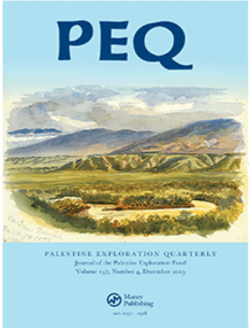

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Charles Warren

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DATES ON WHICH PASCHAL FULL MOONS OCCUR.

By Lieut.-General Sir CHARLES WARREN, K.C.B., F.R.S.

QUESTIONS have been asked (*vide Quarterly Statement*, January, 1898) as to the dates of Paschal full moons about A.D. 33, and it has occurred to me that the tables attached to the calendar of the Book of Common Prayer might be made use of for this purpose if Table II be extended backwards sufficiently far. I have carried it back to the year 5000 B.C. in the form of a supplementary Table II, and by means of its use the dates of full moon can be obtained with the least possible amount of calculation for any period B.C., however remote, with the same facility that the dates of Paschal full moon can be found in the existing Table II. The possible error ought not to be more than a day, but the date will be found according to the Gregorian calendar and a correction will be required to turn this date into Julian years, which are used by astronomers. It is to be observed, however, that for purposes of comparison the Gregorian calendar is the simplest to use, as by it the vernal equinox occurs between March 20th to 22nd for all ages, provided a correction of a day is made for every 3,600 years from the point of departure of the two calendars.

Before giving the results of this use of this supplementary Table II, I will first give some observations on the calendar.

In the year 46 B.C. Julius Cæsar reformed the calendar, adapting it to the sun's course and giving 365 days to the year with 100 leap years in 400 years (*i.e.*, one extra day every fourth year). Owing to this reform the winter solstice in the year B.C. 46 fell on December 24th, and January 1st B.C. 45 coincided with the mean new moon. The months and number of days in each month are the same now as when established in this reformed year, called the *Julian* year.

The Julian calendar supposes the solar or tropical year to consist of 365·25 solar days. The actual length of the tropical year (the interval between two successive passages of the sun through the mean vernal equinox) is about 365·24222 mean solar days. Thus in 400 years there would accumulate (365·25 − 365·24222) × 400 = 3·112 solar days.

This accumulation in the fourth century A.D. amounted to over two days, when the first Æcumenical Council (of Nice) A.D. 325 legislated with regard to the observance of Easter, fixing the vernal equinox on March 21st. In the sixteenth century the accumulation amounted to over nine days, causing great inconvenience with reference to the fixing of the Church festivals, and Pope Gregory in 1582 again revised the calendar, removing 10 days (somewhat more than the accumulation, since the Council of Nice, actually consisted of), and allowing for the future only 97 leap years instead of 100 in 400 years.

This brings the calendar right to ·112 solar day in 400 years, causing an accumulation of one day in 3,600 years. In order to remedy this the French astronomer, Delambre, proposed that the years A.D. 3600 and every multiple of 3,600 should not be a leap year, and a similar correction of this nature is required in using the supplementary Table II beyond B.C. 3600.

The Gregorian calendar coincides with the Julian calendar during the period A.D. 200 to A.D. 300. That is to say, 10 days were taken off for the years A.D. 1100, 1000, 900, 700, 600, 500, 300, so that in adjusting the dates of the first century found in the Gregorian style to the Julian style, two days must be subtracted for the years A.D. 200 and 100.

To carry back the Gregorian style into the period B.C. there should be a leap year at 400 years backwards from A.D. 400 (*i.e.*, in the year before A.D. 1), in other words, in B.C. 1, and from thence the leap years are carried back at intervals of four years, so that the three leap years omitted in every 400 years fall on B.C. 101, 201, 301, 501, 601, 701, 901, 1001, &c., and the days on which leap years remain fall on B.C. 601, 801, 1201, &c. The rule for finding the Sunday letter in any year B.C. is therefore different to that for years A.D.

The rule also for finding the golden number for years B.C. differs from that given in the table for years A.D.

The golden number for the year A.D. 1 is II, for the year B.C. 1 is I, for the year B.C. 2 XIX, and so on: therefore the rule is to divide the year B.C. by XIX and subtract the remainder from XXII, and the result will give the golden number, except when it is above XIX, when XIX must also be subtracted.

