It is for these reasons that I consider that a riveted boiler joint should be laid out so as to have an excess of plate section over that of the rivet.

It is impracticable to proportion the riveted joint so perfectly that the shearing strength of the rivet will be equal to the tearing strength of the plate, for the actual strength of the plates varies more than does the proportion of dimensions of the joint.

[*To be continued.*]

ANALYTICAL DISCUSSION OF THE TIDAL VOLUME

ADMITTED INTO BAYS AND RIVERS UNDER GIVEN CONDITIONS OF WIDTH, DEPTH AND AREA OF BASIN—RELATION BETWEEN WIDTH AND DEPTH IN THE CROSS SECTIONS OF TIDAL STREAMS WHOSE BEDS ARE YIELDING TO THE ACTION OF CURRENTS—VARIATION OF CROSS-SECTIONAL AREA IN SUCH STREAMS.

BY L. D'AURIA.

If a perfect freedom of flow existed in a tidal river of uniform width and uniform depth, the range of tide in such river would also be uniform, and the profile of the tidal wave could be identified with the curve of sines.

Imagine any portion of this ideal tidal river, from its head down to any given cross section, and let us find the analytical expression of the tidal volume which would flow through such cross section, from slack water to slack water.

In the first place we will observe that slack water in any given cross section of the tidal river under consideration would occur when either the crest or the trough of the tidal wave occupies a position midway between the given cross section and the head of the river. When the crest is in such position, we would have what is generally called *high-water slack*, and when it is the trough, we would have the *low-water slack*, through the cross section.

These two phases evidently do not correspond with high water and low water, but occur some time afterward; and it can be seen that the delay between high water and high-water slack, as well as the delay between low water and low-water slack would increase with the distance of the cross section from the river head. Theoretically, in a cross section, of which the distance from the head of the river is equal to one-half the length of the wave, or equal to the distance from crest to trough, the time interval between high water and high-water slack, or low water and low-water slack, would be equal to one-half the duration of the rise or fall of tide, or

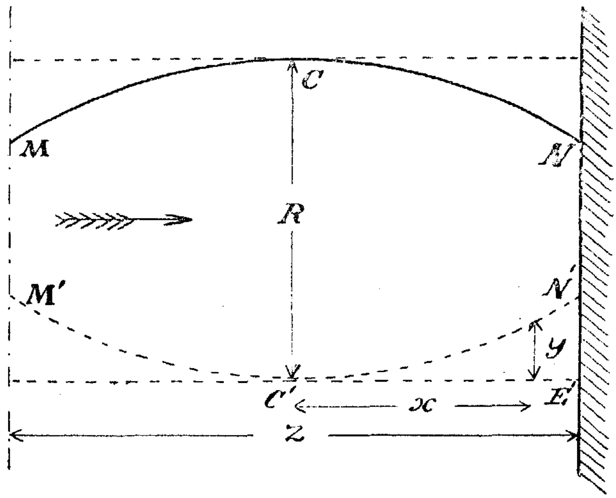

FIG. 1.

a little over three hours. Through such cross section the flood current would continue to flow for all this time after high water, and the ebb current would continue to flow for the same length of time after low water. At any other cross section, either above or below the one already considered, the time interval in question would be shorter. For instance, in a cross section of which the distance from the river head would equal the length of the wave, slack water would occur simultaneously with high and low water, and the same coincidence evidently occurs in the cross section which is right at the head of the river. All these deductions, though theoretical, represent facts generally observed in tidal rivers in a greater or less degree.

Let MCN and $M'C'N'$ (*Fig. 1*), represent the profiles of the tidal wave in the two positions which correspond to high-water slack and low-water slack, respectively, through the cross section MM', whose distance from the head of the river NN' is z; and let λ denote the length of the wave; R the height from crest to trough, and b the width of the river. It is obvious that the tidal volume passing through the cross section MM' from one slack water to the other will be

$$q = b\,(R\,z - 4 \times \text{area } C'E'N')$$

and the problem is reduced to find the area $C'E'N'$.

