ART. X.—*Solar Parallax from the Velocity of Light;* by D. P. TODD, M.A., Assistant Nautical Almanac.

THE opposition of Mars in 1862, and the experimental determination of the velocity of light by Foucault in the same year, mark the beginning of a new era in the history of the determination of the solar parallax. Especial prominence has attached to the subject ever since, not only from its inherent importance, but also from the rapidly multiplying determinations of this constant which have been made, and the vigor of discussion that has been everywhere prevalent in scientific astronomical circles.

If the generally accredited theories of the solar parallax and inter-related facts and phenomena are true, the better class of these determinations should yield values of the parallax in consistent harmony with each other—modified only by deductive consideration of the amount of accidental and systematic error with which they are severally affected. The well known fact, however, is that even the best of these determinations appear, at present, singularly and unaccountably discordant. The solar parallax, 8″·848, derived by Professor Newcomb nearly thirteen years ago, generally replacing Encke's value, 8″·57116, was regarded with caution, only because it was considered too small—the researches of Hansen, of Le Verrier, of Stone, and of Winnecke were thought to have defined the parallax far outside Newcomb's value. Within two or three years, however, the *parallactic pendulum* has swung quite to the lesser extremity of its arc, until the true value of the solar parallax has appeared possibly below 8″·8—and that, too, with good reason. But a slight gravitation toward a central value is already beginning to show itself—and now, in reality, it is not at all possible to say that the mean equatorial horizontal parallax of the sun is so much as a hundredth part of a second different from the ancient figures, 8″·813 [27″·2 centesimal], adopted by Laplace in the Mécanique Céleste, and given by the first discussion of the Transits of Venus in 1761 and 1769.

The method of determining the solar parallax through the velocity of light, though dependent on the results of physical experiments conducted under necessarily limited conditions, has never given a value of the parallax at all inconsistent with a combination of the best of the purely astronomical determinations. And this consideration encourages indulgence of the hope that, at some time in the not far distant future, this method will define the solar parallax within very much smaller limits than astronomers have yet known. To show what the method is competent to at the present moment is the object of this paper.

I bring together all the determinations of the velocity of light which have at all the merit of trustworthiness.

I. Fizeau made the first experimental determination of the velocity of light, in 1849; his experiments, however, hardly signify more than the completion of the first great step of proving the determination to be a physical possibility. The first reliable determination was executed thirteen years later by Foucault. His work has never been published *in extenso*, but brief papers have appeared in the Comptes Rendus, vol. lv, 1862, and in Poggendorff's Annalen, vol. cxviii, 1863. The resulting velocity of light is 298,000 kilometers per second, in which Foucault expresses confidence to about one-six-hundredth part. I think I shall not be regarded far wrong in assigning a probable error twice as great. This I do in consideration of the very unfavorable limitations under which the determination was executed, and quite independently of what has since become known.

II. The first determination by Cornu, related in the Journal de l'École Polytechnique, xliv cahier, vol. xxvii, 1874. The resulting velocity of light is $298{,}500^k \pm 1{,}000$.

III. The second determination by Cornu, related in the Annales de l'Observatoire de Paris, Mémoires, tome xiii, 1876. The resulting velocity of light is $300{,}400^k \pm 300$. Helmert has given a rediscussion of these experiments in the Astronomische Nachrichten, vol. lxxxvii, 1876. His interpretation assigns the velocity 299,990 kilometers, the probable error of which I have estimated at 200 kilometers.

IV. The first determination by Master A. A. Michelson, U. S. Navy.* The resulting velocity of light is 300,100 kilometers. I consider this determination of equal weight with that of Foucault, and with the first determination by Cornu.

V. The second determination by Mr. Michelson. The number of this Journal for November, 1879, contains a brief recital of these experiments. I shall adopt, for the purposes of this paper, the results given on the "corrected slip," for his second determination of this constant of velocity.

The largest result for velocity of light, 300,142 kilometers.

The smallest result for velocity of light, 299,692 kilometers. Giving equal weight to the one hundred separate determinations, the resulting velocity of light is 299,930 kilometers per second. I find that the computed probable error of this determination is no larger than six kilometers. But, in the determination of almost no other astronomical or physical constant should we consider the computation of probable error of the final result from a corresponding number of observations quite so illusory. In consideration of all the sources of error, accidental and systematic, I think the probable error of this result may be estimated at 100 kilometers.

All these several determinations of the velocity of light are now combined as follow :—

I.	298000 ± 1000	Weight 1
II.	298500 ± 1000	1
III.	299990 ± 200	25
IV.	300100 ± 1000	1
V.	299930 ± 100	100

The resulting most probable velocity of light is

299,920 kilometers = 186,360 miles per second.

* Proceedings of the American Association for the Advancement of Science vol. xxvii, 1878.

The next step is the combination of this value with astronomical constants, for the determination of the distance of the center of the sun from the center of the earth.

(I.) Theory and observation of the satellites of Jupiter afford a determination of the time-interval required by light in traversing the mean radius of the orbit of the earth. Only two astronomically precise determinations of this interval have ever been made: the first, by Delambre, in the early part of the present century,* from a discussion of an immense mass of eclipses of the satellites of Jupiter, comprising observations from 1662 to 1802; the second, by Glasenapp, in a Russian thesis,† in which there are discussed the observations of the first satellite of Jupiter from 1848 to 1873. The results of the two determinations are as follow:—

Delambre, $493^{s}\cdot2$; Glasenapp, $500^{s}\cdot84 \pm 1^{s}\cdot02$.

