SCIENCE

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MSS. intended for publication and books, etc., intended for review should be sent to the responsible editor, Professor J. McKeen Cattell, Garrison-on-Hudson, N. Y.

*MEASUREMENT AND CALCULATION.**

IN my address of a year ago I sought, in a summary way, and by concrete illustration, to indicate how science originates in and advances with observation and experiment. I would now invite your attention to a similar consideration of the rôle which measurement and calculation play in the higher developments of science.

All sciences are at first qualitative. They pass in their growth from the fact-gathering stage of unrelated qualities to the orderly stage of related qualities and thence upward to the stage of quantitative correlation under theory. Such, at any rate, has been the course of all sciences hitherto developed, and it seems safe to predict that such will be the course of those which may arise in the future. The recognition of this fact is of prime importance. It helps us to understand the great relative diversity in perfection among the sciences; it affords a basis for rational optimism with respect to the continued progress of science; and it ought to make the specialists of the older sciences less contemptuous than they sometimes are in their attitude toward the newer ones which have not yet passed the 'rock-naming and bug-hunting stage.'

Whenever a quantitative relation be-

* Address of the retiring President of the New York Academy of Sciences, read February 24, 1902.

tween the factors of phenomena is observed, then measurements may be made in response to the question, What is the magnitude of the relation, if constant, or what are the extent and law of variation of the relation if it is not constant? When the law of relation is known, related quantities are subject to calculation, the measured values of some of them sufficing, through computation, to give the values of the others. All calculations, therefore, presuppose a knowledge of the laws of connection of related quantities, or quantitative theories of the phenomena considered.

Measurements and calculations are of all grades of definiteness, ranging from the smallest probabilities of the doctrine of chances up to the rigorous certainties of mathematical deduction. Thus the degree of precision attainable in the measured and computed quantities of a science is commonly taken as a gauge of its perfection. But it would be a mistake to infer complete perfection from the precision attainable in one or more branches of science. Astronomy, for example, is a marvelously perfect science in certain of its branches, but nevertheless some of its fundamental constants, notably the gravitation constant and the aberration constant, are known with only a low degree of precision.* Whether any quantity may be measured or calculated with precision depends, in general, on the degree of complication of its connections with other quantities, and on the applicability of methods already applied in the determination of other quantities. Frequently, a quantity may be measured directly; but it oftener happens, either by reason of the inapplicability or of the disadvantage of a direct method, that resort is had to an indirect method.

It is a remarkable fact, illustrating the essential unity which pervades the apparent diversity of nature, that all of the numerous quantities with which physical science has to deal may be expressed in terms of a certain very limited number of arbitrarily chosen quantities, or units. The units most commonly used, and those which seem best suited to the present requirements of science, are the units of length, mass and time. All other quantities, however complex, may be expressed readily in terms of these arbitrarily as-

* The gravitation constant is the factor by which the product of two masses divided by the square of their distance asunder must be multiplied in order to express the force exerted by those masses on one another. Thus, if m_1 and m_2 denote two masses, s their distance asunder, F the force of attraction between them, and k the gravitation constant, then

$$F = k\frac{m_1 m_2}{s^2}.$$

It should be remarked that k is not a mere numeral, as many eminent writers on the law of gravitation would seem to imply, but that it is the cube of a distance divided by the product of a mass and the squares of a time; or that its dimensions are shown by the exponents in ($L^{+3}M^{-1}T^{-2}$) if L, M, T denote the units of length, mass and time respectively.

It should be remarked also that the above expression of Newton's law of gravitation lacks the precision essential for mathematical calculations. To make the statement definite and general m_1 and m_2 must be regarded as infinitesimals, so that the resultant attraction between two finite bodies requires, in general, a summation, or integration, for its exact expression. A widespread error exists in the notion that the above equation is exact if the distance s is the distance between the centers of gravity of the masses. This is true, indeed, for the class of bodies called centrobaric, like homogeneous spheres; but masses in general are not centrobaric.

