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Solutions of some Topographical Problems, by "One Plane Descriptive Geometry." By John M. Richardson, B. S.

Topographical Problems.

Hills are most correctly represented by cutting them with a system of horizontal planes, and finding the projections of the curves of intersections. These curves are numbered, beginning at the lowest, or each one is marked by a number denoting its height above the lowest or datum plane.

The distance between the secant planes is dependent upon the accuracy with which it is necessary to delineate the surface. With any given distance between the secant planes, it is evident that the projections of the curves of intersection will be nearer together as the surface of the hill is steeper, and farther apart as the surface approaches nearer to horizontality.

The lines of intersection are called "contour lines," or "lines of horizontal section." The methods of determining these lines in the field and of plotting them, belong to Surveying, and cannot be explained here.

Problem I. Fig 1.—To find the intersection of a given plane with a given hill.

Let $(a, 0)$—$(b, 8)$ be the plane, and $(c, 0)$—$(d, 1)$—$(e, 2)$—$(l, 8)$ the surface.

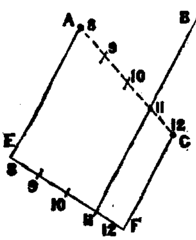
Through each point of division of the scale* of the plane draw a horizontal line in the plane, and find the points in which it intersects the curve of the surface which lies in the same horizontal plane.

The horizontal line through $(m, 4)$ intersects $(g, 4)$ in two points which are projected in r and r'. In the same manner other points of intersection are determined, and the curve is $n\ p\ q\ s\ r\ t\ u\ v\ w\ x\ w'\ v'\ u'\ t'\ r'\ s'\ q'\ p'\ n'$.

Problem II. Fig. 1.—The given surface being that of a hill, to compute the quantity of earth cut off by the plane and lying above it.

Divide the portion $q\ q'$ of the curve $(e, 2)$ into parts so small that they will not differ sensibly from their chords, and take corresponding parts of the curve $(f, 3)$. The trapezium whose projection is $c'\ d'\ e'\ f'$ may be regarded as coinciding very nearly with the surface of the ground, and $c'\ d'\ e'\ f'$ may be regarded as being the right-section of a small truncated prism whose lower base is in the horizontal plane, and

* The scale of a plane is a line divided into parts equal to the horizontal interval between the contour lines of that plane. The contour lines of a plane are of course all straight lines, and their horizontal intervals equal, since the inclination of a plane is the same in all parts.



The scale of a plane may be thus constructed. Let A, B, C, be three points in a plane, their heights above the datum being marked.

Draw A, C, connecting the lowest and highest poiuts—divide in a number of equal parts equal to the difference of height between A and C. Through the point corresponding to the elevation of B draw B 11 and produce it—draw a line E, F, at right angles to B 11. The line E, F., divided as in the cut, and produced if necessary, is called the scale of the plane. It is nothing more than the line of greatest inclination of the plane crossing the contour lines at right angles and divided by them into equal parts.

We must bear in mind that only a plane can have a scale—that an irregular surface cannot.

whose upper base lies in the surface. That portion of this prism which lies above the cutting plane is required. It is evidently equal to the area $c'\ d'\ e'\ f'$ multiplied by the mean height of the four points $(c', d'\ 2)$, $(e', f', 3)$ above the cutting plane.

Knowing the scale according to which the diagram is drawn, it is easy to calculate the area of $c'\ d'\ e'\ f'$, and having the scale of the plane, the height of each of the points $(a', d', 2)$, $(e', f', 3)$ above it, can be readily found. From c' draw $c'\ c''$ perpendicular to the trace* of the plane, and if $\frac{m}{n}$ is the scale of the plane, $\frac{m}{n} \times c'\ c''$ is the height of the point in which the perpendicular through $(c', 2)$ pierces the cutting plane. Hence

$$2 - \frac{m}{n} \times c'\ c'' = \frac{2\ n - m \times c'\ c''}{n}, \text{ is the height of } (c', 2) \text{ above the plane.}$$

Finding the height of each of the other points above the plane in the same manner, there results for there mean height above it,