There are two methods of finding the dates of full moons by means of the tables in the Book of Common Prayer, either by using the supplementary Table II appended to this paper or else by deducting 1,040 or its multiples from the year in which the Paschal full moon is required, so as to bring it within range of the tables in the Book of Common Prayer. This can be done owing to the fact that 1,040 years is a cycle very nearly perfect, in which there are an even number of days, years, and lunations with an error of little over one hour.

According to Bessel ("Chauvenet's Astronomy," vol. i, p. 59), the length of the tropical year in A.D. 1800 was 365·242220027 days, which, multiplied by 1,040, gives 379,851·908828 days. In 1,040 Julian years of 365¼ days there are 379,860·0 days, leaving a difference of 8·091172 days. A correction of eight days in every 1,040 Julian year will thus bring the number of days right to ·0911 day, and in 3,600 years at this rate there will be an error of little more than 6½ hours. In the same number (3,600) of Gregorian years there is an error of just one day.

The year, however, has an annual decrease of about ·595 second per annum, so that about 2,000 to 3,000 years ago 1,040 tropical years were nearly exactly 379,852 solar days.

There are 29·5305887 days in a lunation (29 days 12 hours 44 minutes 3 seconds), or 2,551,442·66368 seconds, which, multiplied by 12,863, gives 1,040 years within two hours), and even nearer when the necessary corrections are applied.

This close approximation of 1,040 years to a perfect cycle was discovered by De Cheseaux, a Swiss astronomer, in the seventeenth century, and called by him the Daniel cycle, because he arrived at it from the examination of the day numbers in the Book of Daniel.

Dr. Grattan Guinness has founded a series of tables of dates of new moons, ranging from B.C. 1622 to A.D. 1934, upon this cycle and that of 2,300 years, and the Superintendent of Nautical Almanac (in letter of March 23rd, 1895), quoted in Guinness's tables ("Creation Centered in Christ"), states that the cycles of 2,300 and 1,040 years are sufficiently accurate to be put to practical use for the computation of the positions of the sun and moon at distant epochs.

In these tables the lunation is taken at 29·5305916 days, which requires no correction and is said to *yield better results* than the best and most accurate measures of the lunation now used by astronomers. In any case the difference over 10,000 lunations only amounts to about 11 minutes, and need not be considered for practical purposes of fixing dates of new and full moon.

The great advantage of Guinness's tables is that at every 1,040 years the mean new moon comes back to the same hour of the mean day, so that these tables can be extended without difficulty for centuries beyond their present limit of B.C. 1622, and I have used them in preference to Ferguson's tables in checking the dates of full moon deduced from the Book of Common Prayer.

In order to show with what accuracy the tables of the Book of Common Prayer will give the date of full moon over long intervals, I will first take the year 1899 and years at intervals of 1,040 years from that date backwards and forwards, and ascertain, by means of Table II with its supplement, on what day the full moon will fall. This is tabulated below (Table A), giving the golden numbers, cypher, day of month, and dominical letter.

It will be seen that, March 26th being the day of Paschal full moon A.D. 1899, the tendency is to go forward to March 27th in the past and back to March 25th and 24th in the future. This is probably owing to the error of a day beyond 3,600 years. Allowing for this error, the Prayer Book gives the same results as Guinness's Tables to a day.

TABLE A.