Let x and y represent the coördinates of any point of the curve $C'N'$ referred to C' as origin. Then will be found

$$y = R\, sin^2 \left(\frac{\pi\, x}{\lambda}\right) \tag{1}$$

and

$$\text{area } C'E'N' = R \int_0^{\frac{z}{2}} sin^2 \left(\frac{\pi\, x}{\lambda}\right) dx$$

Hence,

$$q = b\, R \left(z - 4 \int_0^{\frac{z}{2}} sin^2 \left(\frac{\pi\, x}{\lambda}\right) dx \right)$$

Integrating between the assigned limits we find, after reduction,

$$q = \frac{\lambda\, b\, R}{\pi}\, sin \left(\frac{\pi\, z}{\lambda}\right) \tag{2}$$

Let T represent the mean tidal interval in seconds or $T = 44{,}600$ seconds, approximately. Then if v denotes the mean velocity of wave propagation in the distance z we can put $\lambda = T\,v$, and write.

$$q = \frac{T\,v\,b\,R}{\pi}\, sin \left(\frac{\pi\, z}{T\,v}\right) \tag{3}$$

This expression becomes a maximum when $z = \frac{1}{2}\, v\, T$, which would show that the greatest tidal volume which can be admitted into a tidal basin whose width is b and whose length exceeds the quantity $\frac{1}{2}\ v\ T$ is

$$Q = \frac{T\,v\,b\,R}{\pi} \tag{4}$$

since it would be impossible from a physical point of view to admit that by increasing the length of the tidal basin beyond the limit $\frac{1}{2}\ v\ T$, a diminution of tidal volume would ensue. Hence, from such standpoint equation (4) ought to hold good for all cases where $z = > \frac{1}{2}\ v\ T$, while equation (3) ought to be applied to cases where $z < \frac{1}{2}\ v\ T$.

Now it may be reasonably assumed that two tidal basins of equal area but different shapes, having the same width of entrance, the same average depth and the same range of tide at the entrance would admit an equal tidal volume during each tide. With this assumption, if we denote the area of the basin by A, we can put $z = A \div b$ in equation (3), which then becomes

$$q = \frac{T\,v\,b\,R}{\pi}\ sin\ \left(\frac{\pi\,A}{T\,v\,b}\right) \tag{5}$$

with the limitation of $A < \frac{1}{2}\ v\ T\,b$; and when $A = > \frac{1}{2}\ v\ T\,b$, we have to apply equation (4).

In cases where the widths of the cross sections change from low to high water, the area A of the tidal basin will have to be computed at low water, and the width b at mid-tide. In order to appreciate the correctness of this remark, imagine first a tidal basin of uniform width from low to high water. Now, if without altering the cross section of the entrance to such basin we were to increase gradually the widths of all the other cross sections from low to high water without touching the low-water widths, it can hardly be expected that by such operation the tidal volume would be increased, although a considerable increase in the mean area of the basin has taken place. Such increase would evidently cause a reduction of tidal range in the upper cross sections, and this would compensate for the increased mean area of the basin, as far as the tidal volume is concerned.

If we denote by β the mean low water width of the tidal basin, by l its length, and by θ the average of the time intervals of high water and low water in the distance l, we can put equation (5) under the form

$$q = \frac{T\,l\,b\,R}{\pi\,\theta}\,sin\left(\frac{\theta\,\beta\,\pi}{T b}\right) \tag{6}$$

In order to test this formula, let us apply it to a well-known case where the necessary data have been well determined by observations. This case we find in the portion of the Delaware River, between Bridesburg (above Philadelphia) and Trenton, for which we have $l = 144{,}000$ feet; $b = 3{,}860$ feet; $\beta = 1{,}764$ feet; $R = 5{\cdot}98$, and $\theta = 162$ minutes. Putting $T = 762$ minutes, we obtain by substituting in formula (6)

$$q = 1{,}496{,}000{,}000 \text{ cubic feet.}$$

Now, according to an extremely accurate measurement of the tidal volume in question deduced from simultaneous observations, made by the United States Engineers, it has been found 1,452,000,000 cubic feet, which is only about three per cent. less than the theoretical.