The gravitation constant is, in C. G. S. units, about 667×10^{-12}, with some uncertainty in the last significant figure.

The aberration constant, which is (if it is nothing more than a kinematical quantity) the ratio of the velocity of the earth in its orbit to the velocity of light multiplied by the number of seconds in a radian, is about 20.5″ with some uncertainty in the next significant figure.

sumed fundamental quantities. It is by no means certain, however, that these units will best satisfy the requirements of science in the future. On the contrary, it seems rather probable that advancing knowledge will find some other system of units preferable, if it does not find several different though interconvertible, systems essential. We have, in fact, already attained two such diverse systems in the units of electro-magnetic science.

The study of such systems by the aid of the theory of dimensions, which shows algebraically how the assumed units enter into more complex quantities, is very instructive, not only to the mathematical physicist, but to the general student of physical science.* To illustrate this idea by some simple examples, it is well known that all quantities used in rational mechanics are commonly expressed in terms of length, mass and time. But these quantities might be expressed equally well, so far as algebraical statement is concerned, in many other ways. Thus, we might take energy as one of the fundamental quantities instead of either length, mass or time; in which case our mechanical quantities would be expressed in terms of energy, length and mass; or of energy, length and time; or of energy, mass and time. A consideration of these simple systems shows us, among other things, that rational mechanics might have been developed along lines of thought very different from the lines followed by our predecessors; and the fact that we do not visualize equally clearly all these systems shows that the experience of humanity with physical phenomena has been extremely limited. Most curious and instructive are the system in which length does not appear explicitly and the system in which time does not appear explicitly. May we not see in these systems opportunities respectively for the development of those individuals of our race who seem to possess no realization of distance or no conception of time?

Confining attention to the simpler and more familiar units of length, mass and time, and to a few of the more complex quantities expressed thereby, let us first consider briefly the present status of these fundamental units and the possibility of maintaining their invariability. The stand-

* Designating the units of energy, length, mass and time by E, L, M, T respectively, the dimensions of some of the most frequently used quantities in mechanics are shown in the following tables. In the first of these length, mass and time appear explicitly; in the second, length does not appear explicitly; and in the last, time does not appear explicitly. A glance at the exponents (dimensions) of the symbols shows clearly how definite the meanings of the terms force, energy, power, etc., may be in comparison with the utter ambiguity attaching to them in common parlance.

TABLE I.

Quantity.	Length Factor.	Mass Factor.	Time Factor.
Velocity,	L^{+1}	M^0	T^{-1}
Acceleration,	L^{+1}	M^0	T^{-2}
Force,	L^{+1}	M^{+1}	T^{-2}
Momentum,	L^{+1}	M^{+1}	T^{-1}
Energy,	L^{+2}	M^{+1}	T^{-2}
Power,	L^{+2}	M^{+1}	T^{-3}

TABLE II.

Quantity.	Energy Factor.	Mass Factor.	Time Factor.
Velocity,	$E^{+\frac{1}{2}}$	$M^{-\frac{1}{2}}$	T^0
Acceleration,	$E^{+\frac{1}{2}}$	$M^{-\frac{1}{2}}$	T^{-1}
Force,	$E^{+\frac{1}{2}}$	$M^{+\frac{1}{2}}$	T^{-1}
Momentum,	$E^{+\frac{1}{2}}$	$M^{+\frac{1}{2}}$	T^0
Energy,	E^{+1}	M^0	T^0
Power,	E^{+1}	M^0	T^{-1}

TABLE III.

