Royal United Services Institution. Journal

ISSN: 0035-9289 (Print) (Online) Journal homepage: http://www.tandfonline.com/loi/rusi19

The New Science of Dual Arithmetic Applied to Naval and Military Calculations

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To cite this article: Oliver Byrne Esq. (1866) The New Science of Dual Arithmetic Applied to Naval and Military Calculations, Royal United Services Institution. Journal, 10:38, 37-63, DOI: 10.1080/03071846609417196

To link to this article: http://dx.doi.org/10.1080/03071846609417196

Published online: 11 Sep 2009.

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LECTURE.

Friday, January 19th, 1866.

CAPTAIN E. GARDINER FISHBOURNE, R.N., C.B., in the Chair.

THE NEW SCIENCE OF DUAL ARITHMETIC APPLIED TO NAVAL AND MILITARY CALCULATIONS.

By OLIVER BYRNE, Esq., formerly Professor of Mathematics, Civil Engineers' College, Putney.

GENTLEMEN,—A scientific lecture could only interest or entertain a highly enlightened and experienced audience like the greater portion of the one that I have the honour to address, either by bringing to their minds subjects long forgotten, or by offering materials for thought upon matters engaging their immediate attention; and but for the newness and importance of my subject, it would be my first duty to apologise for entering upon the principia of this or any scientific subject, however profound. Then, without further introduction, allow me to say that

Dual arithmetic is a new art of manœuvring numbers and investigating the relations of quantities with ease and accuracy, with or without the use of tables. In the most general sense dual arithmetic is a new art, and not merely a new method of obtaining results that might be found by arts previously known. The science of dual arithmetic unfolds the capabilities of numbers in an original manner, extends the boundaries of mathematical science, and establishes new rules, by which many difficult problems of the greatest utility and importance are solved with ease, without the aid of tables, cumbersome formulæ, or methods of approximation previously resorted to.

The term *dual* is employed because the art has *two branches*, the bases of each branch being composed of *two parts*, but, chiefly because the digits of a dual number may be subjected to a variety of changes in magnitude and position, while at the same time constantly equal in value to *two unchangeable* extremes, namely, a natural number and a logarithm to a known base.

In combination the system is threefold:—

Natural Number | Dual Number | Dual Logarithm.

Since the digits of a dual number are susceptible of a vast variety of changes without altering its two ultimate values, dual numbers may be said to be changeable without being variable.

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Numbers in the dual system of arithmetic are expressed by the continued product of the powers of one or more of the bases which are seldom introduced into the figurative operations of the art. However, the powers and products are always obtainable by common addition and subtraction.

↓ *Bases of the ascending branch.* ↓

$$+\infty \ldots.(10000+1);\ (1000+1);\ (100+1)\ (10+1);\ (1+1);\ (\tfrac{1}{10}+1);\ (\tfrac{1}{100}+1);\ (\tfrac{1}{1000}+1);\ \ldots\ldots\ 1;$$

more conveniently written

$$+\infty \ldots.\ 10001;\ 1001;\ 101;\ 11;\ 2;\ 1{\cdot}1;\ 1{\cdot}01;\ 1{\cdot}001;\ 1{\cdot}0001;\ldots.\ 1.$$

increasing in magnitude from right to left.

These bases are less and less as they approach 1, but cannot be less than 1.

↑ *Bases of the descending branch.* ↑

$$-\infty \ldots.\ (1-10000);\ 1-1000);\ (1-100);\ (1-10);\ (1-1);\ (1-\tfrac{1}{10});\ (1-\tfrac{1}{100});\ (1-\tfrac{1}{1000});\ \ldots\ldots\ 1;$$

but more correctly written

$$-\infty \ldots.\ -9999;\ -999;\ -99;\ -9;\ 0;\ {\cdot}9;\ {\cdot}99;\ {\cdot}999..1.$$

This scale of bases approaches 1, but cannot be greater than 1.

A single example will make clear anything that may seem too abstract in these general statements.

In order to avoid the common but faulty practice of illustrating by small pet numbers, let us take the supposed diameter of the earth through the poles, which is said to be 7898·8809 statute miles of 5280· feet each; therefore, the diameter = 41706091·152 feet, which, according to usage, is a contracted method of expressing

$$4\times(10)^7+1\times(10)^6+7\times(10)^5+6\times(10)^3+9\times(10)^1+1+(10)^{-1}+5(10)^{-2}+2(10)^{-3}.$$

In common arithmetic the coefficients 4, 1, 7, &c., are termed digits, and do not exceed 9. In dual arithmetic the powers of the bases are only registered; they are also called digits, but they may vastly exceed 9. Thus 41706091·152 is equal to

$$(1-{\cdot}1^5)^3(1-{\cdot}1^6)^2(1-{\cdot}1^7)^5(1-{\cdot}1^8)^8(1-{\cdot}1^9)^9(1-{\cdot}1^{10})^1(1+1)^2(1+{\cdot}1^2)^4(1+{\cdot}1^3)^2$$

when multiplied by 10000000.

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The bases being omitted, this dual number is written

$$
\begin{array}{c}
41706091{\cdot}152 \\
\| \\
'0'0'0'0'3'2'5'8'9'1 \ {}_{7}\!\updownarrow^{2}\ 0,4,2,0,0,0,0,0,0,0, \\
= \\
'3'2'5'8'9'1 \ \overset{2}{\underset{5}{{}_{7}\!\updownarrow^{2}}}\ 4,2, \\
= \\
{}_{7}\!\updownarrow^{2}\ 0,4,1,9,6,6,9,5,6,4, \\
= \\
'8'3'1'4'8'7'3'4'3'4 \ \uparrow_{8}
\end{array}
$$

and equal to thousands of other dual numbers, each expressed by a different set of dual digits, but every one reducible to the same dual logarithm 1754615775,.

In an extended form 41706091·152 is equal to

$'0'0'0'0'3'2'5'8'9'1 \uparrow (10)^{7}(2)^{2} \downarrow 0,4,2,0,0,0,0,0,0,0,$ more conveniently written $'3'2'5'8'9'1 \ \overset{2}{\underset{5}{{}_{7}\!\updownarrow^{2}}}\ 4,2,$

'3'2'5 &c., are called dual digits, and express the powers of the bases involved. The positions are reckoned from left to right before ↑ representing the descending branch, and from left to right after ↓ representing the ascending branch. The powers of 1·1 occupy the first position after ↓; the powers 1·01, the second position after ↓, and so on. The powers of ·9 the first position before ↑; the powers of ·99 occupies the second position before ↑; and so on. According to this conventional arrangement, '3 in the fifth position before ↑ represents $(\cdot 99999)^{3}$, 0, in the first position after ↓ indicates that no power 1·1 is involved, and in general ciphers are employed to indicate that no power of the corresponding bases are involved.

The position of a dual digit before, between, or after the signs ↑ and ↓ points out its value.

$$
\underset{p}{n\uparrow m} \qquad \overset{p}{m\downarrow n}
$$

A small figure placed at p designates the position occupied by a dual digit, and sometimes points out the leading position occupied by the first of more dual digits than one. m expresses 10^{m} and n expresses 2^{n}.

$$
\overset{p}{\underset{p}{m\updownarrow n}}
$$

The comma (,) is employed in the operations of dual arithmetic, while the period (·) is retained to separate whole numbers from decimal fractions.

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General Forms of Dual Numbers of Eight Consecutive Digits.