It may be that taking into account the volume of fresh water discharged by the river, this difference would be somewhat larger, but we only claim to give the theoretical, not the actual tidal volume.

When the basin is shaped like a bay, with narrow entrance, it is rather difficult to obtain the proper values of θ and l, which are required by formula (6), to compute the tidal volume. However, if we knew the value of v in equation (5), we could use this equation instead of the (6) in computing the tidal volume, independently of θ and l. Now, if we denote by (H_0) the average depth of the basin at mid-tide, we can put approximately

$$v = 4{\cdot}81\,\sqrt{(H_0)}$$

[See "A new theory of the propagation of waves in liquids. By the author. *Journal Franklin Institute*, December, 1890.]

Then we can write:

$$q = \frac{4{\cdot}81\,b\,T\,R\,\sqrt{(H_0)}}{\pi}\,sin\left(\frac{\pi\,A}{4{\cdot}81\,b\,T\,\sqrt{(H_0)}}\right) \tag{7}$$

When

$$A = > \frac{1}{2} 4{\cdot}81\, b\, T \sqrt{(H_0)}$$

which is the case of very large bays with narrow entrances, then, instead of equation (4), we can use

$$Q = \frac{4{\cdot}81\, b\, T\, R \sqrt{(H_0)}}{\pi} \tag{8}$$

In order to test formula (7) we have taken the case of Absecom and Brigantine Bay, Atlantic City, N. J., for which we have the following data: $A = 362{,}400{,}000$ sq. ft.; $b = 1760$ ft.; $(H_0) = 9{\cdot}5$ ft.; $R = 4$ ft. Substituting these data in equation (7) we find

$$q = 1{,}482{,}000{,}000 \times sin\ 56^\circ$$

or

$$q = 1{,}228{,}000{,}000 \text{ cubic feet.}$$

By gauging, and with the same range of tide of four feet, Mr. George Daubeney, Assistant Engineer, found in 1880 a tidal volume equal to 1,174,612,000 cubic feet [see *Report of Chief Engineers U. S. Army*, 1881]; and in 1886, by simultaneous tidal observations with the same range of tide was found again, by the writer a tidal volume equal to 1,170,614,000 cubic feet. [See *Report of Chief Engineers U. S. Army*, 1887.]

Both these measurements, surprisingly near to each other, are in the mean only four per cent. less than the theoretical tidal volume computed by formula (7). Such close agreement, we believe, entitle us to view our formulæ from a physical point and trust the conclusions which may thus be reached. Now we may readily see from formula (8) that in very large tidal basins with narrow entrances the tidal volume is directly proportional to the width of entrance, showing that in large tidal bays it would be useless to change the width of entrance with a view to increase the velocity of the currents through such entrance; and as far as the upper navigation of such bays is concerned, it teaches that by narrowing the width of entrance we cut away from the various channels an amount of flow proportionately to the reduction of width of entrance effected.

In tidal rivers, however, where formula (7) is applicable, this formula would show that the tidal volume varies with the quantity

$$\phi = b \sin \left(\frac{\pi A}{4{\cdot}81\ b\ T \sqrt{(H_0)}} \right) \tag{9}$$

other things remaining the same. Now, it can be seen that this quantity is only feebly affected by b when the area of the basin A is small compared with

$$2{\cdot}4\ b\ T \sqrt{(H_0)}$$

In fact, suppose for instance that we should find

$$\frac{A}{2{\cdot}4\, b\, T \sqrt{(H_0)}} = \tfrac{1}{3}$$

then the tidal volume would be proportional to $\frac{1}{2}\ b$. If instead of b we substitute, say $\frac{2}{3}\ b$ in (9), then the tidal volume would become proportional to $0{\cdot}47\, b$, showing, therefore, a loss of only six per cent. of tidal volume against a reduction of thirty-three per cent. of the width of entrance.