Quantity.	Energy Factor.	Length Factor.	Mass Factor.
Velocity,	$E^{+\frac{1}{2}}$	L^0	$M^{-\frac{1}{2}}$
Acceleration,	E^{+1}	L^{-1}	M^{-1}
Force,	E^{+1}	L^{-1}	M^0
Momentum,	$E^{+\frac{1}{2}}$	L^0	$M^{+\frac{1}{2}}$
Energy,	E^{+1}	L^0	M^0
Power,	$E^{+\frac{3}{2}}$	L^{-1}	$M^{-\frac{1}{2}}$

ards of length and mass which are now universally adopted in science are the meter and the kilogram respectively, carefully intercompared copies, or 'prototypes,' of which have been distributed by the international bureau of standards to the nations contributing to the cost thereof. The United States possesses two copies of each of these prototypes, and they are, as a matter of fact, our effective working standards, even for the production of standard yards and pounds. It is to be hoped, therefore, that the end of the barbaric system of 'weights and measures' we have inherited from an unscientific ancestry is near at hand, and this not so much in the interest of men of science as in the interests of those less well fitted to struggle with the ingenious intricacies of the British system.

These prototype meters and kilograms are known in terms of the adopted standards, and hence in terms of one another, with a degree of precision which verges close to the limits of the constancy of matter itself. Thus the lengths of the meters are known with an uncertainty expressed by a probable error of only one part in five millions. This degree of refinement corresponds to about one hundredth of an inch in a mile, or to about nineteen miles in the mean distance of the earth from the sun. But this admirable precision is greatly surpassed by that of the kilograms, whose uncertainty falls to one part in five hundred millions. It is well known, of course, that the operation of weighing by means of the balance secures a precision superior to that of every other species of physical measurement; but it is not easy to visualize directly the five-hundred-millionth part of a kilogram. One may get a tolerably definite idea of this magnitude, however, by observing that with the degree of precision in question it would be essential in comparing two kilogram masses to keep the pans of the balance closely at the same level, for a centimeter difference in their altitudes would be appreciable by reason of the variation of the attraction of the earth with distance from its center.*

For present purposes, therefore, our standards of length and mass leave little, if anything, to be desired. But it is a matter of great importance to the future progress of science that these standards be preserved for an indefinitely long period; and although such a contingency seems remote enough now, one can hardly suppress the query as to what would happen to us if our standards should be lost, or if they should unexpectedly prove unstable with the lapse of time. It is quite certain that our standard of length could be recovered with a high degree of precision if such a calamity should befall us during the next ten thousand, or possibly during the next hundred thousand years. Numerous bars of other metals than the alloy used in the construction of the prototype meters are known in terms of the latter. Many base lines scattered at widely separated points of the earth's surface are also known in terms of the meter with a precision of about one part in a million; and although the foundations of the earth are far from stable, we can hardly expect such lines to become systematically shorter

* Denoting the mass of a kilogram by m_1 and the mass of the earth by m_2, the weight of m_1 by w, and the distance from the balance to the earth's center by s (since the earth is nearly centrobaric), the Newtonian law gives

$$w = k\frac{m_1 m_2}{s^2},$$

whence the relation of a small change Δw in w to the corresponding change Δs in s is expressed by

$$\frac{\Delta w}{w} = \mp 2\frac{\Delta s}{s}.$$

Since $\Delta w/w$ is here 1/500,000,000, and since s is about 630,000,000 centimeters, $\Delta s = \mp 0.63$ centimeter.

or longer in so brief a terrestrial interval as a million years. Better still, probably, is the check on the invariability of the meter afforded by Professor Michelson's measurement of it in terms of the wave lengths of particular rays emitted by the metal cadmium.* In this, apparently, we have a cosmic standard, although it remains to be proved that the wave lengths used will remain invariable in the unexplored parts of the universe into which we are journeying along with the solar system at the rate of some kilometers per second.

Our standard of mass is likewise connected directly with various masses which may serve as checks on its stability, and indirectly with the masses of definite volumes of many substances. It is especially well known in terms of the mass of a cubic decimeter of water at a standard temperature. It is less definitely known in terms of the atomic masses of the so-called elements, and it is roughly known in terms of the enormous though slowly varying mass of the earth.† But on the whole, our standard of mass must be regarded as less secure than our standard of length, although the prototype kilograms are less likely to change in mass with the lapse of time than the prototype meters are to change in length; for while such a general variation in volume as is known to occur in metals, especially alloys, need not affect the former, it would almost certainly affect the latter.

