On Magnetic Precession

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XLIV. *On Magnetic Precession.* *By* ARTHUR SCHUSTER, *F.R.S., Professor of Physics at the Owens College, Manchester* *.

1. IN a previous paper † I discussed the possible effects of electric inertia, confining myself to the case of conductors at rest. But Hertz, in his interesting experiments on the subject, showed that the most delicate method of investigating the influence of inertia is based on the apparent electromotive forces which are introduced by the motion of conductors. If electricity possesses inertia, the rotation of a body through which currents pass, affects the flow of these currents in the same manner as the earth's rotation affects the direction of currents of air on its surface.

Hertz obtained only negative results, but could fix an upper limit to the possible inertia of electric currents. It occurred to me that this inertia, even if below the limits given by Hertz, might show accumulated effects, when the currents last for a sufficient time. If the earth's magnetism be due to electric currents, general considerations suggested to me that the effects of inertia might explain the secular variation. The following investigation shows that indeed inertia would cause a "magnetic precession" precisely of the character of the secular variation, but that this precession would be very much slower than the variations which are actually observed.

2. If m is the *mass* of positive electricity in unit volume, and u the velocity, the energy of an electric current, so far as it depends on this mass, would be mu^2 per unit volume, assuming for the sake of simplicity that positive and negative electricity move with the same velocity. If i is the current-density, we may substitute $\frac{1}{2}\mu i^2$ for this energy, where μ has the same meaning as in my previous paper. If q is the quantity of positive electricity in unit volume the current-

* Read December 14, 1900.

† *Ante*, p. 629.

density is $2qu$, so that

$$4q^2u^2 = i^2,$$

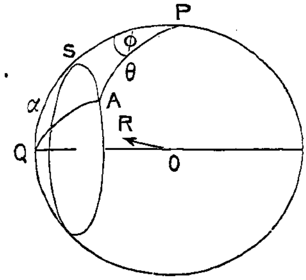
$$\therefore \quad 2q^2\mu = m.$$

The momentum of positive electricity will be mu or $q\mu i$.

If a mass moves relatively to a body which is rotating, we may treat the effect of the rotation as being the same as that due to certain fictitious forces. These fictitious forces are of two kinds. The first, depending on the square of the angular velocity and commonly called centrifugal force, will act equally on positive and negative electricity, and cannot therefore produce any effect on the distribution of electric currents. The second force depends on the relative velocity, it will therefore have opposite effects on positive and negative electricity, in other words it will be equivalent to an electromotive force. Its direction is at right angles both to the direction of relative velocity and to the axis of rotation, and its intensity per unit mass is equal to $2\omega v_r \sin\chi$, where ω is the angular velocity, v_r the vector representing the relative velocity, and χ the angle between v_r and the axis of rotation. The direction of the force is such that the displacement of the body through a right angle in the direction of rotation would bring the direction of the force into the plane containing the axis of rotation and the direction of relative velocity.

If electric currents are confined to the surface of a sphere rotating with an angular velocity ω, we may calculate the components of electromotive force produced by the rotation of the sphere, but it is only the tangential components which will produce any effect. For any point on the earth which has a colatitude θ, the horizontal components will be the same, as if the angular velocity were $\omega \cos\theta$ about the vertical. If the flow is from north to south, the force will be to the west in the northern hemisphere, and its intensity per unit volume will be $2m\omega \cos\theta\, u_S$, u_S denoting the velocity to the south. For flow u_E from west to east the force will be $2m\omega \cos\theta\, u_E$ and act from north to south in the northern hemisphere. Hence, if we take the directions to south and to east as the positive directions the two forces will be $-2m\omega \cos\theta\, u_S$ and $+2m\omega \cos\theta\, u_E$. These are the forces per unit volume; to obtain the forces per unit quantity of electricity we divide by q, and replacing the velocities of flow by the current-densities we have for the two components $-2\mu\omega i_S \cos\theta$ and $2\mu\omega i_E \cos\theta$. These forces are of the nature of electric forces, and their effects may be calculated if the components of i are known.

Fig. 1.



