An Unrecognized Process in Glacial Erosion. WILLARD D. JOHNSON, Washington, D. C.

THE glacial topography of mountains was analyzed, and the more distinctive forms discriminated from those of aqueous erosion. The recognized process, that of scour, its action downward and forward with the glacial advance, was described. Glacial scour and aqueous erosion were regarded as alike incompetent to bring about the results and as a rule inimical to the production of known forms. An unrecognized process was set forth, that of sapping, whose action is horizontal and backward. The tendency of glacial scour is to produce sweeping curves and eventually a graded slope. The tendency of the sapping process is to produce benches and cliffs. Sapping is altogether dominant over scour. Under varying conditions, however, its developing forms become obsolescent; their modification, then, by rounding off of angles, puts them seemingly into the category of scour forms. An hypothesis was advanced as to the cause of glacial sapping. The ultimate effect is truncation at the lower level of glacial generation. A second analysis and a more appreciative classification of transition types terminated the paper.

Before discussion the next paper was read because it dealt with allied phenomena. The hour, however, being late, the discussion went over till the next day.

Geology of the Yosemite National Park. H. W. TURNER, Washington, D. C.

BY means of lantern slides the author illustrated the topography of the granite areas in the high Sierras and the Yosemite and other allied gorges. He developed the view that joints had chiefly caused the precipitous cliffs, and concentric shelling off, the domes. Minor forms were also explained. He opposed the view that faulting had caused the gorges.

Gold Mining in the Klondike District. J. B. TYRRELL, Ottawa, Ont.

BY means of a fine series of lantern slides the author illustrated the geographical situation and the geology of the Klondike gold-bearing gravels. The stream gravels are the usual type of placers, but the bench gravels are small lateral moraines left by glaciers. The gold has not been derived from any distance.

The Nashua Valley Glacial Lake. W. O. CROSBY, Boston, Mass.

BY means of lantern slides from photographs and from maps and profiles based on bore-holes made by the officials of the Boston department of municipal water supply, the speaker described the bed-rock surface, the overlying gravels on the Nashua River, and the characters of the old glacial lake of whose former existence they gave evidence.

On the conclusion of the paper, at 5:45 p. m., the Society adjourned until the following day. In the evening about one hundred Fellows, many with their wives, gathered at the Hotel Logerot for the annual dinner. Under the presiding oversight of Professor B. K. Emerson, the past grand master of all the toastmasters, another enjoyable gathering was added to the list of those previously held.

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(To be Concluded.)

SCIENTIFIC BOOKS.

Theory of Groups of Finite Order. By W. BURNSIDE, M.A., F.R.S., Professor of Mathematics at the Royal Naval College, Greenwich. Cambridge, The University Press. 1897. 8vo. Pp. xvi+388. Price, $3.75.

If, assuming a single but elevated point of view, we describe mathematics as the science of formal law, then the theory of operations easily commands the field, for it is the quintessence of mathematical form, the comparative anatomy, so to speak, of the mathematical sciences. Originally appearing under the special guise of the theory of substitutions and developed in this form by the labors of Galois, Cauchy, Serret and Jordan with reference chiefly to its application to the theory of equations, it has of more recent years overleaped at once its scientific and its national limitations and, receiving new impulse at the hands of Kronecker and Cayley, has been developed largely by Klein and Lie into one of the chief general instruments of mathematical research. In every branch of mathematics the point of view of the theory of operations is now predominant; it is employed in almost every form of mathematical investigation, and by the reaction the science is in turn constantly enriched. Conspicuous instances are Klein's theory of the modular equations and Lie's theory of differential equations.

The number of separate works devoted wholly or in part to the theory of operations is comparatively very small. Serret's Algebra held the field alone down to the appearance in 1870 of Jordan's classical Traité. Netto's Theory of Substitutions, published in 1882, was the first German book on the subject and represents, as regards its special subject, the German (Kronecker) standpoint down to that date. The American translation (1892) of Netto's book was the first separate work in English to touch the field; in fact, it was almost the first presentation of the subject in any form in English. In 1895–96 appeared the two volumes of Weber's Algebra, a work the value of which as a systematic and modern treatment of the various branches of algebraic science cannot be overstated. To this work, rich in other treasures, belongs the distinction of being the first treatise to present the theory of operations in general form independent of the particular content to which the operation might be applied. Closely following the work of Weber, comes now the second English book on the algebra of operations, Burnside's Theory of Groups of Finite Order. Professor Burnside's work is a doubly welcome contribution to the literature of the subject. It not only opens up to the English reader a great and hitherto almost foreign field, but it presents in a form often original and always valuable the most recent developments in that field, to which the author himself has, in fact, made no insignificant additions. Many portions of the subject, otherwise only to be gathered piecemeal from the journals, are here brought together for the first time in orderly sequence. Proofs have been recast and simplified or extended, and the book contains an abundance of those special details and examples, perhaps too familiar in English mathematical works, but very acceptable here in the midst of a highly abstract theory.