Our unit of time is also known with a definiteness that meets in most cases the highest demands of science at the present epoch. The period of rotation of the earth, or the sidereal day, is the standard interval of time, though it has been found convenient for many purposes to use the shorter interval of a mean solar second, of which there are 86,164.1 in a sidereal day. That the earth rotates with wonderful regularity is a fact of the highest importance to science. Without that regularity the development of sidereal and planetary astronomy, with all they have entailed, would have been impossible except by the discovery of some other equally trustworthy timekeeper. But the laws of mechanics, which show us plainly why the earth rotates with such remarkable regularity, also show us that its period of rotation is subject to sources of disturbance, some tending to increase and some tending to decrease that period, whose effects,

* See Tome XI., *Travaux et Mémoires du Bureau International des Poids et Mesures*, Paris, 1895. It is remarkable that the ratios of the three wave lengths used to the meter were measured with a precision requiring seven significant figures, the uncertainty amounting to a few units only in the last figure. Thus the values of the wave-lengths used (designated as red, green and blue respectively) are as follows, in microns, or millionths of a meter:

0.643,847,2,
0.508,582,4,
0.479,991,1.

† If we could measure the gravitation constant with a precision extending to five significant figures, the mass of the earth would at once become known to the same degree of precision, provided only that the law of gravitation is exact to the same number of figures. For I have shown that the product of that constant and the mean density of the earth is known with a precision expressed by five significant figures. Thus, calling the gravitation constant k and the mean density of the earth ρ,

$$k\rho = 36797 \times 10^{-11} / (\text{second})^2$$

This relation may be otherwise expressed by the following theorem: Let τ be the periodic time of an infinitesimal satellite which would revolve about the earth close to the equator (assuming no atmospheric resistance). Then the theorem asserts that

$$k\rho\tau^2 = 3\pi$$

where π is the ratio of the circumference to the diameter of a circle. The value of τ is 1 hour, 24 minutes, and 20.9 seconds. See *Astronomical Journal*, Vol. XVIII., No. 16.

though too minute to be appreciable in such intervals as are known to human history, must certainly become considerable in the course of terrestrial history. Thus, the contraction of the earth due to secular loss of heat tends to shorten the day, while accumulations of meteoric dust and tidal friction tend to lengthen it.* There exists also a graver source of disturbance in the slow rising and sinking of the crust of the earth in different latitudes so often pointed out by geologists. Such movements are only partly compensating in their effects on the day, and it seems highly probable that they may cause irregularities amounting to a few seconds in a century without entailing any noteworthy fluctuations of the relative positions of the land and sea.†

It appears, then, that our time unit is the least stable of the three fundamental units and hence the most in need of checks on its stability. Various other standards of time have been proposed, but none of them meets the requisites of permanency and availability. The interests of astronomical science especially demand that efforts be made to find in the solar system some better timekeeper than the earth. Possibly the fifth satellite of Jupiter may serve as a control on the constancy of rotation of the earth.

Turning now to a consideration of the more complex quantities which are expressed in terms of length, mass and time, we enter the boundless fields of physical science in which measurement and calculation have revealed to us all ranges of magnitudes from the vanishingly small to the indefinitely large. It is in these fields that we learn something definite concerning the limitations of our senses; for while measurements alone carry us but a little way along lines of research, calculation discloses not only the unseen, but also, in many cases, phenomena which are quite beyond the reach of any direct sense perception.*