3. In the simplest and most important case, the currents are such as to produce a magnetic field identical on the outside with that of a uniformly magnetized sphere, the axis of magnetization passing through the equator. Let O Q (fig. 1) be the axis of magnetization. The currents will all lie in planes at right angles to that axis. If A S is a portion of such a current and we write :

α, for the arc QS or QA ;
ϕ, for the angle S P A ;
θ, for the colatitude P A ;
a, for the radius of the sphere;

the current function will be

$$a\mathrm{I}\cos\alpha = \mathrm{I}a\cos\phi\sin\theta.$$

Hence the component currents are :

Easterly component

$$= -\frac{d}{d\theta}\mathrm{I}\cos\phi\sin\theta = -\mathrm{I}\cos\phi\cos\theta \quad . \quad . \quad . \quad (1)$$

Southerly component

$$= \frac{d}{\sin\theta\, d\phi} - \mathrm{I}\cos\phi\sin\theta = -\mathrm{I}\sin\phi. \quad . \quad . \quad (2)$$

These are components of currents per unit length of linear cross-section at any point; to obtain current-densities we must divide by t, the thickness of the shell through which we imagine the currents to flow. If we write Ψ_S and Ψ_E for the electric forces acting towards the south and east respectively, we have therefore

$$\Psi_S = -2\mu\omega\mathrm{I} \,.\, \cos^2\theta\cos\phi/t,$$
$$\Psi_E = 2\mu\omega\mathrm{I}\cos\theta\sin\phi/t.$$

Now

$$2\cos^2\theta\cos\phi = \cos\phi(1+\cos 2\theta) = \cos\phi + \frac{d}{d\theta}(\tfrac{1}{2}\cos\phi\sin 2\theta)$$

and

$$2\cos\theta\sin\phi = \cos\theta\sin\phi - \frac{d}{\sin\theta\, d\phi}(\tfrac{1}{2}\cos\phi\sin 2\theta).$$

Hence we may divide the electric forces into two parts, one of which may be derived from a potential. This latter portion will produce electrification and ultimately be counterbalanced by electric charges having a potential equal to $-\frac{1}{2}\mu\omega a\,\mathrm{I}\cos\phi\sin 2\theta/t$. The remaining portion of the electric forces will be

$$\Psi_E = \mu\omega\mathrm{I}\cos\theta\sin\phi/t, \quad . \quad . \quad . \quad . \quad (3)$$

$$\Psi_S = -\mu\omega\mathrm{I}\cos\phi/t. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

These electric forces will tend to produce currents which are of the same type as those assumed to exist, but turned through a right angle in a direction opposed to that of the angular velocity. This is seen by comparing equations (1) and (2) with (3) and (4), and noting that the latter becomes proportional to the former when $\left(\phi + \frac{\pi}{2}\right)$ is substituted for ϕ. Hence the effect of these forces will be to add a system of currents which will have the same effect as a rotation of the original system in a direction opposite to that of the rotating sphere.

4. To show that the whole system of currents will rotate in the body and to determine the period of rotation some further calculations are necessary. The system of currents we are considering will produce a uniform magnetic field, M, within the sphere, which is equal to $\frac{8}{3}\pi\mathrm{I}$. The energy of the total magnetic field is easily found to be

$$\tfrac{1}{2}\mathrm{M}^2a^3 = \frac{32}{9}\pi^2\mathrm{I}^2a^3.$$

Now imagine such a system of currents as we have been considering, in which the current crossing an element ds_1 of the arc QA (fig. 1) is $\mathrm{I}\sin\alpha\, ds_1$, and let electric forces equal to $\mathrm{A}\sin\alpha$ act at each point on the system of currents. If ds_2 is an element of the line along which the forces act, the rate of doing work in the surface element $ds_1 ds_2$ is $\mathrm{AI}\sin^2\alpha\, ds_1 ds_2$. Hence the rate of doing work over the whole sphere is

$$\int \mathrm{AI}\sin^2\alpha\, ds_1\, ds_2 = 2\pi \mathrm{AI}a^2 \int_0^\pi \sin^3 \alpha d\alpha$$
$$= \frac{8\pi}{3}\mathrm{AI}a^2.$$

The currents will increase in intensity and the rate of doing work must be equal to the rate of increase of energy. Hence, τ denoting time

$$\frac{32}{9}\pi^2 a^3 \frac{d}{d\tau}\mathrm{I}^2 = \frac{8\pi}{3}\mathrm{AI}a^2,$$

or

$$\frac{d\mathrm{I}}{d\tau} = \frac{3\mathrm{A}}{8\pi a}.$$

This equation will determine the rate of increase of the system of currents due to a given system of forces of corresponding type.