Ascending branch.

$$ {}^{m}\!\downarrow^{n} \; u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8, $$

Descending branch.

$$,v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8 \; {}^{n}\!\uparrow^{m} $$

Both branches combined.

$$,v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8 \; {}^{n}\!\uparrow^{m} \; u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8, $$

Nine dual digits give results true to nine places of figures. Ten dual digits give results true to ten places of figures, and so on.

A dual number is easily transformed into another, all of whose digits being reduced to ciphers, except the last. The transformation of a dual number of eight digits into another, whose first seven digits are ciphers, is termed *reducing a dual number to the eight position.*

A dual number reduced to the eighth position is called a dual logarithm.

For example—

$$ 2. = \downarrow 7,2,6,0,7,8,2,6, = \downarrow 0,0,0,0,0,0,0, 69314718, $$
$$ = \downarrow^{8} 69314718. $$

In practice the 8 is omitted, and the expression is written

$$ 2. = \downarrow 69314718, \text{ which represents } (1{\cdot}00000001)^{69314718}. $$

Then 69314718, is termed the dual logarithm of 2· and written

$$ \downarrow,(2\cdot) = 69314718, $$

The dual logarithm of 41706091·152 is\equal to the whole number 1754615775, Dual logarithms are always whole numbers.

By very simple means the operator can find any two of the three corresponding numbers:—

Natural number. | Dual number. | Dual logarithm.

The remaining one being given. Indeed in all cases these reductions may be made by common addition and subtraction.

Then, taking for granted that we can make such reductions when required, I now propose to illustrate the power of this new art by applying it to solve some important practical problems, the solution of which would be either impossible, or so laborious, that no one would attempt to perform the operations by arts previously known.

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Examples.

Ex. 1. Find the 7th root of the cube of 41706091·152.

$$\downarrow,(41706901{\cdot}152)$$
$$\|$$
$$'3'2'5'9 \;{}_{7}\!\updownarrow^{2}_{5,}\; 0,4,2, = 1754615643,$$

$$\frac{1754615775,}{7} \times 3 = 7551978189, \quad = 3\downarrow 6,4,0,3,6,4,2,1,$$

$$3 \downarrow 6,4,0,3,6,4,2,1, = 1844{\cdot}165.$$

Ex. 2. Required the common logarithm of 41706091·152 by a direct calculation, or solve the equation

$$10^r = 41706091{\cdot}152,$$

$$\downarrow, (41706091{\cdot}152) = '3'2'5'9 \;{}_{7}\!\updownarrow^{\overset{2}{2}}_{5,}\; 4,2, = 1754615775,$$
$$\downarrow, (10) = {}_{2}\!\downarrow,2,3,2,6,7,3,2,0, = 230258509,$$

Then by common division

$$\frac{1754615775}{230258509} = 7{\cdot}62019949.$$

Ex. 3. Required the common number answering to the common logarithm 7·62019949 by a direct calculation,

$$7{\cdot}62019949 \times 230258509 = 1754615775,$$
$$1754615775, = {}_{7}\!\downarrow^{5}, 0,4,1,9,6,6,9,1, = \downarrow,(41706091).$$
$$\therefore 7{\cdot}62019949 = \text{the common logarithm of } 41706091.$$

Ex. 4. Given $34{\cdot}56789x^5 - 2345{\cdot}678x^4 - 123{\cdot}4567x^3 + 456{\cdot}7891x^2 + 56789{\cdot}12x = - 415978976{\cdot}065$ to find a value of x, true to nine places of decimals.

In my work on the "Art and Science of Dual Arithmetic," it is shown that if $r\downarrow u_1,u_2,u_3, \ldots$ be a root of the equation

$$ax^5 + bx^4 + cx^3 + dx^2 + ex = f,$$

then u_1,u_2,u_3 may be formed from

$$u_1 = \frac{f - f_1}{\dagger\, 5ar^5 \dagger\, 4br^4 \dagger\, 3cr^3 \dagger\, 2dr^2 \dagger\, er}.$$

It requires but little observation to see that a value of x lies between 0 and 100, and on a closer inspection it will be found that a value lies between 10 and 30. x may be found conveniently by putting any number from 22 to 12 for r; 20 is selected, because its square, cube, &c., are easily obtained and operated with.

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$ar_5 = a_1$ $\quad br^4 = b_1$ $\quad cr^3 = c_1$ $\quad dr^2 = d_1$ $\quad er = e_1$

+ 110617248· − 375308480· − 987653·6 + 182715·64 + 1135782·4

+ 550	5 times	+ 110........	
− 1500	4 times	− 373........	
− 950	3 times	−	
	2 times	+	
	1 time	+ 1........	
		− 264........	(f_1) take
		− 415........	(f) from

−950) − 151 (+ ↓ 1, $= \downarrow u_1$,

$a_1\downarrow 5u_1, = a_2$ $\quad b_1,\downarrow 4u_1, = b_2$ $\quad c_1\downarrow 3u_1, = c_2$ $\quad d_1\downarrow 2u_1, = d_2$

+ 178150185· − 549489146· − 1314566·942 + 221085·924

$e_1\downarrow u_1, = e_2$

+ 1249360·64

+ 8905	5 times	1781........	
− 21976	4 times	5494........	
− 39	3 times	13........	
+ 4	2 times	2........	
+ 12	1 time	12........	
− 1309)		− 3712........	(f_2) take
		− 4159........	(f) from
		− 447........	(+ ↓ 0,3, $= u_2$,

$a_2\downarrow 5u_2, = a_3$ $\quad b_2\downarrow 4u_2, = b_3$ $\quad c_2\downarrow 3u_2, = c_3$ $\quad d_2\downarrow 2u_2, = d_3$ $\quad e_2\downarrow u_2, = e_3$

+ 206826835· − 619178123· − 1437722·51 + 234687·163 + 1287217·52

+ 103410	5 times	+ 20682........	
− 247668	4 times	− 61917........	
− 429	3 times	− 143........	
+ 46	2 times	+ 23........	
+ 128	1 time	+ 128........	
144513)		− 41227........	(f_3) take
		− 41597........	(f) from
		− 370	(+ ↓ 0,0,2,3, = + ↓ u_3, u_4,
		− 289	
		81	

The root being thus far determined by contracted operations, let 20 ↓ 1,3,2,5, be substituted for x in the given equation.

The succeeding operation by using the coefficients of the given

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equation is independent of those before employed to determine $20\downarrow 1,3,2,5,$. What follows not only determines x to the required degree of accuracy, but also corrects errors, if any be committed.

$$
\begin{array}{rcrcl}
\downarrow,1,3,2,5,0,0,0,0, & = & 12766017, & & \\
\text{square} & = & 25532034, & = & \downarrow,2,6,5,0,0,0,5,0, \\
\text{cube} & = & 38298051, & = & \downarrow,4,0,1,7,4,0,2,9, \\
\text{4th} & = & 51064068, & = & \downarrow,5,3,4,2,4,0,7,9, \\
\text{5th} & = & 63830085, & = & '0'5'4'5'9'2'6'3'|2
\end{array}
$$

$$
\begin{array}{llll}
'0'5'4'5'9'2'6'3|2r^5 & r^4\downarrow 5,3,4,2,4,0,7,9, & r^3\downarrow 4,0,1,7,4,0,2,9, & r^2\downarrow 2'6'5'0'0'0'5'0, \\
+\ \ 34\cdot56789 & -\ \ 2345\cdot678 & -\ \ 123\cdot4567 & +\ \ 456\cdot7891 \\
+\ 209427346\cdot & -\ 625398452\cdot & -\ 1448541\cdot55 & +\ 235863\cdot66
\end{array}
$$