Let ϕ_1 represent the value of ϕ corresponding to the width $b_1 < b$; then the loss of tidal volume incurred would be expressed by

$$j = \left(1 - \frac{\phi_1}{\phi} \right) q$$

In the above argument we have tacitly supposed that the reduction of width amounts to a contraction of the river at a particular point, forming a kind of bay above it; but where only a rectification of the banks of the river is contemplated no loss of tidal volume can possibly follow.

In projecting improvements of tidal rivers it has always been a perplexing question how far the tidal volume might be affected. We can answer this question now, fully, by consulting our formulæ (9) and (10), when the necessary data are given; and in a general way we have learned that where the ratio

$$A \div 2{\cdot}4\ b\ T \sqrt{(H_0)}$$

is considerably smaller than *one*, say $\frac{1}{4}$ or $\frac{1}{3}$, which would be the case of the upper sections of tidal rivers, then a contraction of width may be effected, practically without loss of tidal volume, a conclusion which may save a great amount of discussion relating to the improvement of such sections of tidal rivers.

In tidal rivers whose beds are yielding to the action of currents the depth of the cross sections must bear some relation to the mean velocity of the stream. Let u represent this velocity through a cross section Ω whose depth is H, and put $H = cu^n$, in which n is to be determined, and c is a constant. Now, from formula (7), we can compute

$$u = \frac{4{\cdot}81\ R \sqrt{(H_0)}}{\pi H} \sin\left(\frac{\pi A}{4{\cdot}81\ b\ T \sqrt{(H_0)}}\right) \qquad (10)$$

and consequently we find

$$H = c_1\ R^{\frac{n}{n+1}}\ (H_0)^{\frac{n}{2(n+1)}}\ \sin^{\frac{n}{n+1}}\left(\frac{\pi A}{4{\cdot}81\ b\ T \sqrt{(H_0)}}\right) \qquad (11)$$

This formula when applied to the Delaware River led the writer to the conclusion $n = 2$, hence

$$H = c_1\ R^{\frac{2}{3}}\ (H_o)^{\frac{1}{3}}\ \sin^{\frac{2}{3}}\left(\frac{\pi A}{4{\cdot}81\ b\ T \sqrt{(H_0)}}\right) \qquad (12)$$

Changing the width b of the cross section Ω, this formula offers the corresponding mean depth when other things are given. Let, for instance, b_1 and H_1 represent the new width and the new depth, we would have

$$\frac{H_1}{H} = \left(\frac{R_1}{R}\right)^{\frac{2}{3}} \cdot \frac{\sin^{\frac{2}{3}}\left(\frac{\pi A}{4{\cdot}81\ b_1\ T \sqrt{(H_0)}}\right)}{\sin^{\frac{2}{3}}\left(\frac{\pi A}{4{\cdot}81\ b\ T \sqrt{(H_0)}}\right)} \qquad (13)$$

When A is small compared with

$$2{\cdot}4\ bT \sqrt{(H_0)}$$

and the ratio does not exceed, say $\frac{1}{2}$, we can write simply

$$\frac{H_1}{H} = \left(\frac{R_1}{R}\right)^{\frac{2}{3}} \left(\frac{b}{b_1}\right)^{\frac{2}{3}} \qquad (14)$$

If we multiply equation (12) by b we have

$$\Omega = c_1\ b\ R^{\frac{2}{3}}\ (H_0)^{\frac{1}{3}}\ \sin^{\frac{2}{3}}\left(\frac{\pi A}{4{\cdot}81\ b\ T \sqrt{(H_0)}}\right) \qquad (15)$$

or, remembering that we have expressed the velocity of propagation by

$$v = 4{\cdot}81 \sqrt{(H_0)}$$

and that $A = z\,\beta$ in which z is the length and β the average width of the basin, we can readily find

$$\Omega = c_2\, b\, R^{\frac{2}{3}} \left(\frac{z}{\theta}\right)^{\frac{2}{3}} \sin^{\frac{2}{3}} \left(\frac{\beta\, \theta\, \pi}{b\, T}\right) \tag{16}$$

