

THURSDAY, APRIL 4, 1918.

GOURSAT'S "COURSE OF ANALYSIS," AND OTHER MATHEMATICAL WORKS.

- (1) *A Course in Mathematical Analysis. Differential Equations.* Being part ii. of vol. ii. By Prof. E. Goursat. Translated by Prof. E. R. Hedrick and Otto Dunkel. Pp. viii+300. (London: Ginn and Co., n.d.) Price 11s. 6d. net.
- (2) *Finite Collineation Groups, with an Introduction to the Theory of Groups of Operators and Substitution Groups.* By Prof. H. F. Blichfeldt. Pp. xi+194. (Chicago, Ill.: University of Chicago Press; London: Cambridge University Press, 1917.) Price 1.50 dollars net, or 6s. net.
- (3) *Introduction to the Calculus of Variations.* By Prof. W. E. Byerly. Pp. 48. (Mathematical Tracts for Physicists.) (Cambridge, Mass.: Harvard University Press; London: Oxford University Press, 1917.) Price 2s. 6d. net.

(1) THE American translation of Prof. Goursat's "Course of Analysis" will be welcome to those who may be unable to read the original easily. The present instalment covers ground in which the author is an acknowledged adept, and it illustrates his remarkable power of illuminating obscurities and giving charm to discussions which, although unavoidable, are apt to be dull. Thus his chapter on existence theorems is not only a model of rigour, but actually entertaining as well; §30, on the Cauchy-Lipschitz method, is most instructive, and illustrates the value of a diagram when properly used—not as a vehicle for a sham "intuitive proof," but as an image corresponding to a set of analytical data and deductions. Geometrical imagery of this kind is frequently used throughout, and with the happiest results—especially, it seems to us, in the part dealing with partial differential equations of the first order.

There are some features of special interest in the earlier part, which deals with ordinary equations. As an isolated gem we may note the integration of Euler's equation (pp. 23–28), especially the method which leads to Stieltjes's form of solution. The third part of the chapter on linear equations gives a capital summary of the main results obtained by Fuchs, Picard, and others; illustrations are afforded by the hypergeometric series and Lamé's equation. In the chapter on non-linear ordinary equations of the first order there are a number of valuable results, especially those based on Briot and Bouquet's researches as to equations of the form $(dy/dx)^n = R(y)$, where $R(y)$ is a polynomial in y . Here we have a list of all the cases of this type which can be satisfied by a one-valued function of x , and also—which is more important—a clear proof that there are no others.

Next comes a section on singular solutions, and an Englishman cannot help feeling surprised to find no reference to Cayley here (or, indeed, anywhere else in the volume). Readers should notice the last paragraph of §71; the point is that, if we equate the p -discriminant to zero, the normal meaning of the result is a cusp-locus (or tac-locus, or both) which does not yield a singular solution; the reason that mathematical students so often obtain a singular solution from the p -discriminant is that so many equations of the type $f(x, y, p) = 0$ are made up by eliminating a constant c from the equation of a set of algebraic curves $\phi(x, y, c) = 0$, which have an envelope.

The discussion of Charpit's method seems to us to be as good as any that can be put into a text-book. What makes it so unusually clear is that the author proves in a separate article (§81) that the condition for the compatibility of $f(x, y, z, p, q) = 0$, $\phi(x, y, z, p, q) = 0$, $dz = p dx + q dy$, is $[f, \phi] = 0$, where the symbol on the left is that introduced by Jacobi. Later on we have discussions of Cauchy's method (pp. 249–64) and of Jacobi's method (pp. 265–78). It should be added that there is a very brief account (pp. 86–98) of Lie's theory of transformation-groups.

From time to time the author pauses to make a general remark on this or that aspect of his subject, and these *obiter dicta* deserve the most careful attention. For instance: "Although this reduction is not, in many cases, of any practical utility, it nevertheless possesses great theoretical interest, for it enables us to determine just how difficult the problem is" (p. 214). Most text-books on differential equations are very misleading, because they give the student the impression that the subject is very much better understood than it really is. The most simple-looking partial differential equations may baffle the most eminent mathematicians, and it would scarcely be too much to say that there is no extensive *theory* of differential equations except for linear ordinary equations the coefficients of which are of certain specified types. This assertion is not so paradoxical as it looks; all the fundamental functions of analysis (not of arithmetic) can be defined by very simple ordinary differential equations; for instance, $\exp(x)$ is that solution of $dy/dx = y$ which has the value 1 when $x = 0$. All the properties of $\exp(x)$ can be deduced from this, and the whole of analytical trigonometry is then only a corollary.