$$
\begin{array}{l}
r\downarrow 1,3,2,5,0,0,0,0, \\
+\ \ 56789\cdot12 \\
+\ 1290438\cdot27
\end{array}
$$

$$
\begin{array}{rlrl}
+\ 107413 & \text{5 times} & +\ 209427346\cdot & \\
-\ 250159 & \text{4 times} & -\ 625398452\cdot & \\
-\ \ 434 & \text{3 times} & -\ \ 1448541\cdot55 & \\
+\ \ 47 & \text{2 times} & +\ \ 235863\cdot66 & \\
+\ \ 129 & \text{1 time} & +\ \ 1290438\cdot2_7 & \\
\hline
-\ 145704) & & -4158|93344\cdot62 & (f_4)\ \text{take} \\
\ldots\dagger & & -4159|78976\cdot065 & (f)\ \text{from} \\
\hline
 & & -\ |85631\cdot & (+\downarrow^5 5,8,7,7, = \downarrow^5 u_5,u_6,u_7,u_8, \\
 & & 72852 & \\
\hline
 & & 12779 & \\
 & & 11656 & \\
\hline
 & & 1123 & \\
 & & 1020 & \\
\hline
 & & 103 &
\end{array}
$$

$$\therefore\ x = 20\downarrow 1,3,2,5,5,8,7,6, = 22\cdot7246716$$

Common number. **Dual number.** **Dual logarithm.**

$$
x = 22\cdot7246716 \left\{
\begin{array}{l}
22\downarrow 0,3,2,5,5,8,7,5, \\
20\downarrow 1,3,2,5,5,8,7,6, \\
15\downarrow 4,3,4,3,1,1,3,8, \\
12\downarrow 6,6,6,9,8,4,4,7,
\end{array}
\right\} = 312345121,
$$

&c.

Ex. 5. Find the three sides of a right angled triangle that will have one of its acute angles equal $21^\circ\ 19'\ 37''\cdot8$.

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The length of an arc of 21° 19′ 37″·8 = ·37222928 radius = 1.
In solving questions similar to this, the operative numbers

	1	3	6	10	15	21	28	36	..	..
and	2	6	12	20	30	42	56	72	..	..

will be required, as well as the use of the following ~~rules~~. (See "The Young Dual Arithmetician," pp. 116, 117.)

Rule I.

Set down 10000· times the length of the arc, that measures the least of the acute angles, and divide it by the square root of 2. The quotient will be the dual logarithm of the hypothenuse.

Rule II.

Let h represent the dual logarithm of the hypothenuse, and from the square of h take h; then $h^2 - 3h + 2$; $h^2 - 5h + 6$; $h^2 - 7h + 12$; &c., are found by merely subtracting $2h$ and adding at each step a term of the series 2 6 12 20 . . . .

In what follows $[h^2 - h]$ is put for $h^2 - h$, divided by 10^8;
$[h^2 - 3h + 2]$ is put for $h^2 - 3h + 2$, divided by 10^8, and so on.

Put A = the length of the arc to radius 1, and $B = [h^2 - h]$; $C = [h^2 - 3h + 2]$; $D = \frac{B}{6}[h^2 - 5h + 6]$; $E = \frac{C}{10}[h^2 - 7h + 12]$; $F = \frac{D}{15}[h^2 - 9h + 20]$; &c.

Then A − C + E − G + I − &c., gives the base, and
I − B + D − F + H − &c., gives the perpendicular.

These rules are demonstrated in the author's work on the science of dual arithmetic applied to trigonometry.

↓,(·37222928)
‖
'3'|½|,0,2,1,0,8,3,3,0 = '98824528

$\frac{↓,(2.)}{2}$ = '34657359

'133481887 = ↓,(·2632058)
2

'266963774 = ↓,(·06927732)

∴ ↓,(h) = 2632·058   ↓,(h^2) = 6927732.

But ↓,(1·00002632) = 2632,

∴ Hypothenuse = 1·00002632

$$
\begin{array}{rl}
h^2 = & 6927732\cdot \\
h = & 2632\cdot \\
\hline
& 6925100\cdot = h^2 - h \\
& 5264\cdot \\
\hline
& 6919836\cdot = h^2 - 3h \\
& 5264\cdot \\
\hline
& 6914572\cdot = h^2 - 5h \\
& \&c. \qquad \&c.
\end{array}
$$

$$
\begin{array}{rcl}
\text{Radius} & = & 1\cdot00000000 + \\
A & = & \cdot37222928 + \\
B & = & \cdot06925100 - [h^2 - h] \\
C & = & 898589 - [h^2 - 3h + 2]\frac{A}{3} \\
D & = & 79808 + [h^2 - 5h + 6]\frac{B}{6} \\
E & = & 5930 + [h^2 - 7h + 12]\frac{C}{10} \\
F & = & 876 - [h^2 - 9h + 20]\frac{D}{15} \\
G & = & 19 - [h^2 - 11h + 30]\frac{E}{21} \\
H & = & 1 + [h^2 - 13h + 42]\frac{E}{28}
\end{array}
$$

Perpendicular.			Base		
A	37222928		Radius	100000000 +	
C	$\bar{1}141411$	ar. co.	B	$\bar{1}3074900$	ar. co.
E	5930		D	79808 +	
G	$\bar{1}81$	ar. co.	F	1623	ar. co.
			H	1 +	
	·36370250			·93154342	

$\therefore$ Natural sine of $21^\circ\ 19'\ 37''\cdot8 = \dfrac{\cdot36370250}{1\cdot00002632} = \cdot36369293$

Ex. 6. Required the log. sine, log. cosine, log. tangent, log. cotangent, log. secant and log. cosecant of

$$21^\circ\ 19'\ 37''\cdot8$$

↓,(·36369293) = '3'0'2'2'|½↓,⁵1,0,7,0, = '101141904
↓,(·93154342) = '1'0'0'4'0'0'5'3↓,0,3,5, = '7091256

	'101141904	'101141904
h	'2632,	7091256,
	'101144536 = log. sine	'94050648 = log. tangent.

'7091256
h '2632

'7093888 = log. cosine 94050648, = log. cotangent.

101144536, = log. cosecant. 7093888, = log. secant.

Ex. 7. Given the equatorial semidiameter of the earth = unity, and the polar semidiameter $= r = \frac{304}{305}$. Required l, the latitude of the central zenith, $l' = 32° \ 29' \ 46''$, the latitude of the apparent zenith, or the spheroidal or geographical latitude being given.

Required also the radius of the place of observation and its logarithm.

Put ρ = radius of place of observation.

RULE.