It is quite impossible to judge with certainty just how these two widely discordant values should be combined. The former determination rests on a much greater number of observations than the latter; but it is difficult to form a just estimate of the worth of an average last-century observation of an eclipse of a satellite of Jupiter. And, moreover, astronomers have no means of knowing the process of discussion which led the distinguished French astronomer to his result—which he has adopted in his own tables of the satellites, and which was adopted by Damoiseau in his Tables Écliptiques, published in 1836. The latter determination rests upon a mass of observations of definite excellence, which have been discussed after the modern fashion. I combine the two values giving weight unity to the first, and weight two the second. The adopted value of k is, therefore, $498^{s}\cdot3$, which, combined with the constant of light-velocity just deduced gives the mean radius of the orbit of the earth equal to

149,450,000 kilometers = 92,866,000 miles.

If, now, we combine this result with the value of the equatorial radius of the earth derived by Listing,‡

$$a = 6377^{k}\cdot377 \; [3\cdot8046421]$$
$$= 3962^{m}\cdot790 \; [3\cdot5980011]$$

there results the mean equatorial horizontal parallax of the sun, $8''\cdot802$.

(II.) The velocity of light, the constant of aberration, and appropriate elements of the terrestrial orbit are combined, the equation of connection being,

* Tables Écliptiques des Satellites de Jupiter, par Delambre, Paris, 1819.

† Сравненіе Наблюденій Затмѣній Спутниковъ Юпитера Съ Таблицами Затмѣній и Между Собою . . . С. Глазенапа . . . С.-Петербургъ. . . . 1874.

‡ Neue geometrische und dynamische Constanten des Erdkörpers. Eine Fortsetzung der Untersuchung: über unsere jetzige Kenntniss der Gestalt und Grösse der Erde. Von Johann Benedict Listing. Göttingen, 1878.

$$\frac{\delta g_{,,,}}{\iota t}\frac{a}{\sin \pi} = V\theta \cos \varphi_{,,,}$$

wherein $g_{,,,}$ denotes the mean anomaly of the earth,
$\varphi_{,,,}$, the angle whose sine is the eccentricity of the earth's orbit,
θ, the constant of aberration.

Struve's constant of aberration, 20″·4451, with Listing's value of a, leads to the following results: The mean radius of the orbit of the earth equal to

149,293,000 kilometers = 92,768,000 miles;

and the mean equatorial horizontal parallax of the sun, 8″·811.

It remains to consider the probable variations of the elements of this computation, and the effect of such variations on the derived parallax. The following elements of sensible uncertainty are considered:—

(1.) Uncertainty in the determination of the terrestrial velocity of light. In the process of experimental determinations of the velocity of light, almost no sources of mechanical error have been encountered which cannot, under the most favorable conditions and methods, be reduced to a minimum. What these conditions and methods of experiment are will not now be considered. Let it suffice, for the present, to remark that in approaching the utmost refinement in a determination of such supreme nicety, experimenters are like to be confronted with modifying physical circumstances—which, in all probability, however, need not be considered in connection with any experimental determination of the velocity of light heretofore executed. For the detail of uncertain elements affecting the result of each series of experiments, reference must be had to the individual papers themselves. I am disposed to think that the limit of uncertainty of the velocity of light concluded above may be fairly taken at seventy kilometers.

(2.) Uncertainty in the coefficient of the light-equation of the satellites of Jupiter. The same circumstances, unfavorable in considering the proper combination of the two independent determinations of this constant, hold here. The amount of uncertainty is probably not far from one second of time.

(3.) Uncertainty in the constant of sidereal aberration. I conceive that a variation of 0″·025 in this well determined constant will not be regarded far from the limit of uncertainty. However, this estimate of its variation cannot reasonably be adhered to, except on the supposition that the accepted value of the constant of aberration holds for stars in every part of the celestial sphere,—that is, that the motion of translation of the solar system in luminiferous ether is not sufficiently great to affect the astronomical accuracy of determination of this constant. This question pertains to the astronomy of the future.

(4.) Uncertainty in the relation of the absolute terrestrial velocity to the velocity in space. On this matter, much difference of opinion exists. As the history of the Greenwich water-telescope shows that the constant of aberration is not affected by the passage of the light of the determining star through refracting media, this element of uncertainty exists only in the derivation of the solar parallax through the light-equation of the satellites of Jupiter. The impossibility of an experimental determination of this relation renders the assumption of identity necessary.

(*a*.) Combining the maximum velocity of light with the maximum coefficient in the light-equation, and the minimum velocity with the minimum coefficient, the following relations exist:—

Limiting k	Limiting V	Distance of Sun In kilometers.	Distance of Sun In miles.	Solar Parallax.
$499^s \cdot 3$	299990	149,785,000	93,074,000	$8'' \cdot 782$
$497^s \cdot 3$	299850	149,115,000	92,658,000	$8'' \cdot 822$

(*b*.) Combining the maximum velocity of light with the maximum value of the constant of aberration, and the minimum velocity with the minimum constant, the following relations exist:—

Limiting θ	Limiting V	Distance of Sun In kilometers.	Distance of Sun In miles.	Solar Parallax.
$20'' \cdot 47$	299990	149,510,000	92,903,000	$8'' \cdot 798$
$20'' \cdot 42$	299850	149,076,000	92,633,000	$8'' \cdot 824$

It will be remarked that all these combinations are made in the most unfavorable manner, so as to give the limiting values of the solar parallax with the variations of the elements previously adduced. The probable errors of the intermediate values of the parallax are about one-third these variations.

In conclusion, then, all the experimental determinations of the velocity of light hitherto made give, when combined with astronomical constants, the mean equatorial horizontal parallax of the sun, $8'' \cdot 808 \pm 0'' \cdot 006$.

The corresponding mean radius of the terrestrial orbit is

149,345,000 kilometers = 92,800,000 miles.

Nautical Almanac Office, Washington, Nov. 10, 1879.