To begin with quantities near the lower limit of determination, think, for a moment, what is going on in the air which for the present is the main medium of communication between us. No one has ever seen the particles of the atmosphere in the sense that we have all seen the particles, or corpuscles, of the blood. But we probably know more about the molecules of gases than we do about blood corpuscles. By actual count it is known that there are four to six millions of the latter in a cubic millimeter; and with equal definiteness calculation shows us that there are about a million million million molecules in a cubic

* I have discussed the effects of secular cooling and meteoric dust on the length of the day in a paper published in the *Astronomical Journal*, Vol. XXI., No. 22, July, 1901. From this paper it appears that the change in length of the day from secular cooling cannot be perceptible during any such brief interval as that of human history (twenty centuries, say); but that in the course of complete cooling, or in a million million years, say, the change in length of the day may amount to as much as six per cent. of its original length.

From the same paper it appears that accumulations of meteoric dust will only begin to be perceptible in their effects on the length of the day when the process of secular cooling has been substantially completed. In a subsequent number of the *Astronomical Journal* (Vol. XXII., No. 11), Dr. G. Johnstone Stoney has shown that if the compression produced by a layer of meteoric dust is taken into account the effect will be still less than that just indicated.

† See 'Mathematical and Physical Papers of Lord Kelvin,' Vol. III., pp. 333–335, Cambridge University Press, London, 1890.

* The reader may be referred to a very instructive paper by Dr. G. Johnstone Stoney entitled 'Survey of that Part of the Range of Nature's Operations which Man is Competent to Study.' *Scientific Proceedings of the Royal Dublin Society*, Vol. IX., No. 13; *Philosophical Magazine*, Fifth Series, No. 294, November, 1899; published also in *Report of Smithsonian Institution* for 1899.

millimeter of the air around us. Notwithstanding this apparently crowded assemblage, the individual molecules move about in the liveliest manner, their average speed being about five hundred meters per second, and this in spite of the fact that the average length of an unimpeded journey is barely visible by the aid of the best microscopes. Each molecule must therefore collide with its neighbors astonishingly often, the encounters occurring, in fact, about five thousand million times per second.*

More surprising still than the properties of assemblages of molecules forming gases are the properties of the individual molecules, especially when they are made up of two or more atoms. Such miniature systems, comparable, probably, in complexity with the Martian and Jovian subsystems of the solar system, exhibit degrees of constancy which rival the invariableness of the fixed stars themselves. This is particularly the case with their rates of vibration as disclosed by the spectroscope. These rates afford one of the most delicate tests of the properties of matter, whether it is found on the earth or on the most distant star; and yet the vibrations, which recur with a regularity equal to, if not surpassing, the regularity of the rotation of the earth, are executed at the rate of some hundreds of millions of millions per second.† Herein, perhaps, we may find a cosmic unit of time as well as a cosmic unit of distance, though both appear to be inconveniently small for terrestrial purposes.

But the smaller bodies of the universe do not end with molecules and atoms of gases. Recent investigations point to the conclusion that there is another order of bodies of much smaller dimensions and possessing still more wonderful properties. These have been called corpuscles.* Their density is only about one thousandth as great as that of the lightest gas, hydrogen; they are freely given off by several of the so-called radio-active substances; and they move about with speeds of the same order as the velocity of light. It appears not improbable that they play a most important rôle in cosmic as well as in terrestrial physics, and the amount of attention being given to them justifies the hope that their study may illuminate many obscure corners in the realm of molecular science.

Passing per saltum from the smallest measureable and calculable quantities to those with which we have an every-day familiarity, I would direct your attention to the great number of articles of commerce

* See, for example, 'The Kinetic Theory of Gases,' by Dr. Oskar Emil Meyer, translated by Robert E. Baynes, Longmans, Green and Co., New York, 1899.

† The number of vibrations per second corresponding to any given wave-length of light may be easily computed. For the velocity of light is about 300,000 kilometers, or 3×10^{14} microns per second, and this divided by the wave-length in question gives the number of vibrations per second. Thus the average wave-length of the cadmium rays used by Professor Michelson (cited above) is about half a micron. The material sources of these rays must vibrate, therefore, about six hundred million million times per second.