To the reader whose vocation or avocations have not lead him into this remote region of serene thought a short excursion among the groups may be instructive and more or less agreeable. Let him, then, first become familiar with the idea of the 'product' of two operations. This is simply the single operation which alone produces the same effect as the successive performance of the two given operations. If it be asked: "What sort of operations do you mean?" I reply with unction: "Any kind you please, and the more general the conception the better." Algebraic, geometric, physical, chemical, even metaphysical or 'sociological' operations, if nothing better offers, all are taken in one net. But to condescend from this lofty altitude, let us take for an example the rotations of a sphere about its diameters. Choosing any two of them, and applying them successively to the sphere, regarded as a rigid body, the resulting, or *resultant*, displacement of the sphere is equivalent to a third rotation about a proper diameter. This third rotation is, then, the *product* of the two given ones. The rotations of the sphere, taken all together as a system, serve also to exemplify the next important notion, that of a 'group.' When a system of operations is so constituted that the product of any two of them is itself an operation of the system, so that the system is a *closed* one with respect to the process of forming products, then if a couple of minor conditions are also satisfied, the system forms a group. And now the theory of operations in its present form concerns itself not with all kinds of operations, but with these groups. Examples of groups are not far to seek, after the idea is grasped. No science is exempt from them; in mathematics they simply tumble over each other. Transformations of coördinates in geometry form a group; so do the projections of a plane or of space; the motions of space as a rigid body form the Euclidean group of motions; the $n!$ permutations of n letters form a group; the eight permutations of x_1, x_2, x_3, x_4 which leave the function $x_1 x_2 + x_3 x_4$ unchanged in form, form a group; the multiplication table, the operations of the post office, the theory of the tides, psychological phenomena, all embody characteristic groups. A specially important class of groups, which may serve to close the list, is that of the linear transformations (which are formally identical with geometric projections and with various other operations). Thus the equation

$$z' = \frac{\alpha z + \beta}{\gamma z + \delta}$$

may be looked upon as defining an operation by which any number z is connected into a corresponding number z'. If we have two of these operations, and if, having applied the one to z, getting z' as a result, we apply the other to z', getting z'' as a result, then an examination will show that z'' is itself a linear function of z, *i. e.*, the product of two linear transformations is a linear transformation.

Prepare now for a step into the abstract. In expressing ourselves in terms of 'operations' we have been walking on the crutches of the concrete. But if we designate the operations of a group by A, B, C, . . . , their products AB, BC, . . . have a definite mode of formation, constituting an *algebra*, and we will now throw away the 'operations' and keep the symbols and their algebra. The symbols are now 'elements,' and if these elements form a group the product AB is identified by the algebra with some element C of the same group. Two other properties have to be added to make the definition of a group precise: (1) the algebra must be associative, *i. e.*, $(AB).C = A.(BC)$, and (2) if $AB = AC$ then $B = C$ and if $AB = CB$ then $A = C$. Algebras can, of course, be constructed which omit these conditions, but they are not algebras of groups.

The *order* of a group is the number of its elements. A group may be of finite or infinite order, *e. g.*, all the rotations of a sphere about its diameter form an infinite group; those of them which turn into itself a regular polyhedron inscribed in the sphere form a finite group. Infinite groups are only touched on in Burnside's book. Access to their theory is most readily had through Lie's works. Burnside's opening chapter on abstract groups (Chapter 2) is not so happily executed as Weber's treatment (Vol. II., Chapter 1), which is a masterpiece (*Cf.* also Frobenius's 'Ueber endliche Gruppen,' *Berliner Sitzungsberichte*, 1895, p. 163). Burnside retains the operations and makes use of their concrete qualities in discussing properties which are better treated in the pure abstract.