In the system of currents with OQ as axis, we may take the current-intensity I, at the points at which $\alpha = \frac{\pi}{2}$, to be the variable, which is now to be considered a function of the time τ. We also take I′ to be the corresponding variable of a system of similar currents having OR as axis. Introducing the forces due to the rotation of the sphere, we find that

$$\frac{d\mathrm{I}'}{d\tau} = \frac{3}{8\pi a t}\mu\omega \mathrm{I},$$

$$\frac{d\mathrm{I}}{d\tau} = -\frac{3}{8\pi a t}\mu\omega \mathrm{I}'.$$

From which we deduce

$$\mathrm{I} = \mathrm{I}_0 \cos \Omega\tau,$$

$$\mathrm{I}' = \mathrm{I}_0 \sin \Omega\tau,$$

where $\Omega = \frac{3}{8\pi}\frac{\mu\omega}{at}$, and I_0 is the initial value of I. Ω is the angular velocity with which the whole system of currents revolves.

5. The rate of rotation of a system of currents in a rotating spherical sheet which has been determined in the

simplest case, can also be calculated when the distribution of currents is of a more complicated character. Let there be a current function $a\Phi$ on a sphere of radius a from which the current-intensities are derived, so that if u_S and u_E are the currents in the direction of the axes of X and Y respectively:

$$u_S = a\frac{d\Phi}{dy},\ u_E = -a\frac{d\Phi}{dx}.$$

Introducing polar coordinates and taking $dx = a d\theta$, $dy = a\sin\theta d\phi$, we find as in the previous paragraph that the electric forces Ψ_E and Ψ_S acting towards the south and east respectively are expressed by

$$\Psi_S = 2\mu\omega u_E \cos\theta/t$$
$$\Psi_E = -2\mu\omega u_S \cos\theta/t,$$

or

$$\Psi_S = -\frac{2\mu\omega\cos\theta}{t}\frac{d\Phi}{d\theta}, \quad . \quad . \quad . \quad . \quad (5)$$

$$\Psi_E = -\frac{2\mu\omega\cot\theta}{t}\frac{d\Phi}{d\phi}. \quad . \quad . \quad . \quad . \quad (6)$$

Put

$$\cos\theta\frac{d\Phi}{d\theta} = -\kappa\frac{d^2}{\sin\theta d\phi^2}\Phi + \mathrm{H}, \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\cot\theta\frac{d\Phi}{d\phi} = \kappa\frac{d^2}{d\theta d\phi}\Phi + \mathrm{K}, \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where κ is a constant and H and K functions of ϕ and θ, which are to be determined.

If an operation is performed with the equations (7) and (8) which may be represented by the symbol

$$\frac{d(7)}{d\phi} - \sin\theta\frac{d(8)}{d\theta},$$

the left-hand side of the resulting equation becomes

$$\frac{d}{d\phi}\cos\theta\frac{d\Phi}{d\theta} - \frac{d}{d\theta}\cos\theta\frac{d\Phi}{d\phi} = \sin\theta\frac{d\Phi}{d\phi};$$

on the right-hand side we have

$$-\kappa\left[\frac{d^3}{\sin\theta d\phi^3} + \frac{d}{d\theta}\sin\theta\frac{d^2}{d\theta d\phi}\right]\Phi + \frac{d\mathrm{H}}{d\phi} - \frac{d\mathrm{K}\sin\theta}{d\theta}$$

Hence

$$\frac{d\mathrm{H}}{d\phi}-\frac{d\mathrm{K}\sin\theta}{d\theta}=\kappa\sin\theta\frac{d}{d\phi}\left[\frac{d^2}{\sin^2\theta d\phi^2}+\frac{d}{\sin\theta d\theta}\sin\theta\frac{d}{d\theta}+\frac{1}{\kappa}\right]\Phi$$

The right-hand side vanishes, if Φ is a surface harmonic of degree n, provided that $n(n+1)=1/\kappa$.

Hence we may put in that case,

$$\mathrm{H}=\frac{d\mathrm{Q}}{d\theta},\quad \mathrm{K}=\frac{d\mathrm{Q}}{\sin\theta d\phi}.$$

Equations (7) and (8) will now become

$$\Psi_{\mathrm{S}}=\frac{2\mu\omega}{t}\left(\frac{1}{n\,.\,n+1}\frac{d^2}{\sin\theta d\phi^2}\Phi-\frac{d\mathrm{Q}}{d\theta}\right),\quad . \quad . \quad (9)$$

$$\Psi_{\mathrm{E}}=-\frac{2\mu\omega}{t}\left(\frac{1}{n\,.\,n+1}\frac{d^2}{d\theta d\phi}\Phi+\frac{d\mathrm{Q}}{\sin\theta d\phi}\right). \quad (10)$$