* See a paper by Professor J. J. Thomson, 'On Bodies Smaller than Atoms,' *Popular Science Monthly*, August, 1901.

See also a paper by Professor John Cox on 'Comets' Tails, the Corona and the Aurora Borealis,' *Popular Science Monthly*, January, 1902.

A fact of great interest in connection with the 'corpuscles' considered in these two papers is the repulsion of light impinging on bodies, the amount of which has been actually measured recently by several observers. This repulsion between the sun and the earth is very great, amounting to about a hundred million million dynes; but the gravitational attraction between these bodies is about forty million million times as great as that repulsion.

which are now weighed, measured and rated with precision and sold at a cost which, a half century ago, would have been thought quite impossible. Standard yards, meters, pounds and kilograms, and pocket time-pieces that will run within a few seconds per day, are available at prices within the reach of all who need them. Screws and screw gauges which will easily measure a hundredth of a millimeter (or four ten thousandths of an inch) are articles of trade; beautifully true spheres of steel or bronze may be had for a few cents each; helical springs of the finest steel and of remarkable uniformity are sold for a dollar a dozen; while articles like wire, tubing, sheet metal, and an indefinite variety of tools and machinery are made with a degree of perfection and at a cheapness of cost which would have been regarded as quite unattainable by the founders, for example, of the New York Academy of Sciences. The ready availability of, and the constant demand for, all these products to meet the daily needs of the complex civilization of our time affords a sufficient answer to him who would question the efforts spent in attaining those products or the efforts applied in subjecting new objects of study to the rigorous tests of measurement and calculation.

But the principles of measurement and calculation are not limited in their application to external objects, or to the properties of what we are sometimes pleased to call 'gross matter.' They apply equally aptly in many ways to man himself, and it is clear that with advancing civilization we may confidently expect such application to be greatly extended. While we have not yet attained formulas which will comprehend the vagaries of the individual, we have many formulas which will accurately express the resultant of those vagaries as manifested in racial types. A life insurance company, for example, may not assert at the beginning of a year that any individual of ten thousand men of the same class will die within the year, but it may assert with practical certainty that a definite number of this class will die within the year. Such 'facts and figures' are trite enough, of course, but what we commonly fail to see and appreciate is the solid basis on which they rest, and how greatly it would be to our advantage to extend the same sort of reasoning that has built up great systems of fire and life insurance into other departments of human affairs. Most people, I fear we must infer, are, like Thomas Carlyle, still scoffers at statistics, and few, even of the educated, have any adequate conception of the order which the principles of probability will bring out of the apparent disorder of statistical data.

Of the larger objects of the universe to which measurement and calculation have been applied with success, the earth easily surpasses all others in interest and importance. So great has been this success that one may assert that we know more of the earth than we do of any other body to which science has given attention. Its size, its shape, the amount and arrangement of its mass, its magnetic properties, its speeds of rotation and translation, its precession and nutation, and the lately discovered wabbling of its axis of rotation are all known with a definiteness which is truly surprising when one considers its magnitude and the degree of complexity of those properties. That the eight thousand miles in its diameter should be known within a few hundred feet, that the two hundred millions of square miles in its surface should be known within a few hundred square miles, or that the acceleration of gravity at any point on its surface should be known within a few millimeters per second per second, are results little short of marvelous when one reflects that they have all been attained within the brief interval of two hundred and fifty years. It would be quite wrong, however, to consider these achievements of geodesy as marvelous from the point of view of science. They are, rather, just such results as persistent scientific investigation has always produced, and such as we may safely predict will be uniformly produced by persistent scientific investigation in the future. The element of the marvelous comes in only when one takes account of the fact that these grand results were attained by a very small number of men, mostly members of academies, struggling, like our own, to maintain an existence, in whose work the general public took little interest, and whose names, even now, are much less known than the names of the obscure philosophers and the obscene poets of antiquity.