Take $\frac{1}{3}$ of twice the sine of double either latitude, multiply by 1000· and the dual number ↓ 0,1,6,0,1,0,2,0, the result gives the seconds to be added or substracted.

$$\rho = \sqrt{\frac{\cos l'}{\cos l \cos (l' - l)}}$$

$$\therefore \downarrow,(\rho) = \tfrac{1}{2} \left\{ \downarrow, \frac{\cos l'}{\cos l} - \downarrow,(\cos (l' - l) \right\}$$

Sin 2 l' = ·906 2 50 4
2
3)1·81 2 5 00 8
60|4·1|66|9 ↓ 0,1,6,0,1,0,2,0,
|6·0|41|7
61 0|2 08|6
3|6 61|2

$l' - l = 10'\ 14 = 614''$ ∴ $l = 32° \ 19' \ 32''$

cos $(l' - l)$ = ·99999560
∴ ↓, (cos $(l' - l)$) = '440

cos l = ·845 0 2 3 50
cos l' = ·843|6 2 8|00
1|6 8 6|86
|84
845 1|1 5 70 '0'0'0'1'0'9'0'9 ↓ 0,0,2,0,0,0,0,0,
9|2 20 = 188991,
8|4 51
7|69
7|61
8|
8|

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$$\begin{array}{r} '188991 \\ 440, \\ \hline 2)'188551 \\ \hline \downarrow'94276 \end{array}$$

$$\tfrac{1}{2}\left[\frac{\cos l'}{\cos l} - \downarrow, \cos (l' - l)\right] = \downarrow'94276 = \downarrow, (\rho)$$

$$\frac{94276\cdot}{230258509} = \cdot 0004095$$

Ar. co. $= 9\cdot9995905 =$ the common log of ρ.

$$\downarrow, (\rho) = '94276 = '1 \overset{5}{\underset{3}{\downarrow}}, 5,7,7,4, = \downarrow, (\cdot99905768)$$

$$\therefore \rho = \cdot99905768.$$

Ex. 8. Given the apparent altitude of the moon's centre 8° 26′ 13″ (a) the true altitude 9° 20′ 45″ (A), the apparent altitude of a star 35° 40′ (a_1), the true altitude 35° 38′ 49″ (A_1), and the apparent distance 31° 13′ 26″ (d); required the true distance (D), so as to find the longitude at sea.

It is well known that

$$\text{Cos D} = [\cos d + \cos (a + a_1)] \frac{\cos A \cos A_1}{\cos a \ \cos a_1} - \cos (A + A_1)$$

$$\begin{array}{lll} \cos d = \cdot 8551482 & \cos A = \cdot 9867261 & \cos A_1 = \cdot 8126236 \\ \cos (a + a_1) = \cdot 7180824 & \cos a = \cdot 9891779 & \cos a_1 = \cdot 8124229 \\ \hline 1\cdot 5732306 & & \end{array}$$

$$\cos (A + A_1) = \cdot 7071959$$

$$\cos D = [1\cdot5732306] \frac{\cdot9867261}{\cdot9891779} \times \frac{\cdot8126236}{\cdot8124229} - \cdot7071959$$

·989 1 7 7 9 0
·986|7 2 6|1 0 ↓ 0,0,2,4,8,2,7,2,
1|9 7 3|4 5
|9 9
·988 7|0 0 5 4
3|9 5 4 8
6
989 0 9|6 0 8
8|1 8 2
7|9 1 3
2|6 9
1|9 8
7|1
6|9
2|

Logarithm
248172,

·8126 2 3 60
8124|2 2 90 ↓ 0,0,0,2,4,7,0,2,
1|6 2 48
1
8125 8|5 39
† 5|8 21
3|2 50
5|71
5|69
2|

Logarithm
24702,

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```
   24702,
  '248172
 ________
 '223470 = '0'0'2'2'3'3'7'0 | [1·5732306] = 1·5697188