As Φ only depends on ϕ in so far as it contains terms which have $\cos\sigma\phi$ or $\sin\sigma\phi$ as factors, the effect of differentiating with respect to ϕ is the same as a multiplication by σ and a change of $\sigma\phi$ to $\sigma\phi+\frac{\pi}{2}$. Hence the terms depending on Φ will be proportional to the original current-intensities if $\sigma\phi$ is replaced by $\sigma\phi+\frac{\pi}{2}$. In other words the currents which the forces Ψ_{S} and Ψ_{E} tend to produce are of the same type as the original currents, but turned through an angle $\frac{\pi}{2\sigma}$ round the axis of rotation, in a direction opposite to that of the angular velocity of the body. If the original current function has been proportional to $\cos\sigma\phi$ or to $\sin\sigma\phi$, inertia will tend to produce currents of the same type but proportional to $-\sin\sigma\phi$ or to $\cos\sigma\phi$ respectively. The final effect of these will be a rotation of the system of currents.

6. The terms depending on Q in (9) and (10) will not produce any permanent currents, but an electrification having $-2\mu\omega\mathrm{Q}a/t$ for potential. We obtain Q from (8), by substituting $\mathrm{K}=d\mathrm{Q}/\sin\theta d\phi$. After integration with respect to ϕ, it is thus found that

$$\mathrm{Q}=\Phi\cos\theta-\kappa\sin\theta\frac{d\Phi}{d\theta}.$$

This may be put into the standard form of tesseral harmonics, if we write

$$\Phi = T_n^\sigma \cos \sigma\phi$$

and

$$T_n^\sigma = \sin^\sigma\theta \frac{d^\sigma P_n}{d\lambda^\sigma},$$

where $\lambda = \cos\theta$ and P_n stands for the zonal harmonic of degree n.

By differentiation we obtain

$$\sin\theta \frac{d}{d\theta} T_n^\sigma = \sigma \cos\theta \, T_n^\sigma - \sin\theta \, T_n^{\sigma+1}.$$

We have also the following general equations :—

$$(2n+1)\sin\theta T_n^{\sigma+1} = (n+\sigma+1)(n+\sigma)T_{n-1}^\sigma - (n-\sigma+1)(n-\sigma)T_{n+1}^\sigma ;$$

$$(2n+1)\mu T_n^\sigma = (n-\sigma+1)T_{n+1}^\sigma + (n+\sigma)T_{n-1}^\sigma.$$

Combining these equations we obtain

$$n \, . \, n+1 \, . \, 2n+1 Q = (n-\sigma+1)n^2 T_{n+1}^\sigma \cos\sigma\phi + (n+1)^2(n+\sigma)T_{n-1}^\sigma \cos\sigma\phi.$$

7. To obtain the angular velocity of the system of currents, we may proceed as in the simple case, which has already been discussed.

A current-function $a\Phi_n$ of degree n produces a magnetic potential which inside the sphere is equal to

$$-\frac{n+1}{2n+1} 4\pi a \left(\frac{r}{a}\right)^n \Phi,$$

and in the outer space

$$\frac{n}{2n+1} 4\pi a \left(\frac{a}{r}\right)^{n+1} \Phi.$$

The energy of magnetic stress is easily calculated from this and found to be

$$4\pi a \frac{n \, . \, n+1}{2n+1} \int \Phi^2 dS.$$

If there is a force-function Φ' from which the electric forces are derived in the same way as the currents from the current-function, the rate of doing work in a rectangular element bounded by the linear elements $ad\theta$ and $a\sin\theta d\phi$

will be

$$\left(\frac{d\Phi}{\sin\theta d\phi}\frac{d\Phi'}{\sin\theta d\phi}+\frac{d\Phi}{d\theta}\frac{d\Phi'}{d\theta}\right)dS.$$

But

$$\int\frac{d\Phi}{\sin\theta d\phi}\cdot\frac{d\Phi'}{\sin\theta d\phi}dS=\int\frac{\sigma^2}{\sin^2\theta}\Phi\Phi' dS,$$

as Φ and Φ' only contain ϕ in the form of a factor $\cos\sigma\phi$ or $\sin\sigma\phi$.

