistance was 76·52 ohms, and the inductance, measured independently, was ·212 henry.

$$\frac{E_1}{10^8} = 54{\cdot}13 \sin(\omega t - 10{\cdot}5) - 5{\cdot}36 \sin 3(\omega t - 12{\cdot}5) + 1{\cdot}37 \sin 5(\omega t - 13{\cdot}2).$$

$$C_1 = 1{\cdot}907 \sin(\omega t - 52{\cdot}1) - {\cdot}065 \sin 3(\omega t - 34{\cdot}7) - {\cdot}017 \sin 5(\omega t - 63{\cdot}1).$$

$$n_1 C_1 + n_2 C_2 = 11{\cdot}18 \sin(\omega t - 48{\cdot}37) + 1{\cdot}82 \sin 3(\omega t - 42{\cdot}14) + {\cdot}31 \sin 5(\omega t - 35).$$

$$F = 154900 \sin(\omega t - 99{\cdot}9) + 5200 \sin 3(\omega t - 101{\cdot}7) + 800 \sin 5(\omega t - 98).$$

$$C_2 = {\cdot}299 \sin(\omega t - 232{\cdot}38) - {\cdot}015 \sin 3(\omega t - 215{\cdot}4) - {\cdot}002 \sin 5(\omega t - 235).$$

$$E_2 = 312{\cdot}9 \sin(\omega t - 189{\cdot}08) - 31{\cdot}8 \sin 3(\omega t - 189{\cdot}5) - 8{\cdot}3 \sin 5(\omega t - 223)$$

6. The same wave-tracer deflexions were individually multiplied by their proper factors to reduce them to absolute measure, and the products plotted as wave ordinates against the corresponding wave-tracer divided-circle readings (*i. e.* against ωt where $\omega = 2\pi/\text{period}$) as abscissæ. Fig. 2 (Pl. X.) represents correctly in amplitude and relative phase the different periodic quantities for the transformer at no load; fig. 3 (Pl. X.) for the transformer at (q.p.) full non-inductive load, and fig. 4 (Pl. X.) for the transformer at (q.p.) full inductive load.

Obviously in figs. 2, 3, and 4 the same periodic quantities are graphically represented as are analytically expressed in series 1, 2, and 3 respectively of the preceding paragraph.

In addition to the sets of related waves, there is, in each of the three diagrams, an area very similar to the well-known hysteresis indicator-diagram. In the present case these closed curves were obtained by plotting the flux as ordinate against the corresponding value of the magnetizing-current turns (i. e., $n_1 C_1 + n_2 C_2$) as abscissa for a complete period.

Thus the area enclosed (A say) is

$$\int^{t+T} (n_1 C_1 + n_2 C_2) dF,$$

where T is the period, and this integral can be shown to be equal to the total core loss per cycle due to both hysteresis and eddy currents as follows:—

Neglecting rc^2 losses, the energy entering the transformer on the primary side in any element of time dt is $eC_1 dt$, where

e is the back E.M.F. due to variation of the flux ; and as

$$e = n_1 \frac{dF}{dt}$$

this energy is equal to

$$n_1 C_1 \frac{dF}{dt} dt.$$

Similarly the energy leaving the transformer on the secondary side in the same element of time dt is equal to

$$-n_2 C_2 \frac{dF}{dt} dt ;$$

hence in the time dt the transformer absorbs energy to the amount

$$(n_1 C_1 + n_2 C_2) \frac{dF}{dt} dt,$$

so that in one cycle, of duration T, the energy absorbed is equal to

$$\int_t^{t+T} (n_1 C_1 + n_2 C_2) \frac{dF}{dt} dt = \text{area A}.$$

7. It is easy to show that when $n_1 C_1 + n_2 C_2$ and F are expressed in the forms

$$n_1 C_1 + n_2 C_2 = m_1 \sin(\omega t - \mu_1) + m_3 \sin 3(\omega t - \mu_3) + m_5 \sin 5(\omega t - \mu_5) + \&c.,$$
$$F = f_1 \sin(\omega t - \phi_1) + f_3 \sin 3(\omega t - \phi_3) + f_5 \sin 5(\omega t - \phi_5) + \&c.,$$

the integral or area A and therefore the total core loss, in ergs, per cycle is equal to

$$\pi\{m_1 f_1 \sin(\phi_1 - \mu_1) + 3 m_3 f_3 \sin 3(\phi_3 - \mu_3) + 5 m_5 f_5 \sin 5(\phi_5 - \mu_5) + \&c.\},$$

which when divided by 10^7T, where T is the period, gives the total power lost in the core in watts.