Geodesy is undoubtedly the most advanced of the sciences in which measurement and calculation have attained a high order of certainty. It has made modern commerce possible, and it seems destined to play a still more important rôle than it has hitherto in the advancement of terrestrial affairs. It has also made modern astronomy possible, for the certainty of its data enables us to measure not only the dimensions of the solar system, but also the approximate dimensions of the visible universe.

Not less important to the progress of science and to the general advance in human enlightenment are the achievements of the allied science of geology. It cannot boast, as yet, like geodesy, of a high degree of precision in measurement and calculation, for it deals, in general, with phenomena which have not yet been reduced to simple laws. But, on the other hand, its subject-matter is more obvious and tangible, and it appeals therefore more forcibly and continuously to the average mind. No science seems comparable with geology in the completeness with which its history and its main processes are contained in the subjects and objects of investigation. Whoso would read the story of the earth's crust will find it written and illustrated in infinite detail in the rocks themselves. No vivid or perfervid imagination of the historian has concealed the facts or misinterpreted their sequence; they are all recorded with a truthfulness that shames the straightest human testimony and with a permanency which permits comparison and verification in endless repetition.

Geology illustrates more clearly, perhaps, than any other science the value of measurement and calculation when the order only of the quantity sought can be attained. The determination of the fact, for example, that nothing short of a million years is a suitable time unit for measuring the age of the earth, was an achievement whose importance can hardly be overestimated; indeed, our race may yet require decades, if not centuries, to appreciate its full significance, for in spite of the great advances in our times it appears probable that not one in a thousand of the good people with whom we live realizes how profoundly definite acceptance of such a fact must modify thought.

A criticism which the devotees of the so-called humanistic learning often apply to such matters of fact, and which is still occasionally accepted by men of science, helps us to see the absolute need of countless recurrences to the evidence so well exhibited in the crust of the earth. "Ah!" says the humanist, "I observe that the physicists and the geologists do not agree on the age of the earth. Some say it is ten million years, others that it cannot be more than two hundred million years, and others that it cannot be less than a thousand million years. I conclude, therefore, that so long as your doctors disagree in this manner, we may continue to accept the age recorded in our sacred books." Thus easy is it to mistake the order of a quantity for the quantity itself.

When we pass from terrestrial limitations to celestial phenomena the field for measurement and calculation is immensely enlarged, though the results attainable are less easy of ready appreciation. The Jovian, the Saturnian and the Martian subsystems, which have been pretty thoroughly explored by the observer and the computer, present to us the type, apparently, not only of the solar system, but of the galaxy of systems within telescopic view. And the surveys of the heavens now in progress indicate likewise that isolated stars are the exception rather than the rule, and that the visible stars are generally attended by one or more satellites, which are probably oftener dark than bright bodies. Visual and photographic measurements have, in fact, united in recent years in the demonstration that the number of material bodies in the universe is enormously greater than we have hitherto imagined. Here again, however, as in the case of the geological phenomena just referred to, we must be content to a great extent for the present with a knowledge of the order of the quantities measured and calculated. But to be able to state what is the order of the distances which separate the fixed stars from one another, the order of the volume of the visible universe, the order of the quantity of mass in that volume, and the order of the time unit requisite for the expression of the historical succession of celestial events, seems little short of a stupendous contribution to knowledge when one reflects on the obstacles, material and intellectual, that have stood in the way of its attainment.