For the upper portion of a tidal river we find θ considerably smaller than T, and therefore, for such case, we can assume the sine proportional to

$$\left(\frac{\beta\, \theta}{b}\right)$$

and write

$$\Omega = c_3\, R^{\frac{2}{3}}\, z^{\frac{2}{3}}\, \beta^{\frac{2}{3}}\, b^{\frac{1}{3}} \tag{17}$$

and

$$\frac{\Omega_1}{\Omega} = \left(\frac{R_1\, \beta_1}{R\, \beta}\right)^{\frac{2}{3}} \left(\frac{z_1}{z}\right)^{\frac{2}{3}} \left(\frac{b_1}{b_1}\right)^{\frac{1}{3}} \tag{18}$$

When the cross sections are not too far apart we can consider the mean width of the basin and the range of tide to remain constant, and then we have

$$\frac{\Omega_1}{\Omega} = \left(\frac{z_1}{z}\right)^{\frac{2}{3}} \left(\frac{b_1}{b}\right)^{\frac{1}{3}} \tag{19}$$

In order to test this relation we have selected three pairs of cross sections from the Delaware River, each pair exhibiting a great variation of width in a short distance, and the results are as follows:

Fisher's Point:

$\Omega = 52{,}927$ sq. ft.; $b = 2{,}610$ ft.; $z = 144{,}000$ ft.

Elevator Wharf:

$\Omega_1 = 67{,}380$ sq. ft.; $b_1 = 4{,}240$ ft.; $z_1 = 154{,}000$ ft.

$$\frac{\Omega_1}{\Omega} = 1{\cdot}27;\ \left(\frac{z_1}{z}\right)^{\frac{2}{3}} \left(\frac{b_1}{b}\right)^{\frac{1}{3}} = 1{\cdot}23.$$

Gloucester:

$\Omega = 53{,}625$ sq. ft.; $b = 1{,}800$ ft.; $z = 186{,}000$ ft.

Horse Shoe:

$$\Omega_1 = 91{,}380 \text{ sq. ft.}; \; b_1 = 5{,}850 \text{ ft.}; \; z_1 = 196{,}000 \text{ ft.}$$

$$\frac{\Omega_1}{\Omega} = 1{\cdot}70; \left(\frac{z_1}{z}\right)^{\frac{2}{3}} \left(\frac{b_1}{b}\right)^{\frac{1}{3}} = 1{\cdot}53.$$

Deep Water Point:

$$\Omega = 140{,}500 \text{ sq. ft.}; \; b = 4{,}860 \text{ ft.}; \; z = 326{,}000 \text{ ft.}$$

New Castle:

$$\Omega_1 = 182{,}200 \text{ sq. ft.}; \; b_1 = 8{,}400 \text{ ft.}; \; z_1 = 344{,}000 \text{ ft.}$$

$$\frac{\Omega_1}{\Omega} = 1{\cdot}30; \left(\frac{z_1}{z}\right)^{\frac{2}{3}} \left(\frac{b_1}{b}\right)^{\frac{1}{3}} = 1{\cdot}24.$$

On examination it will be found that at Gloucester the bottom is rather hard compared with the other cross section, a circumstance which seems to account for the difference of ten per cent. found in the second case between the actual and the theoretical result. The small difference found in the other two cases, though utterly insignificant in computations of this kind, still is in a direction which is in accordance with the factor neglected in formula (18). Had this factor been computed a still closer agreement would have been found between the results of observation and those obtained by theory.

ON A MAXIMUM STEAM-JACKET EFFICIENCY.

BY ROBERT H. THURSTON.

(1) IDEAL EFFICIENCY.—The fact is sufficiently well known that the steam jacket, as employed on the steam engine, of whatever form and arrangement, is intrinsically a wasteful element, and that its use only gives, in certain cases, an economical advantage by its repression of wastes of larger magnitude. It checks a serious and unavoidable waste, more or less completely, by a process which as inevitably involves a waste which is commonly, but, perhaps, not invariably, a lesser one. The ideal steam engine, such as is treated of in the purely thermodynamic study of the steam engine, has a lower efficiency with, than it has with-