This form of expression has the advantage of giving separately the power absorbed by the iron by means of the harmonics of different orders, and from the analytic expressions for E.M.F.s and currents we can also obtain for the different orders of harmonics the input, output, and copper losses. Hence we can draw up a debit and credit account for the individual harmonics which will afford a good test of the accuracy of the wave-tracer, as the account for each order should balance.

This has been done for the first and third harmonics of the three series given in this paper and the results shown in the following tables. The quantities for the fifth harmonics are negligible. The figures represent watts.

TABLE I.—No Load.

	1st Harmonic.	3rd Harmonic.	Total.
Output	0	0	0
Iron loss	29·93	–·28	29·65
Copper loss, I.	·06	·002	·06
Copper loss, II.	0	0	0
Sum	29·99	–·28	29·71
Input	29·95	–·30	29·65

TABLE II.—Non-inductive Load.

	1st Harmonic.	3rd Harmonic.	Total.
Output	499·80	2·53	502·33
Iron loss	25·63	·24	25·87
Copper loss, I.	12·79	·07	12·86
Copper loss, II.	9·71	·05	9·76
Sum	547·93	2·89	550·82
Input	543·8	3·02	546·82

TABLE III.—Inductive Load.

	1st Harmonic.	3rd Harmonic.	Total.
Output	340·5	·51	341·01
Iron loss	23·18	·01	23·19
Copper loss, I.	12·29	·01	12·30
Copper loss, II.	9·13	·02	9·15
Sum	385·10	·55	385·65
Input	386·0	·69	386·69

The output in Table III. is the mean value of E_2C_2. A second determination of it, obtained from the mean value of RC_2^2, where R is the external resistance (76·52 ohms), gives 342·86 watts as against 341.

8. Other tests can be applied to the harmonic expressions given in § 5, and though their results may not be so good as those in § 7, yet they will prove fairly satisfactory when the number of independent determinations involved in obtaining each series is considered.

Thus the pressure drop from E_1 to $E_2/6$ is too small in both series 2 and 3, and this is probably due to a common error or errors. Moreover the speed never remained quite constant throughout a complete series. Again in series 3, knowing the external secondary resistance, and assuming as correct the value given for the external inductance (·212 henry), C_2 can be calculated from E_2. When this is done we find that

$$C_2 = \cdot 2966 \sin(\omega t - 232 \cdot 58) - \cdot 014 \sin 3(\omega t - 213) - \cdot 002 \sin 5(\omega t - 239),$$

which agrees well with the value of C_2 obtained with the wave-tracer.

Conversely from E_2 and C_2 obtained with the wave-tracer, both the external resistance R and the external inductance L can be calculated, or if R is known two determinations of L can be obtained from each order of harmonic.

If, however, as in the present case $\omega L/R$ is considerable, the first order is the only one suitable for this purpose; for if ψ_1, ψ_3, and ψ_5 be the difference of phase of the first, third, and fifth harmonics respectively of E_2 and C_2, then, since

$$\tan\psi_1 = \frac{\omega L}{R}, \qquad \tan\psi_3 = \frac{3\omega L}{R}, \qquad \tan\psi_5 = \frac{5\omega L}{R},$$

and $\omega L/R$ is large, a small error in the observed values of ψ_3 or ψ_5 will cause a large error in the value of L deduced from them.

For the first harmonics, as $\psi_1 = 43^\circ{\cdot}3$, $R = 76 \cdot 52$, $\omega = 342 \cdot 4$, we find from

$$\tan\psi_1 = \frac{\omega L}{R}$$

that $L = \cdot 2106$ henry; and from

$$C = \frac{E}{\sqrt{R^2 + \omega^2 L^2}}$$

that $L = \cdot 2084$ henry.