The distances asunder of the stars are so great that the hundred and ninety odd millions of miles in the diameter of the earth's orbit about the sun make an inconveniently small base line for the measurement of the least of those distances and a hopelessly inadequate one for the measurement of the greatest of them. It would appear more fitting, in fact, to express such distances indirectly in the number of years it takes light moving at the rate of 300,000 kilometers per second to traverse them. Assuming with Lord Kelvin that the visible universe is comprised within a sphere whose radius is equal to the distance of a star whose parallax is one thousandth of a second, this distance would require light about three thousand years to pass over it, while the average distance asunder of the visible stars is considerably less, but still of the same order. Lord Kelvin has shown also in a profound mathematico-physical investigation recently published* how we may assign limits to the amount of mass in the visible universe. It appears from this investigation that there are something like a thousand million masses of the magnitude of our sun within that universe. The figures for this amount of mass have little meaning to most of us when expressed in ordinary units. The mass of the earth, for example, with its $6,000 \times 10^{18}$ metric tons,† is a mere trifle, for the sun has about 327,000 times as much mass as the earth. The mass of the sun therefore is the obviously convenient unit in this case; and we have only to imagine our solar system surrounded by a thousand million such suns, each in all probability attended by a group of planets, to get a sufficiently clear idea of the quantity of mass within visual range of our relatively insignificant

* 'On Ether and Gravitational Matter through Infinite Space,' *Philosophical Magazine*, August, 1901. 'On the Clustering of Gravitational Matter in any Part of the Universe,' *Nature*, Vol. 64, No. 1669.

† The metric ton of 1,000 kilograms, or 2,205 pounds, is about the same as our 'long ton' of 2,240 pounds.

terrestrial abode. And the time scale for the varied events which take place in the interaction of these millions of suns is not less imposing when expressed in familiar terms. A million years is the smallest unit suitable for estimating the history of a star, although the record of that history is transmitted to us through the interstellar medium by vibrations whose period is so brief as to almost escape detection.

Measurements and calculations have thus made known to us a range of phenomena which is limited only by our sense perceptions, sharpened and supplemented by the refinements of mathematical analysis. In space and mass relations these phenomena exhibit all gradations from the indefinitely small to the indefinitely large; and in time they point backward to no epoch which may be called a beginning and forward to no epoch which may be called an end. Dealing chiefly with those aspects of phenomena which possess permanence and continuity, or at least a permanence and a continuity compared with which all human affairs appear ephemeral and fleeting, measurement and calculation tend to raise man above the level of his environment. They bid him look forward as well as backward, and they assure him that in a larger study of the universe lies boundless opportunity for his improvement.

But while that sort of knowledge which has been reduced to quantitative expression has done more, probably, than all else to disclose man's place in and his relations to the rest of the universe, it would appear that mankind makes relatively little use of this knowledge and that we are not yet ready, as a race, to replace the indefinite by the definite even wherein such substitution is clearly practicable. It is a curious and a puzzling, though perfectly obvious, fact that mankind as a whole lives less in the thought of the present than in the thought of the past, and that as a race we have far more respect for the myths of antiquity than we have for the certainties of exact science. Our ships, for example, are navigated with great success by aid of the sextant, the chronometer, and the nautical almanac; but what company would dare set Friday as the day for beginning the transatlantic voyage of a passenger steamer? From time immemorial tradition has dominated reason in the masses of men. Each age has lived, not in the full possession of the best thought available to it, but, rather, under the sway of the thought of some preceding age. We are assured even now, by some eminent minds, that the highest sources of light for us are nearly all found in the distant past; and a few go so far as to assert that modern science is merely furbishing up the half-lost learning of ages long gone by.

The work of academies and other scientific organizations is therefore nowhere near completion. Great strides toward intellectual emancipation have been made during recent times, but they have served only to enlarge the field for, and to increase the need of, that sort of knowledge which is permanent and verifiable. Measurement and calculation have furnished an invaluable fund of such knowledge during the two centuries just past, and we have every reason to anticipate that they will furnish a still more valuable contribution to such knowledge in the centuries to come.

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'NATURAL HISTORY,' 'ŒCOLOGY' OR 'ETHOLOGY'?

A STUDY of recent literature reveals the fact that zoologists are much in need of a satisfactory technical term for animal behavior and the related subjects which go to make up what is variously known as 'natural history,' 'œcology' and 'biology'