   1·5697188
    ·7071959
   _________
    ·8625229 = cos 30° 23′ 56″·3, the true distance D.
```

This number, 41706091·152 Ex. 1 is generally written, scientifically speaking, in the form—

$$4 \times (10)^7 + 1 \times (10)^6 + 7 \times (10)^5 + 6 \times (10)^3 + 9 \times (10)^1 + 1 + 10^{-1} + 5 \times (10)^{-2} + 2 \times (10)^{-3}.$$

I never question the conventional contraction of that great number to 41706091·152. Observe, I omit the fourth and also the second power of (10). In all cases with common arithmetic the power of the base, and also the base, are unrecorded. We merely set down the coefficient, its position before or after the decimal point marks its value. It is readily shown that the dual system of notation is not as compound as the common system.

This system possesses a series of bases, although not employed more frequently than in the old system. I only employ my bases necessary to show what they are.

I will begin with $+ \infty$ and go on to $(10{,}000 + 1)$: $(1{,}000 + 1)$: $(100 + 1)$: $(10 + 1)$: $(1 + 1)$. One of these numbers decreased one-tenth; the other remaining stationary, which is 1 — . Proceeding to $(\frac{1}{10} + 1)$, and so on, until it approaches unity, but never becomes so small. These are more conveniently written by the second line.

$$+ \infty \; \ldots . \; 10001; \; 1001; \; 101; \; 11; \; 2; \; 1·1$$

Observe 2 is one of my bases . . . 1 + 1. It would not appear to be so at first view.

This is called the ascending branch, the bases of the ascending branch being positive, and the ↓ pointing towards a greater number. A comma , and an arrow ↓ are the only two signs I employ to do all my work. In this place I do not use any other innovation in mathematics.

The other base commences with $- \infty$; and goes on $(1 - 10000)$; $(1 - 1000)$; $(1 - 100)$; $(1 - 10)$; $(1 - 1)$; $(1 - \frac{1}{10})$; $(1 - \frac{1}{100})$; $(1 - \frac{1}{1000})$; . . . . 1.

Observe, 0 is a base of my system, because the bases may be written thus :—

$$- \infty \; \ldots . \; - 9999; \; - 999; \; - 99; \; - 9; \; 0; \; ·9; \; ·99; \; ·999; \; .. \; 1$$

It may approach 1, but never arrive to be as great. The old base,

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10, has no such power of introducing 0 between positive and negative numbers. Therefore is deficient in range.

The arrow points up ↑ in the descending branch, downwards in the ascending ↓. The powers of 10 and 2 are placed at the centre of the shaft. n is the power of 10, while m is the power of 2. p stands for position.

When ten digits are used the results are true to ten places of figures; when twenty digits are used the results are true to twenty places of figures; when one hundred digits are used the results are three hundred places of figures. The accuracy can be carried as far as you please.

With reference to Example 1, page 38, that is no power of 11; that is in the first position. There is the fourth power of 101, and the square of 1001 in the third position.

The first base. We will say we count from 1 + 1. There is 1 + 10, first, second, and third, and so on. Powers of 1·1 in the first position, powers of 1·01 in the second position, powers of 1·001 in the third position.

So that the position of the digit shows the base that is raised to the power, and the powers are only employed in these cases, not the coefficients as in a common number. In a common number they use the coefficient and not the power of the base. In my system I use the indice only. It is the continued product of the powers of the base that forms the dual number, and not the sum. The dual number may assume a thousand different forms, but it has two ultimate values, the dual logarithm on one side, and the natural number on the other:—

Natural number (dual number) dual logarithm. These two extreme values never change. What I have termed a dual logarithm is a logarithm that is ten times more accurate than one taken from a table of seven figure logarithms. I have, therefore, used eight positions.

This is another form of the same number:—'8'3'1'4'8'7'3'4'3'4 ↑ 8. (See Ex. p. 39).

That is the eighth power of 10. Before the above arrow the counting begins, and after this the digits, '8'3'1'4, &c.

The eighth power of 9, the cube of 99, the first power of 999, the fourth power of 9999, and so on.

These are the descending numbers. Yet very different from the dual number previously mentioned, yet amounting to the same common number, and to the same dual logarithm. The following is another form of the same dual number, all positive, all on the right of the arrow (See Ex. p. 39):—

1↓2 0,4,1,9,6,6,9,5,6,4,

The seventh power of 10, the square of 2, no power of 11, the fourth power of 1·01, the first power of 1·001, so the position of the digits tells the power of the base. It must be again observed, I never use the bases in figurative operations.

This number is called a deal number because the base is composed of two parts; it has two ultimete values that never change. But as a whole it is threefold, because the dual number is flexible, capable of an infinite variety of forms, and has two values.

If logarithms have to be used, a boy can make them himself. And he can do more than that; he can tell whether Mr. Babbage's logarithms are right or wrong, or Mr. Taylor's, or Mr. Brigg's. I have supplied large tables, which can be looked at when the lecture is over, of dual logarithms for the use of business men, sailors, and those who have to make calculations, but who do not wish to make such calculations for themselves. But there is this difference between the ordinary tables, that any man can tell whether they are right or whether they are wrong. He can check them in fifteen minutes, and can tell whether a dual logarithm, or a dual number, or a sine, cosine, or secant, or anything else, is right or wrong, and in order.

But if this system did nothing more than calculate logarithms, it would be a very useless thing, indeed.

I want to cube that number, 41706091·152, and I want to take the seventh root of the cube. I at once find the dual logarithm of that number; first, the dual number, then the dual logarithm. I multiply the dual logarithm by 3, and divide it by 7, and reduce it back again to its dual number, and then back again to the common number. So that I can take the cube of a number, the seventh root of that, or raise it tc any power.

My first example in this synopsis is, how to extract the seventh root of a number, the first problem on page 41.

Find the seventh root of the cube of 41706091·152.

$$\downarrow,(41706091{\cdot}12) = 1754615643,$$

$$\frac{1754615643, \times 7}{3} = 7551978189,$$

$$= \downarrow, (1844,165) \text{ the root required.}$$

I require to calculate in a direct way the logarithm of that number, being the second example, or to solve the equation $10^x = 41706091{\cdot}152$ (see Ex. 2, page 41).

Now, in this solution, I do not beg the question. I do not get any number before or behind; I reduce it to a dual number at once, and then to a dual logarithm, I divide that by the dual logarithm of 10, which I also find. The quotient is the common logarithm, by common division.

So that you can take up any number you like, calculate the logarithm from the number itself. This problem was impossible before I discovered dual arithmetic; with the formula generally given for the purpose, it was necessary to dodge from 1 up, and then, by a series of differences, interpolate, either by machinery or other means, the intermediate numbers.

In the third problem, I wish to find the number corresponding to the logarithm. There is no formula to do it, and no one has ever attempted to give one that I am acquainted with. It is very easily done by the dual method, because we have nothing to do but to take the common

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logarithm, and multiply it by the dual logarithm of 10, which gives the dual number; bring it to a common number, and the required value is found.

We can test any common logarithm at once, and see whether the logarithm be correct or not. Even the first number given in Baron Von Vega's system of logarithms, the logarithm of the mean distance of the earth from the sun is wrong. For that alone, dual arithmetic requires some little commendation.

In the higher branches of mathematics, its value is shown in such, as finding the area of the curve of probabilities between certain ordinates, and the roots of large equations. The fourth example is an example of the fifth power of the equation, in which the unknown quantity is evolved with very large coefficients. This is the general expression for the fifth power of an equation:—

$$u_{,} = \frac{f - f_{,}}{\updownarrow 5ar^5 \updownarrow 4bp^4 \updownarrow 3cr^3 \updownarrow 2dr^2 \updownarrow cr}$$

(See page 41.)

The first dual digit must always be equal to that. The denominator of that fraction is something like the differential of the equation, because you can see the powers 5, 4, 3, 2, 1 become coefficients. But the sign there means either *plus* or *minus*, of *plus* more things than 1. We have nothing more to do but to select any number we like between 10 and 30, say 20, 25, 30, and substitute it for the unknown quantity in the original equation. I take 20, and then find out by taking five times the first, of which I put only the first three figures, four times the second, three times the third; I then divide the difference between f and $f1$ by 950, which gives me $\downarrow 1$, — (see Ex. 4, page 41).

But supposing I should make the first digit 2, it does not make any difference; the next would correct it. Then the second digit will be 3 by the same simple process. But suppose I make it 4, 5, or 6, it does not matter, the next step will always correct it. I continue that process, at last come to a stand-still. Now, I do not care whether I am right or wrong, I take one decided step, and I find exactly the value of x to the seventh place of decimals, disregarding whether all that work is right, or whether it is wrong. I take for granted that I have assumed that to be the proper value of x. I square it, cube it, take the fourth and fifth power of it, because we employ the logarithm of the root. I immediately take the original coefficients, substitute these for the original coefficients, and no matter what has been done before this, whether right or wrong in all that has been done, I determine exactly the value of x; and x is shown to be equal to 20 times that dual number.

$$\therefore x = 20 \downarrow 1,3,2,5,5,8,7,6 = 22{\cdot}7246716.$$

Now, any mathematician that will undertake to find the value of x in that fifth power of the equation by any known process, I need not explain to many here the labour he would have to go through. Sup-

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posing I commenced my operations with 22, I should get the dual number, 22↓0,3,2,5,5,8,5,7. That is the first I commenced with, simply because 2 is easily raised to any power. Had I commenced with 15, I would have got the dual number, 15↓4,3,4,3,1,1,3,8; had I commenced with 12, I would have got the dual number, 12↓6,6,6,9,8,4,4,7. I am not limited to what I should begin with. I am sure always to get the same dual logarithm, and the same common value of x to the seventh decimal place.

In my larger work, when I enter into the limits of roots of equation, I show the shortest path. Although I show that the mere tyro cannot go astray, yet I also show how he can save all his time by confining his operations to the narrowest limits. In my works I then show how a dual number may always begin with a digit not higher than 3; that every second dual digit may be an 0; that it is not necessary to do anything but add or subtract, not to multiply by a digit higher than 5.