These give a mean value for L of ·2095 henry, which is as likely to be correct as the value ·212 already given and which was obtained by the Wheatstone's bridge method. As ω or $2\pi/T$ was determined by the chronograph attached to the wave-tracer, the above constitutes a new method of measuring an inductance in terms of a resistance and a time.

9. In Table I. § 7, it will be noticed that the iron loss for the 3rd harmonic is negative (−·28 watt). This means that all the watts put down as iron loss for the 1st harmonic

are not dissipated as heat, but that some of them are transformed by the iron to 3rd harmonic power; a portion of these are dissipated as heat, while the remainder ·28 watt are given out as electrical power by the iron to the copper circuits. This phenomenon has already been drawn attention to in a former paper * by the author.

10. In the diagrams, figs. 2, 3, and 4 (Pl. X.), it is worth drawing attention to the approximate permanence of form of the magnetizing current wave, and to the permanence of the relative phase relations of E_1, $n_1C_1+n_2C_2$, and F, for the three essentially different conditions of working for which the diagrams were obtained.

Discussion.

Mr. W. Duddell said the paper was an interesting and important one, and congratulated the author upon the ingenious double mutual induction method by which the vector sum $(n_1C_1+n_2C_2)$ was determined electrically. In the paper the core loss was nearly constant for no load and full load, but the author had used an E.M.F. wave which was practically a sine curve. He suggested that experiments should be carried out using distorted E.M.F. waves such as occur in many generators and transformers.

Mr. A. Campbell remarked in respect to the title of the paper that it seemed a wanton confusion of nomenclature to use in this connexion the term "indicator diagram" which had such a definite and accepted meaning in mechanics; we might as well talk of hysteresis loops as "iron indicator diagrams." Prof. Lyle's methods were interesting, particularly that for obtaining the curve of effective ampere turns $(n_1C_1+n_2C_2)$ by the use of a double mutual inductance which performed the vector addition. The author had wisely kept the mutual inductance small: without that condition the double air-core transformer might alter C_1 and C_2. It would probably be better to split it up into two separate pairs of coils. The setting of the ratio of the mutual inductances on the triple coil could be more easily done by Maxwell's null

* "Variation of Magnetic Hysteresis with Frequency," Phil. Mag. Jan. 1905.

method, which was perfectly applicable to a pair of secondaries having a common primary. If various values of n_2/n_1 had to be dealt with, the two primary coils might be fixed at right angles and the secondary mounted so as to be capable of rotation with regard to the primaries in order to allow of variation in the ratio M_2/M_1. In connexion with Prof. Lyle's methods of determining inductances by the help of the wave-tracer, the somewhat similar method of Dr. E. B. Rosa might be mentioned. It was interesting to see the actual hysteresis loops given; a comparison of these with ballistic tests on the same transformer would add valuable information.

Mr. A. Russell said that the author's results proved that to a first approximation the magnetizing force acting on the core followed the same law and had the same amplitude at all loads. This theorem was the starting point in the ordinary engineering theory of the transformer, but this was the first time that a careful experimental proof for a particular case had been given. The magnetic leakage in the transformer experimented on must have been very small. The formula $\int_t^{t+T}(n_1C_1+n_2C_2)\frac{dF}{dt}dt$ only gave the energy absorbed per cycle when the magnetic leakage was negligibly small. In many commercial transformers it would not be safe to use this formula. The core-loss diagrams were interesting, but their value would have been greater if the transformer had been so constructed that the magnetic flux density was approximately uniform over the cross section of the core. The internal diameter of the centre-hole iron stampings employed was only about half the external diameter, hence the magnetic force to which the iron was subjected was almost twice as great at the inner as at the outer circumference. As the values of the permeability of the iron at these forces might vary widely, we could not tell the distribution of the magnetic flux. This prevented us from making calculations as to the hysteresis and eddy-current losses in the core. The complete solution of the transformer problem could not be obtained until we knew more about hysteresis. Apparently there were molecular "frequency changers" in the core, and any theory that would elucidate their action would be of great value to electricians. In this connexion he referred to a paper read by Prof. J. Perry in 1892.