From Book of Common Prayer, with Sup. Table II.						Obtained directly or deduced from Guinness's Tables. Jerusalem Civil Time.																
—	Golden Numbers.	Cypher.	Dominical Letters.	Gregorian Years.		Julian Years.								Gregorian Years.								
				Full Moon.	Days.	M. Vernal Equinox.				M. New Moon.				M. New Moon.				M. Vernal Equinox.				
							d.	h.	m.		d.	h.	m.		d.	h.	m.		d.	h.	m.	
B.C. 4342..	XI	4	—	March 27	− 36	A.	26	7	57	A.	17	1	17	M.	12	1	17	M.	21	7	57	
3302..	VI	9	—	„ 27	− 28	A.	18	6	19	A.	9	1	17	M.	12	1	17	M.	21	6	19	
2262..	I	13	—	„ 26	− 20	A.	10	4	41	A.	1	1	17	M.	12	1	17	M.	21	4	41	
1222..	XV	17	—	„ 26	− 12	A.	2	3	3	M.	24	1	17	M.	12	1	17	M.	21	3	3	} Guinness's Tables. (1222–1899)
182..	X	23	—	„ 27	− 4	M.	25	1	22	M.	16	1	17	M.	12	1	17	M.	21	1	22	
A.D. 859..	V	26	—	„ 25	4	M.	16	23	42	M.	8	1	17	M.	12	1	17	M.	20	23	42	
1899..	XIX	1	A	„ 26	12	M.	8	22	2	F.	28	1	16	M.	12	1	16	M.	20	22	2	
2936..	XIV	6	I	„ 26	20	F.	28	20	24	F.	20	1	16	M.	12	1	16	M.	20	20	24	
3979..	IX	10	—	„ 25	28	F.	20	18	46	F.	12	1	16	M.	12	1	16	M.	20	18	46	
7099..	XIII	24	—	„ 25	52	J.	27	17	8	J.	19	1	16	M.	12	1	16	M.	20	17	18	
8139..	VIII	28	—	„ 24	60	J.	19	15	30	J.	11	1	16	M.	12	1	16	M.	20	15	30	

It will be seen from the above that the date of full moon can be obtained to a day.

NOTE.—It is to be noted that the Prayer Book gives full moon, while Guinness's Tables give new moon; the difference being between 14 and 15 days.

TABLE B.

I will now take certain dates which are known or are derived from Guinness's Tables:—

From Book of Common Prayer, with Sup. Table II.						Derived from Guinness's Tables. Jerusalem Civil Time.											
				Gregorian Years.		Julian Years.						Gregorian Years.					
—	Golden Number.	Cypher.	Dominical Letter.	Full Moon.	Days.	Vernal Equinox.			New Moon.			New Moon.			Vernal Equinox.		
						April. d.	h.	m.	April. d.	h.	m.	March. d.	h.	m.	March. d.	h.	m.
B.C. 4142 ..	II	5	E	April 6 ..	33	24	18	24	25	17	41	23	17	41	22	18	48
4017 ..	XIII	5	D	„ 5 ..	33	23	1	31	23	0	46	21	0	46	21	1	31
3702 ..	V	7	D	„ 5 ..	30	21	8	52	21	5	12	22	5	12	22	8	52
2662 ..	XIX	12	E	„ 6 ..	22	13	7	12	13	5	12	22	5	12	22	7	12
1622 ..	XIV	16	D	„ 5 ..	14	5	5	32	5	5	12	22	5	12	22	5	32
1285 ..	VI	18	D	„ 5 ..	11	2	3	20	2	2	11	22	2	11	22	3	20
						March.											
935 ..	XVII	19	D	„ 5 ..	9	30	15	36	30	15	41	21	15	41	21	15	36
601 ..	IX	21	D	„ 5 ..	6	27	13	24	27	12	40	21	12	40	21	13	24
248 ..	I	22	C	„ 4 ..	4	25	1	40	25	2	11	21	2	11	21	1	40
A.D. 87 ..	XII	23	C	„ 4 ..	2	22	23	28	22	23	9	20	23	9	20	23	28
440 ..	IV	25	C	„ 4 ..	1	19	11	44	19	12	40	20	14	40	20	11	44
774 ..	XV	26	C	„ 4 ..	4	17	9	32	17	9	38	21	9	38	21	9	32
1108 ..	VII	28	C	„ 4 ..	7	14	7	20	14	6	37	21	6	37	21	7	20
1461 ..	XVIII	29	C	„ 4 ..	9	11	19	36	11	20	8	20	20	8	20	19	36
1795 ..	X	1	C	„ 4 ..	11	9	17	24	9	17	6	20	17	6	20	17	24

These are dates on which the hour of new moon approach most nearly to the hour of the vernal equinox. *See* Note, Table A.