By partial integration, if again $\lambda=\cos\theta$,

$$\int_1^{+1}\sin^2\theta\frac{d\Phi}{d\lambda}\cdot\frac{d\Phi'}{d\lambda}d\lambda=-\int_{-1}^{+1}\Phi\frac{d}{d\lambda}\sin^2\theta\frac{d\Phi'}{d\lambda}d\lambda;$$

so that the total rate of doing work will be

$$\int\Phi\left\{\frac{\sigma^2}{\sin^2\theta}\Phi'-\frac{d}{d\lambda}\sin^2\theta\frac{d\Phi'}{d\lambda}\right\}dS,$$

which by the characteristic equation of tesseral harmonics becomes

$$n\,.\,n+1\int\Phi\Phi' dS.$$

The rate of doing work is equal to the rate of increase of energy; hence

$$\frac{8\pi a}{2n+1}\frac{d\Phi}{d\tau}=\Phi'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This equation connects the force-function Φ' with the corresponding current-function Φ.

Now let there be two current-functions, $a\Phi_1$ and $a\Phi_2$, both of degree n and differing only in so far as Φ_1 contains the factors $\cos\sigma\phi$, and a function of the time, and Φ_2 contains the factors $\sin\sigma\phi$, and some other function of the time.

The current-function $a\Phi_1$ will call forth a force-function $-\frac{2\mu\omega}{tn\,.\,n+1}a\Phi_1$ of the same type as Φ_2, and the current-function $a\Phi_2$ a force-function $\frac{2\mu\omega}{tn\,.\,n+1}a\Phi_2$ of the same type as Φ_1.

So that substituting in (11),

$$\frac{8\pi a}{2n+1}\frac{d\Phi_1}{d\tau}=\frac{2\mu\omega}{t}\cdot\frac{1}{n\,.\,n+1}\Phi_2,$$

$$\frac{8\pi a}{2n+1}\frac{d\Phi_2}{d\tau}=-\frac{2\mu\omega}{t}\cdot\frac{1}{n\,.\,n+1}\Phi_1.$$

It follows from this, that Φ_1 and Φ_2 are proportional to $\cos \Omega\tau$ and $-\sin \Omega\tau$ where

$$\Omega = \frac{\mu}{at} \cdot \frac{2n+1}{n \,.\, n+1} \cdot \frac{\omega}{8\pi}. \quad . \quad . \quad . \quad . \quad (12)$$

This gives the periodicity of the forces. The complete current-function will now be proportional to

$$\cos \sigma\phi \cos \Omega\tau - \sin \sigma\phi \sin \Omega\tau = \cos (\sigma\phi - \Omega\tau),$$

$$= \cos \sigma\left(\phi - \frac{\Omega\tau}{\sigma}\right);$$

so that the angular velocity with which the currents revolve is Ω/σ.

8. I have so far only dealt with electric currents confined to a spherical shell, as this problem adapts itself most easily to mathematical treatment. But in the simplest and most important case we may without difficulty obtain the solution for currents which circulate in the body of a conducting sphere. We take the case that the currents are such as would produce outside magnetic effects which can be represented by a magnetic potential of degree $-(n+1)$. Dividing the sphere into concentric shells we may neglect radial currents, and the currents within each shell may be represented by a current-function which is proportional to a surface harmonic of degree n. If we write $t\Phi$ instead of Φ in the previous investigation, the differential coefficients of this function will give current-densities instead of currents per unit linear cross-section, and we may apply equations (5) and (6), leaving t out of the denominator. The investigation of § 7 may be replaced by a simple application of a result given in Prof. Lamb's paper on electric currents in a sphere*. If electric currents are once started in a solid they will gradually die out owing to electric resistance, and if these currents are represented by a current-function, as assumed, the time-factor will be of the form $e^{-\lambda\tau}$, where the value of λ is given by Prof. Lamb for some of the simpler types of currents. The forces per unit volume which act on the currents under these circumstances are $-\rho i$, where i is the

* Phil. Trans. 174, p. 519 (1888).

current-density and ρ the resistivity ; the corresponding rate of diminution of current will be λi.