If it were nothing but finding the roots of these common equations, t would be of some use. But I undertake to solve the equation $x^x = a$, never solved before. I also solve $a^x + b^x = c^x$; an equation that Auguste Comte, in his Philosophy of Mathematics, has stated that to solve this last equation has defied the will of mathematicians, and that it would be impossible to try to solve it.

I shall now go into the more practical part of dual arithmetic.

The Chairman: Would you explain one or two preliminary stages. Show the change of a natural number into a dual number, and a dual number into a dual logarithm and the reverse.

Mr. Bryne: Very well. I will put in the first position any digit you like. I will put down the arrow and the digits.

↓3,4,5,7,6,2,5,8,

↓3,4,5,7,6,2,5,8,
 1 5 2 0 2 5 0 = 3,04,05,0 × 5
―――――――
3 3 0 5 6 0 0 8

The first three digits multiplied by 5, with 0 after each which subtract. I have only two more numbers to operate with to reduce a dual number to a dual logarithm. I will put them down, 31018 and 33.

33056008
93054 = First digit times 31018
Second digit times 33 = 132
―――――――
33149194 =

Dual logarithm of ↓3,4,5,7,6,2,5,8.

That is a dual logarithm. Any schoolboy would acquire the rule in five minutes. The reversing of this process would bring a dual logarithm back to a dual number, a process which never could be done before. It was possible to calculate a logarithm with great labour, but to get a number back again was quite a different matter. I will

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put down another dual number, composed of both ascending and descending branches.

	'2'2'7'4'3'2'6'5	$\updownarrow$	3,4,7,6,5,3,6,2,	
Add	101035		152035	Subtract
	23753615		33245012	
36052 × 2 =	72104		93054	= 31018 × 3
34 × 2 =	68		132	= 33 × 4
	'23825787		33338198,	

Supposing I wish to reduce a dual number composed of two branches to a dual logarithm at one and the same time, the difference of these numbers have to be taken:

33338198,
'23825787

9512411,

The constants employed for the descending branch are 36052 and 34.

Taking it for granted, therefore, that the operator can convert natural numbers into dual numbers, dual numbers into dual logarithms, and the reverse, it is, indeed, a kind of gymnastics in calculation very easily acquired. He, then, has to recollect that the arrow tells the position in which the first digit stands. Thus $\downarrow^{3}3$ indicates that the second 3 is in the third position; but as there is no figure on the arrow thus written $\downarrow 3$, the 3 is in the first position. Had 2'2'7'4'3'2'6'5'$\downarrow$3,4,7,6,5,3,6,2, been reduced to $\downarrow$0,0,0,0,0,0,0,9512411, then let me define what this dual logarithm is; that the dual number corresponding to it would be an 0 in the first position, an 0 in the second, an 0 in the third, an 0 in the fourth, an 0 in the fifth, an 0 in the sixth, and 0 in the seventh position; and the dual logarithm in the eighth position is 9512411, if I put 9512411, in the eighth position, we have a dual logarithm when all the other digits are zeros, which is nothing more than the base 1·00000001 raised to the power 9512411. All dual logarithms are whole numbers; there are no fractions. Therefore, 1·00000001 is the base that I have selected, and is much more correct than logarithms to seven places of figures, and is true to the single digit in the eighth place; altogether true to seven places.

But a common logarithm is this: 10 being the base raised to the power of ·30103 is nearly equal to 2; 10 raised to the power of 0·0103 a decimal, ·30103 called the logarithm of 2. That is the common logarithm of 2. The base of a common logarithm being 10, the power to which 10 is to be raised to produce 2, that power is termed the common logarithm of 2. The hyperbolic system of logarithms are employed in the differential and integral calculus, and is of this sort:—

2·7 18 28 18 28 18 28

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raised to the power of $\cdot 69314718 = 2$. It happens that the dual logarithm of a number and the hyperbolic logarithm are made up of the same figures; but one of them is divided by 100000000. So when I have the dual logarithm of a number I have only to cast off 00000000, eight decimals on the right, and I have the hyperbolic logarithm. The dual logarithm, therefore, answers all scientific purposes without any transmutation or change, while it has to be changed in other systems; $(1\cdot 00000001)^{69314718} = 2$. If we take 2·718281828 for a base it is very awkward. Now, to compare my base with 2·718281828. If I require greater accuracy, I take 1·000000001, the base beyond 1·00000001; if I require greater still I take the base beyond 1·000000001. If I want a calculation true to 20 places of figures, I take 100000000000000000001, which you will see explained in my book. But there is no way in which you can change a fixed number like 10, or 2·718281828; my base will give it the power of a series of bases, and so the flexibility of a dual number is of great advantage in many ways.

I have taken the Nautical Almanac for 1836, and the plan of calculating longitudes by eclipses or transits. It is necessary first to ascertain the radius of the place where I stand, also the longitude of the place where I stand, together with the latitude of it. In this work the expression for the radius ρ; Ex. 7, page 46. That is $\rho = \left(\frac{\cos l'}{\cos l(\cos l' - 2)}\right)^{\frac{1}{2}}$; $\cos l$ is the cosine of the centre latitude; the other l' is the greater latitude, that given by the line perpendicular to the horizon. Now, to find the value of that expression, I give a simple rule, page 10, which is nothing more than to divide double the cosine by 3; multiply that by 1000· and the dual number ↓0,1,6,0,1,0,2,0, and the result gives the seconds. I need only go as far as 604·, having only used the first and second dual numbers; and I find the number of dual seconds to be 614, consequently the two latitudes differ by 614″. Such a calculation as that is very easily made by dual arithmetic. Now, the cosine of l' is very nearly equal to the cosine of l. The details of the solution of this problem is given at pp. 10, 11. If you observe, the cosine of l' is so very near the cosine of l, you can tell by a few additions and a few subtractions that that is the dual logarithm of the cosine of n, and nearly the cosine of l. I can tell the difference at once between the dual log cosines of these two latitudes.

Sir John Hay: You say a few subtractions. Will you point out the subtractions.

Mr. Byrne: I have to do something with the number ·84362800 until it becomes equal to ·86502350, Ex. 7. If you observe, I add twice to it in periods of three figures. Thus—

843|628|00
1|686|86

Sir John Hay: You multiply the first three figures.

Mr. Byrne: Yes, that is by 1, 2, 1, for 2. That number differs from the original number, reference being made to the numbers

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·84502350
·84511570

9220 difference

(See page 46.)

Common division gives the remaining numbers, but they are obviously on the descending side. These are all found by subtractions, and only one by addition. That, then, is the dual number, which represents at once a dual logarithm. I do not want to retrace my steps. I would have been approximating towards these results had I to turn back. Now I wish to find the radius of the earth in the proposed latitude. There is the dual logarithm '1885991, page 47, already found. Divide by 2 for the square root. I have reduced my formula into the logarithm of the difference of the cosines. When I have got the cosine I divide that, and find the common number. By dividing the dual number by the logarithm of 10, there is the common logarithm of ρ, see page 47. In one minute I find the common logarithm of ρ, an operation which gives astronomers a great deal of trouble; because I bring the logarithm back to the dual number. The length of

ρ is ·99905768

Supposing it is necessary to obtain this with greater accuracy. It is no matter, if you want a hundred places of figures it is all the same, without employing any extraneous number. Now I take a more extensive number than that. Here is a problem (Example 8, page 47) that has engaged the attention of naval men for a long time, that of determining the longitude at sea by lunar observations. The solution of this question called the Nautical Almanac into existence, and, I believe, a large book-case might be filled with works on this problem. I will show how to solve it without any logarithm whatever, without a table, without any operation except the operations exhibited in Ex. 8 and Ex. 5.

The CHAIRMAN: The additions and subtractions alluded to are explained in this little book.

The Rev. WALTER MITCHELL: Having been a pupil of Mr. Byrne, I think I know what the gentlemen present wish to have explained. Perhaps it would be interesting to them to see how readily a knowledge of Mr. Byrne's method may be acquired.

There are two branches of Mr. Byrne's art, an ascending and a descending. You may work with either alone, or save a considerable amount of arithmetical computation by combining the two. For the sake of simplicity I shall confine my attention to the ascending branch, and show with what ease any dual number of that branch can be converted into a common number, and conversely, how any common number can be changed into a dual one.

I will write down a dual number—

$\downarrow$7,2,6,0,7,

The arrow $\downarrow$ is a sign that the numbers following it represent a dual number of the ascending branch, and not a common number. The figures 7,2,6,0, &c., following the arrow, are called dual digits, each digit being separated by a comma, these digits may be any number greater or less than 10. The digits again are said to be in the 1st, 2nd, 3rd, or 4th position, according as they stand in the 1st, 2nd, 3rd, or 4th position to the right of the arrow.

This number $\downarrow$ 7,2,6,0,7, is a contracted representation of the continued product.

$$(1{\cdot}1)^7(1{\cdot}01)^2(1{\cdot}001)^6(1{\cdot}0001)^0(1{\cdot}00001)^7$$

The first practical value of the dual arithmetic which we may notice, depends upon the ease with which any number may be multiplied by any of the powers of numbers of the form 1·1, 1·01, 1·001, &c.

Thus, according to our notation,

$$\begin{array}{lr}
\downarrow 1, = 1{\cdot}1 \;= & 1{\cdot}1 \\
 & 11 \\ \cline{2-2}
\downarrow 2, = (1.1)^2 = & 1{\cdot}21 \\
 & 121 \\ \cline{2-2}
\downarrow 3, = (1{\cdot}1)^3 = & 1{\cdot}331 \\
 & 1331 \\ \cline{2-2}
\downarrow 4, = (1{\cdot}1)^4 = & 1{\cdot}4641 \\
 & 14641 \\ \cline{2-2}
\downarrow 5, = (1{\cdot}1)^5 = & 1{\cdot}61051 \\
 & 161051 \\ \cline{2-2}
\downarrow 6, = (1{\cdot}1)^6 = & 1{\cdot}771561 \\
 & 1771561 \\ \cline{2-2}
\downarrow 7, = (1{\cdot}1)^7 = & 1{\cdot}9487171 \\
 & 19487171 \\ \cline{2-2}
\downarrow 8, = (1{\cdot}1)^8 = & 2{\cdot}14358881 \\
 & 214358881 \\ \cline{2-2}
\downarrow 9, = (1{\cdot}1)^9 = & 2{\cdot}357947691
\end{array}$$

In the above example it will be readily seen, that we have obtained the first nine dual digits in the 1st position, or, in other words, the first nine powers of 1·1 by simple addition.

This method of multiplying by dual digits, using simple addition, may be thus generalised. Suppose we multiply the number 3·14159 by $\downarrow$ 0,0,1, or the dual digit 1 in the third position, which stands for the number 1·001.

Now by common multiplication—

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```
  3·14159
    1·001
 --------
   314159
 314159
 ----------
 3·14473159
```