TABLE C.

I now give the dates of full Paschal moon in Gregorian years from A.D. 22 to A.D. 37, and compare the results with Guinness's tables corrected to Gregorian years:—

From Book of Common Prayer, with Sup. Table II.					From Guinness's Tables. Jerusalem Civil Time.		
—	Golden Number.	Cypher from Table II.	Dominical Letter.	Paschal Full Moon.	Vernal Equinox.	Mean New Moon.	—
					March. d. h. m.	d. h. m.	
A.D. 22 ..	IV	23	A	April 2	21 5 34	19 8 51	March.
23 ..	V	23	D	March 22	21 11 23	8 17 40	,,
24 ..	VI	23	B	April 10	20 17 12	26 15 13	,,
25 ..	VII	23	E	March 30	20 23 1	16 0 1	,,
26 ..	VIII	23	C	April 18	21 4 49	3 21 31	April.
27 ..	IX	23	F	,, 7	21 10 38	24 6 23	March.
28 ..	X	23	B	March 27	20 16 27	12 15 11	,,
29 ..	XI	23	G	April 15	20 22 16	31 12 44	,,
30 ..	XII	23	C	,, 4	21 4 5	20 21 33	,,
31 ..	XIII	23	F	March 24	21 9 54	10 6 21	,,
32 ..	XIV	23	D	April 12	20 15 42	28 3 54	,,
33 ..	XV	23	G	,, 1	20 21 31	17 12 42	,,
34 ..	XVI	23	C	March 21	21 3 20	6 21 31	,,
35 ..	XVII	23	A	April 9	21 9 9	25 19 4	,,
36 ..	XVIII	23	D	March 29	20 14 58	14 3 52	,,
37 ..	XIX	23	B	April 17	20 20 47	2 1 25	April.

See Note, Table A.

Supplementary Table II, giving the cypher correction to be used in Table III, Book of Common Prayer, and also the number of days to be added to or deducted from a Gregorian year to make it a Julian year :—

Cypher. (*See* Table III.)	Gregorian years B.C.	To be added for Julian years.	Cypher. (*See* Table III.)	Gregorian years B.C.	To be added for Julian years.	Cypher. (*See* Table III.)	Gregorian years A.D.	To be deducted for Julian years.
4	4401 B	35	14	2001 B	17	—	—	—
5	4301	34	15	1901	16	—	—	—
5	4201	33	16	1801	15	24	200	0
6	4101	32	16	1701	14	25	300	1
6	4001 B	32	16	1601 B	14	25	400 B	1
6	3901	31	17	1501	13	25	500	2
7	3801	30	17	1401	12	26	600	3
7	3701	29	18	1301	11	26	700	4
7	3601 B	29	18	1201 B	11	26	800 B	4
8	3501	28	18	1101	10	27	900	5
9	3401	27	19	1001	9	28	1000	6
9	3301	26	19	901	8	28	1100	7
9	3201 B	26	19	801 B	8	28	1200 B	7
10	3101	25	20	701	7	29	1300	8
10	3001	24	21	601	6	29	1400	9
11	2901	23	21	501	5	0	1500	10
11	2801 B	23	21	401 B	5	0	1600 B	10
12	2701	22	22	301	4	1	1700	11
12	2601	21	23	201	3	1	1800	12
13	2501	20	23	101	2	2	1900	13
13	2401 B	20	23	1 B	2	2	2000 B	13
13	2301	19		A.D.		2	2100	14
14	2201	18	24	100	1	3	2200	15
15	2101	17	24	200	0	4	2300	16

Note.—The letter B denotes the years to be accounted leap years in new style. The corrections for Julian years and for using the cypher for the years B.C. are to be applied for the 100 years *below* and including the number indicated.

The correction for turning Gregorian years into Julian years should be 8 days in 1,040 years, giving $27\frac{2}{3}$ days in 3,600 instead of 27 days, as given in this table according to new style, reckoning from new moon to new moon; but reckoning from vernal equinox to vernal equinox the error in the new style amounts to about a day in 3,600 years instead of $\frac{2}{3}$ day.