Hence if Φ represents the current-function and Φ' the corresponding force-function, we may put $\Phi' = -\rho i$ and $\frac{d\Phi}{d\tau} = -\lambda i$, and derive

$$\frac{d\Phi}{d\tau} = \frac{\lambda}{\rho}\Phi'.$$

This shows that in equation (12) we must replace $\frac{2n+1}{8\pi at}$ by $\frac{\lambda}{\rho}$. The angular velocity of the rotating currents then becomes

$$\frac{\Omega}{\sigma} = \frac{\mu\omega}{n \,.\, n+1} \cdot \frac{\lambda}{\rho\sigma}.$$

For $n = 1$, we have for the simplest case $\lambda^{-1} = \frac{4}{\pi}\frac{a^2}{\rho}$, so that

$$\Omega = \frac{\mu\pi}{2a^2}\omega. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

9. We may now apply the results obtained to the case of electric currents which we may imagine to circulate in the earth. If terrestrial magnetism is due to such currents, we may represent them by a superposition of different systems, each system producing magnetic forces on the outside, the potential of which is represented by a spherical harmonic. As regards the currents which give rise to zonal harmonics, they must flow in circles at right angles to the axis of rotation and the revolution of the earth cannot affect them. The tesseral harmonics will revolve relatively to the earth in a direction opposite to that of its own rotation. The effects are therefore such as are actually observed. But the calculated angular velocity is much too small to explain the secular variation.

Taking the case of a solid sphere first. If $\mu = 1$, which is a possible case if the conductivity is of an electrolytic character,

$$\Omega = \frac{\pi}{2a^2}\omega = \frac{\pi}{8 \times 10^{17}}\omega.$$

Hence the time of revolution of the magnetic system would be $2 \cdot 5 \times 10^{17}$ days or about 7×10^{14} years. No admissible value of μ could reduce this number very materially. If the currents could be imagined to be confined to a sheet of thickness t, the angular velocity would be increased to

$$\Omega = \frac{3}{8\pi}\frac{\mu}{at}\omega.$$

Taking $\mu/t=1$ this would give for the period of revolution $8\pi a/3$ or as $2\pi a = 4\times 10^9$ the periodic time would be 5×10^9 days, or 14×10^6 years, which is still of a much bigger order of magnitude than the period of the secular variation. To produce a revolution of the magnetic system of *e.g.* 500 years t/μ would have to be equal to 36×10^{-6}, so that for possible values of μ, t would have to be reduced to molecular dimensions.

10. The fact that a current sheet of molecular dimensions would show a magnetic precession of angular velocity comparable with that of the secular variation suggests the possibility of the phenomenon being rather of a molecular than a molar character. If terrestrial magnetism is due to the rotation of the electron round the atom, a precession of the required amount might be produced. But the subject is not capable of theoretical treatment without making some assumptions of too speculative a character to be introduced here.

An experimental investigation would be more likely to help, for it might give an answer to the question :—" Does the magnetic axis of a transversely magnetized sphere or of a steel disk magnetized along a diameter, lag behind when the sphere or disk is rotating rapidly ? " Some trials which were made, under not very suitable conditions, have given me a negative answer to the question, but I hope to be able to arrange for more decisive experiments.

11. In the above investigation it has been assumed that electric currents are conveyed equally by the two opposite electricities, so that the current as a whole has no moment of momentum, but as far as I can see without detailed calculation the general result of the investigation would not be altered, even assuming the existence of angular momentum. As far as the results of this investigation are concerned, we must conclude that the inertia of electric currents, which doubtless does exist, is too small to account for the observed secular variation of terrestrial magnetism.

Discussion.

Prof. Rücker congratulated the author upon his attempt to solve the problem of terrestrial magnetism, and expressed the hope that further calculation would throw more light upon this difficult subject.

Mr. Blakesley asked if the time of the secular variation would be altered if the interior of the earth were liquid or solid.

Prof. Lodge observed that the precession was rapid in the case of a thin enough layer, and mentioned J. J. Thomson's notion that the electrons were thrown out by centrifugal force, and formed a layer of molecular thinness. Hertz, in his experiments on electricity, had looked for material inertia besides electromagnetic inertia. In the present theory the distinction disappears, and there is only one inertia, and that electromagnetic, though not in the ordinary sense as contributing to self-induction.

Prof. Ayrton said if the two forms of inertia were electromagnetic, he would like to know why in detecting the second form it was necessary to associate it with an absorption of energy as in the case of an electrodeless discharge. In the case of ordinary self-induction there is no absorption of energy, and if there is absorption in the second form, and if they are both electromagnetic, he would like to know the difference between the two.

Prof. Schuster, replying to Mr. Blakesley, said that if the interior of the earth were treated as liquid, the period of the cycle would be about one hundred times less. In reply to Prof. Ayrton, he said he had only cited one experiment to show that a phenomenon explained by the gas being a good conductor could also be explained by its electric inertia. It was impossible to say in general whether self-induction caused an absorption of energy or not.