(2) Prof. Blichfeldt collaborated with Messrs. G. A. Miller and L. E. Dickson in a work on finite groups reviewed in these columns on November 23, 1916 (vol. xcvi., p. 225). The present work, dealing with collineation groups, so far departs from abstract group-theory as to choose a special imagery, or, if you will, a drapery, for the sets of abstractions considered. Every group may be imaged as a substitution group; not every group can be represented by a collineation group. So Prof. Blichfeldt has restricted his field of inquiry, and deliberately tried not to use abstract group-theory any more than he can help. For the purpose he has in

hand this has undoubted advantages. One of these is that collineation groups form a very extensive family, which admits of geometrical or quasi-geometrical interpretation. (This book has no figures, but the reader should make illustrative figures and models for himself, and think out the arguments in as geometrical a form as possible.) So far as we can judge, the treatment is sound, though it involves some rather artificial arrangements; e.g. chap. ii. contains a good deal of *abstract* group-theory, and so far as we can see, the term "group" in its technical sense has not been anywhere defined, and on p. 31 it seems to be confounded with "set" and "class," which, if meant, is very unfortunate.

The discussion goes as far as linear groups of four variables; there is a chapter on group-characteristics (mainly, of course, after Frobenius and Burnside); there are numerous references, and a moderate number of examples. We hope the book will have a wide circulation; every advance in the theory of groups is bound to result in an advance in many other branches of mathematics.

(3) Prof. Byerly's tract will be useful to those who are interested in the classical problems of the brachistochrone, etc., and also, it may be hoped, to physical students engaged in their first struggles with the Hamiltonian equations, least action, least constraint, and so on. So far as we know, a really good elementary treatise on the calculus of variations has yet to be written; meanwhile, such an outline as this is better than many big and pretentious productions. G. B. M.

MEDICAL HISTORY AND SCIENTIFIC METHOD.

Studies in the History and Method of Science.
 Edited by Dr Charles Singer. Pp. xiv + 364.
 (Oxford: At the Clarendon Press, 1917.) Price
 21s. net.

DURING the last ten years there has been a notable revival of the study of the historical development of medicine by the scientific methods which have been applied to other branches of history. In this country there has been no more active worker than Dr. Charles Singer, who for some time has been employed in unearthing for convenient reference the medical historical treasures of the Bodleian Library. The present volume is the outcome of some of the studies of Dr. Singer and his co-workers, and must be regarded as a notable contribution to certain branches of medical history and evolution. The book is splendidly got up, and in addition to forty-one plates, many of which are excellently reproduced in colour, there are large numbers of figures in the text. It is almost remarkable that such a work should make its appearance in the fourth year of the war, and especially at the moderate price of 21s.

The text contains seven articles and studies, most of them of immediate historical interest. Dr. Singer himself contributes two of these, the first

a very learned account of the scientific views and visions of Saint Hildegard, the German religious mystic of the twelfth century. From the extensive literature which has collected round this complicated personality, Dr. Singer has managed to create a study of great interest, and has dealt in particular with her views on anatomy and physiology. In his second essay, "The 'Anothomia' of Hieronymo Manfredi," he has dealt with the hitherto unknown account of the body written in manuscript by Manfredi at the end of the fifteenth century. As he points out, this is the most complete post-medieval account of anatomy until we come to the first of the anatomists, Berengario da Carpi, who published his work in 1521. Dr. Singer publishes the whole of the Italian manuscript, and leads up to it with a masterly account of the Early Renaissance anatomy, profusely illustrated.

In his "Blessing of Cramp Rings" Dr. Raymond Crawford writes exhaustively of a treatment of epilepsy which was in vogue for hundreds of years. Although it is often regarded as springing up in the time of Edward the Confessor, it cannot be denied that the idea of applying some kind of constriction to inhibit the convulsions of epilepsy can be traced back to classical times.

One of the most interesting studies is Dr. E. T. Withington's on "Dr. John Weyer and the Witch Mania." Herein are traced the origin and development of the most extraordinary superstition which has ever disgraced the human mind, and led to the sacrifice and mutilation of vast numbers of unfortunate human beings even so late as the seventeenth century. It is particularly remarkable that the witch mania should have reached its height at a time when the Renaissance was in full tide and learning was opening men's minds. Dr. Withington considers that at least two causes co-operated for the development of this madness, viz. the development of heresies and the increasing prominence given to the supposed operations of the Evil Spirit, a doctrine supported by the pronouncements of Pope Innocent VII. in 1484. It was then that the Church called upon the civil powers to exterminate witches, and Europe rang with the cries of the innocents perishing daily on the rack and at the stake. Amidst all the ghastly shambles we have the vision of Dr. John Weyer, of Arnheim, trying to stem the tide of this mania, but with ill-success. Although his great work was published in 1563, the practice of torturing witches progressed or increased, and only finally began to die down in the seventeenth century, being finally extinguished in England so late as the beginning of the eighteenth century.

Mr. Reuben Levy contributes an article on "The 'Tractatus de causis et Indiciis morborum' attributed to Maimonides." This work was said to be by the Jewish philosopher, and was considered to be his chief claim as a medical writer. By a complete examination of the only manuscript known, Mr. Levy proves that it was by another writer altogether, and thus clears away an error.

In his essay on "Scientific Discovery and Logi-