But this multiplication can easily be performed by addition in two lines, by simply repeating the number to be added, placing its digits in the second line three figures to the right, thus—

```
3·14|159
    |314159
-----------
3·14473159 = 3·14159 × ↓0,0,1
```

This process can easily be expressed in general terms, thus—To multiply any number by a dual digit 1, in any position write the number under itself, placing its digits as many figures to the right as the number of the position of the dual digit. This very easy rule will enable us at once to determine the common number which corresponds to the dual number ↓ 7,2,6,0,7. To avoid needless figures we shall only carry out our result to eight places of decimals, we have already found the value of ↓ 7 to be 1·9487171.

```
1·9|487171   = ↓7,
   |1948717
-----------
1·9|6820427  = ↓7,1,
   |1968204
-----------
1·98|788631  = ↓7,2,
    |198789
-----------
1·98|987420  = ↓7,2,1,
    |198987
-----------
1·99|186407  = ↓7,2,2,
    |199186
-----------
1·99|385593  = ↓7,2,3.
    |199386
-----------
1·99|584979  = ↓7,2,4,
    |199585
-----------
1·99|784564  = ↓7,2,5,
    |199785
1·9998|4349  = ↓7,2,6,
      |2000
-----------
```

1·9998|6349 = $\downarrow$7,2,6,0,1,
|2000

1·9998|8349 = $\downarrow$7,2,6,0,2,
|2000

1·9999|0349 = $\downarrow$7,2,6,0,3,
|2000

1·9999|2349 = $\downarrow$7,2,6,0,4,
|2000

1·9999|4349 = $\downarrow$7,2,6,0,5,
|2000

1·9999|6349 = $\downarrow$7,2,6,0,6,
|2000

1·9999 8349 = $\downarrow$7,2,6,0,7,

According to the rules of decimal arithmetic, if we neglect any number in the 9th place, we make the number preceding it greater by one if the neglected digit be greater than 4.

The same method may evidently be extended to a dual number of any number of digits.

It must not be supposed that this is the only or most expeditious method of converting a dual number into a common number, I only use it to show that this conversion can be made by simple addition without uming any law of binomial coefficients, or any arithmetical process but that of simple addition.

I shall now show, conversely, how to find the dual number corresponding to a common number. Suppose we require the dual number which represents the number 1·12347036.

Referring back to our list of the values of the first nine dual digits, we see that our number must be between $\downarrow$ 1, and $\downarrow$ 2, or we might make our table till our first two digits corresponded with the number

1·1| = $\downarrow$1,
|11

1·1|11 = $\downarrow$1,1,
|1111

1·12211 = $\downarrow$1,2,

Now, it is evident if we increase the second dual digit by 1, our new number will become too great, we therefore proceed to the third digit:—

1·12|211
|112211

1·12323211 = $\downarrow$1,2,1

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Any increase of the third digit will make the number too great, we therefore proceed to the fourth and following digits:—

```
1·123|23211
     |11232
-----|-----
1·123|34443 = ↓1,2,1,1,
     |11233
-----------
1·1234|5676 = ↓1,2,1,2,
      |1123
-----------
1·12346|799 = ↓1,2,1,2,1,
       |112
-------|---
1·12346|911 = ↓1,2,1,2,1,1,
       |112
-----------
1·123470|23 = ↓1,2,1,2,1,2,
        |11
-----------
1·1234703|4 = ↓1,2,1,2,1,2,1,
         |1
-----------
1·1234703|5 = ↓1,2,1,2,1,2,1,1,
         |1
-----------
1·12347036  = ↓1,2,1,2,1,2,1,2,
```

I have thus shown how any common number can be reduced to a dual number, and conversely how any dual number can be changed into a common number by simple addition. Now, the next thing I propose to do is to convert a dual number into a dual logarithm. Here I may notice a very curious fact. Any common number may be represented by a vast number of dual numbers, each differing from each other as to their digits, but all, when reduced, producing the same common number. For instance, the dual number ↓7,2,6,0,7,8,2,6, may be represented by another dual number, whose first digit is zero, and its third digit some number greater than 70. This may again be reduced to another whose second digit is also zero; and so on we may reduce the original dual number into one whose first seven digits are zero's, and its eighth digit some large number. I cannot detain you by demonstrating the method of this reduction, but shall content myself by stating how it may be done by a simple arithmetical rule. Write the dual number as a common number; add to this the number 31018 multiplied by the first digit, and 33 multiplied by the second digit; subtract from this sum five times the first three digits, each followed by zero, written as a common number.

Thus to find the dual logarithm of the dual number—

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$$
\begin{array}{r l}
\downarrow 7,2,6,0,7,8,2,6 & \\
2\,1\,7\,1\,2\,6 & = 7\times 31018 \\
6\,6 & = 2\times 33 \\
\hline
7\,2\,8\,2\,5\,0\,1\,8 & \\
-\,3\,5\,1\,0\,3\,0\,0 & = 702060\times 5 \\
\hline
6\,9\,3\,1\,4\,7\,1\,8 &
\end{array}
$$

$$\text{or, } \downarrow 7,2,6,0,7,8,2,6 = \downarrow 0,0,0,0,0,0,0,69314718$$

which also means that the number $\left(1+\frac{1}{10^8}\right)$ raised to the power indicated by the whole number 69314718 will produce the common number represented by the dual number $\downarrow 7,2,6,0,7,8,2,6$, which by the method of reduction I have already demonstrated can be shown to be equivalent to the common number 2.

We, therefore, call the number 69314718 the dual logarithm of the number 2.

Now, I think I have succeeded in showing you how the dual logarithm of any common number may be obtained. These dual logarithms possess all the powers and properties of any other logarithms, and are in addition available for operations in which common logarithms would fail. Besides the hyperbolic or Napierian logarithms, and the Briggs' or common logarithms can at once be calculated from them.

To convert a dual logarithm into a hyperbolic one count off eight figures and place a decimal point.

Thus—69314718 is the dual logarithm of 2.

·69314718 is the hyperbolic logarithm of 2.

230258509 is the dual logarithm of 10.

2·30258509 is the hyperbolic logarithm of 10.

To convert a dual logarithm into a common logarithm, that is, one whose base is 10; all we have to do is to divide the dual logarithm by the number 230258509, which is the dual logarithm of 10, which can readily be done by the rule for contracted division by decimals.