From the mere definition of a group it is possible to raise a considerable crop of properties without any artificial fertilizer. Add the ideas of isomorphism and transformation, and consider the groups whose elements are commutative (Chapter 3), and those whose orders are powers of single prime numbers (Chapter 4), and the wilderness fairly blooms. Even the non-specialist may rapidly make his way through the easy roads and add valuable ideas to his stock as he goes. He can hardly do better than to read this book, which gives a very clear and straightforward treatment of these general matters. But this is mere surface production. Underneath is gold, but only the Frobenius brand of dynamite will reach that. More than twenty-five years ago a solitary prospector, Sylow, found the lode and worked it with good results as far as he could follow it. Others have tried new leads, but none have accomplished anything remarkable until the work of Frobenius, who in the past ten years or so, and more particularly in his articles published in the *Berliner Sitzungsberichte* for 1895–6 has opened up a vast wealth of new relations, at the same time revising and enriching the earlier methods, nomenclature, and general point of view. Some of the most prominent of Frobenius's results are discussed in Chapter 6. Another line of ideas, which, however, dates back in its beginning as far as Galois, and has been improved especially by Hölder, the theory of composition of a group, is discussed in Chapter 7. The three following chapters are devoted to an extensive discussion of substitution groups, whose theory has also been considerably extended of recent years. The theory of isomorphism of a group with itself, also a very recent notion, is given a full chapter. The scene then shifts to the graphical representation of groups, exploited by Klein in his treatment of the automorphic functions, and treated separately by Dyck, whose methods are here employed. Cayley's color groups also receive attention. A chapter follows on the linear group, following Jordan's classical discussion. Finally, Sylow's theorem and its derivatives are applied to the determination of the composition of groups whose order are resolved into prime factors.

The book concludes with a useful trilingual table of equivalent technical terms and a still more useful Index. The publishers have done their full duty; the type is large and clear, and the paper gives a good impression. The text would have been improved by the introduction of descriptive section headings, and frequently the reader is not kept comfortably informed of what the author has in view, and must suspend judgment for a too lengthy interval.

The small public to which such a work appeals makes it unlikely that books on the theory of groups should ever become very numerous. It is fortunate, therefore, that in Professor Burnside's treatise we have a work of genuine and permanent value from which many a future student may draw wholesome inspiration.

F. N. COLE.

Elements of Sanitary Engineering. By MANSFIELD MERRIMAN. John Wiley & Sons. 1898.

The book opens with an interesting and, for a student, instructive series of historical notes. This is followed by a section dealing with 'classification of disease,' wherein may be found the novel proposition that 'disease is normal and health ideal—' a view that will call forth much opposition.

The illustrations distinguishing between contagion and infection are good, but the suggestion that goitre is probably due to the use of limestone water is hardly warranted; for, were it a fact, the hard waters of southern England should produce the disease abundantly.

An excellent and timely statement is given in the table on page 17, showing how much more serious is consumption than sundry other diseases against which we take far greater pains to guard.

The relation of filth to disease is well put, and the illustrations are striking. The chapter on 'drinking water and disease' is in terse form, suitable for class-room work, but the remarks concerning the Hamburg cholera epidemic need to be supplemented by a map of the city, in order to grasp fully what may be learned from that instructive outbreak.

The book is evidently intended for use as a student's text-book, and excellent questions are inserted at frequent intervals, which require the student to make use of a reference library. This is a very valuable feature, and one but rarely found. There is, unfortunately, no index.

M.

Bush Fruits: A Horticultural Monograph of Raspberries, Blackberries, Dewberries, Currants, Gooseberries and other Shrub-like Fruits. By FRED. W. CARD, Professor of Horticulture, Rhode Island College of Agriculture. The Macmillan Company. 1898. Pp. xii + 537. Price, $1.50.

Under this concise and somewhat descriptive title another book is added to the list upon small fruits, from which, in this instance, are excluded the grapes, strawberries and cranberries.

The contents are divided into three parts, namely, (I.) General Considerations, (II.) The Brambles and (III.) The Groselles. The last name is adopted from the French, includes both the currants and gooseberries, and is a convenient term as a heading for a book division, but will scarcely be of much service elsewhere.

Under brambles, of course, the red raspberries, black raspberries, blackberries and dewberries are considered each with its separate chapter.

Part I. deals with the consideration of location, fertilizers, planting, tillage tools, pruning, propagation, thinning, spraying, picking, packing and marketing of fruit, with a few closing pages upon the methods of crossing and the results of such blending of the varieties and species.

Many of the above-mentioned points are again more specifically treated under the chap-