If our number be greater than 2 it is evident that our first dual digit will be greater than 7, but if our number be greater than 2 by dividing it by 2, 4, or 8, and some power of 10, we can always reduce it to a number of the form of a decimal whose first digit does not exceed 1, and is followed by the decimal point.

Now, if Professor Byrne had taught nothing more than what I have now shown you that I, as his pupil, can do, and any of you can do for yourselves by the simplest arithmetical processes, he would have a just right to claim the merit of doing what all his predecessors, as mathematical professors, have failed to do, from the days of Napier, Briggs, and Newton, to the present time. But, in addition to this very elementary part of his art, Mr. Byrne has shown you the application of it to the solution of questions of vast importance, hitherto defying the power of the calculator.

Mr. Byrne: I feel very proud of my pupil, and I think he has done it a great deal better than I could do it myself. But had I to stop here,

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and could I only have shown you what has been discussed in this lecture I would not have made my appearance here. But I have satisfied myself that, in the five volumes already printed, of which you will find on this table an account, I have framed a system to supersede mathematics as it is by the introduction of a new science called the *Calculus of Form*, out of which the dual calculus has grown. The books which I have in the press will supersede the differential and integral calculus, and clear away fallacies upon which they are based.

The CHAIRMAN: We have had an extraordinary lecture. It evidences extreme boldness if it is untrue, and if it is true, it has extreme ingenuity, when we remember that any extension of the powers of analysis under the old systems, has but increased the difficulty and mystification, for it was no uncommon thing for mathematicians not to be able to read their own answers. Now, this system is really very simple. I have myself gone through a great deal of this little book, and the operations are as simple as those that have been exhibited to you by Mr. Mitchell; in fact some of the process exhibited by Mr. Byrne, and which seemed so very difficult, are merely an application of the simple process explained by Mr. Mitchell. It is a subject so important that it ought to be investigated. (Hear hear). Mr. Byrne has thrown down the gauntlet, and such immense consequences in every department of mathematical science are involved, that the system ought to be fully tested. If any gentleman has any objection to offer, he ought to come forward and substantiate that objection, if not, he ought to admit the system to be correct. It makes mathematical investigations possible to the million. The system is easy to be comprehended by the lowest intelligence; it is just as simple as any of the operations of common arithmetic, and less abstruse and difficult than some. There are a thousand occasions which occur to us when we should like to be independent of the actuary, the mathematician, and the naval architect. We professional men would like to be able to calculate for ourselves, and the fact of our professional knowledge would give us powers of analysis, or means of addressing ourselves to obtain results that we have not now, because mathematicians in looking at things from their own stand-point and not from ours, do not really arrive at the result we want—I mean the practical result. Now, this system will render every man independent of the actuary and the mathematician in every direction. I beg gentlemen not to be dismayed because there is a new system of notation which seems difficult, but which is not really so. But though new it is very simple and definite, and enables you much better to keep in view your chase. A little attention to it, throwing overboard what you have been indoctrinated with to a great extent, and beginning as it were with a blank sheet, you will find it simple, and, as you go on, you will be astonished at the immense power which you will find in your hands. I beg to commend it to this assembly. We have to thank not only Mr. Byrne for the invention of it and for bringing it forward, but also Mr. Mitchell for the very lucid way in which he has illustrated it. We have also to thank

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Messrs. Bell and Daldy for the public spirit with which they have taken up this scheme. Without any immediate prospect of results, disregarding vested interests, they have invested a considerable sum of money in its publication. I contend that they are national benefactors. The Council will be anxious to afford every facility for investigation, and no doubt would give a meeting for the purpose, when questions might be asked; and to facilitate the matter some of the questions should be put in writing. Mr. Byrne has been soaring in the highest regions of mathematics until he has got quite transcendental. We unfledged ones must try to bring him down to us till we are able to fly.

Rear-Admiral Sir Frederick Nicolson, Bart., C.B.: Mr. Byrne has stated that his system will supersede the integral and the differential calculus.

The Chairman: Practically it comes to that, it is more accurate and much more simple in its determinations; and, as he stated to you in one or two cases, which might be multiplied indefinitely, there are equations which cannot be worked out by any other method; therefore, as far as that goes, the other systems are valueless.

Sir Frederick Nicolson: I simply wanted to put a question. As far as I understand the question it is an easier way of solving problems, which have hitherto been solved by the differential and integral calculus. That is as I understood it. At any rate, it is quite a novel proposal in mathematics. Mr. Byrne has given great attention and great labour to it, and has published elaborate books on the subject. The question I was going to ask was simply this, whether these calculations and these books have been submitted to any mathematician of well-known capacity, in order to give a deliberate and an independent opinion on them?

Mr. Scott Russell: Perhaps you will allow me to answer that question. I say that Mr. Mitchell is himself one of the highest mathematicians in the land. I believe the subject came before him, not as a pupil, but as a judge; and I believe the consequence of its so coming before him as a judge, was that he entered himself as a pupil.

The Rev. Walter Mitchell: I will endeavour to answer Sir F. Nicolson's question. The whole of the dual arithmetic, and all the works now passing through the press, and all the tables, I have looked through myself, and I have not found a faulty demonstration. Every problem is worked out by methods and principles as simple and as easy as anything I have shown you; and that, I think, must be saying something about things which belong to transcendental analysis, and therefore hitherto only to be understood by those who can enter into the highest branches of mathematical science. If this be true, it is something to have the most abstruse speculations the human mind can enter into, reduced to simple addition and subtraction. In Mr. Byrne's book you will find problems solved by the aid of La Place's theorem, and by the aid of La Grange's, and you will find the highest methods of the differential calculus leading you to equations of most difficult solution, and you will be inclined to say, "What a very awkward system this is in obtaining the truth."

Then you have these same problems solved by processes as simple as those I have shown you. But Mr. Byrne goes further, and gets simple solutions of problems where the theorems of La Place and La Grange and all other known methods, utterly fail to give any result.

The CHAIRMAN: Having worked out the results by Mr. Byrne's method, any one can establish by existing means that the answers are perfectly true. I repeat that it is perfectly within the compass of any one here present, to establish that the answers are true. The Council will be glad to give the meeting an opportunity to ask questions.

Adjourned to Wednesday evening, January 24th.

ADJOURNED MEETING.

Wednesday, January 24th, 1866.

REAR-ADMIRAL SIR F. E. NICOLSON, Bart., C.B., in the Chair.

The CHAIRMAN: We are about to resume the discussion of Mr. Byrne's paper, which was read the other day. I believe Mr. Byrne wishes to make some additional statements.

Mr. BYRNE: Viewing this subject from different stand-points, I rapidly passed over one development after another without entering upon minor details. It is very important at starting upon any subject that we start right; therefore, I left it for a separate explanation, and I have taken three examples, the solutions of which are exhibited on the black board. All the figures employed are set down. Although they are common place questions they could not be done without great labour before I discovered this art and science. I have not taken the easiest plan, because that would not bring with it the method of investigating, and you will find that that method of teaching which approaches the method of investigating most nearly is always the best.

The numbers exhibited in the tabulated form I call operative numbers. They are used sometimes as binomial coefficients; they are employed in the calculus of differences; and found useful in calculations respecting the doctrine of chances. This tabulated form is sometimes called the arithmetical triangle. I use it in three different ways. When I employ these numbers in one way in raising a power, it differs materially in the use of them, in passing from step to step, whether positive or negative. Any one can form this table in a minute or so. I set down a line of one's perpendicularly, and one horizontally. I commence with a line of units in a horizontal direction, and another line in a perpendicular direction.

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