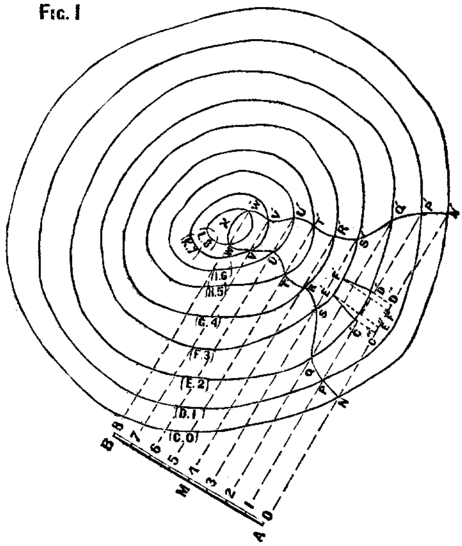
$$\frac{10\ n - m\ (c'\ c'' + d'\ d'' + e'\ e'' + f'\ f''),}{4\ n}$$

and for the volume of the truncated prism above the plane,

$$c'\ d'\ e'\ f' \times \frac{10\ n - m\ (c'\ c'' + d'\ d'' + 'e'\ e'' + f'\ f'').}{4\ n}$$

Finding in the same manner that part of each of the elementary truncated prisms which lies above the cutting plane and adding them all together, their sum will be volume cut off by the plane.

FIG. I



Of course this is only an approximate solution, and is more or less correct according as the horizontal planes which determine the scale of the cutting plane and the horizontal contour lines of the surface, are nearer together or farther apart, and as the chords of the elementary curves correspond more or less nearly with them.

If the curves are very near in space, the chords of the elementary portions of their projections may be regarded as parallel, and $c'\ d'\ e'\ f'$ will then become a trapezoid.

PROBLEM III. Fig. 2.—To draw a plane through a given line tan-

* The TRACE of a plane is its intersection with the plane of projection, and is of course a straight line lying in the plane of projection.

gent to a given hill, the plane to lie entirely above the surface of the hill.

Let (*a*, 0)—(*b*, 6) be the line, and (*c*, 0)—(*d*, 1)—(*k*, 6) the given surface.

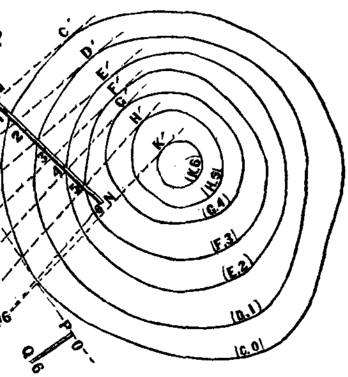
Through each point of division of the line draw horizontal lines tangent to the horizontal sections of the surface; *a e'*, 1 *d'*, 2 *e'* and *c.*, are their projections.

Then, the plane which passes through the given line, and that horizontal line whose projection makes the least angle with *a b*, (the angle being estimated from the projection towards the direction in which the line descends), is the required tangent plane, *a* 5 *h'* being the least angle, 5 *h'* is the projection of a line of the required plane, and its scale will be perpendicular to 5 *h'*.

Drawing through *a*, 1, 2, 3, &c-, lines parallel to 5 *h'*, (*m*, 0)—(*n*, 6) is the scale of the required plane.

It is evident that (*m*, 0)—(*n*, 6) is a tangent plane; for if through *m*, 1, 2, &c., lines be drawn perpendicular to *m n*, none of them will cut the horizontal sections of the given surface, and one of them, 5 *h'*, is tangent to the section of the surface which lies in the same horizontal plane.

FIG. 2



It is also evident, that if planes be passed through the given line and the other horizontal lines which were drawn tangent to the horizontal sections of the surface, that they will all cut the surface. From a given point in the plane of a curve two tangents can generally be drawn to the curve; it follows, then, from the construction, that two planes can be drawn through the given line tangent to the given surface. (*p*, 0)—(*q*, 6) is the scale of the other plane.

PROBLEM IV. Fig. 3.—To draw a plane parallel to a given plane and tangent to a given hill, the plane to lie entirely above the hill.

Since the planes are to be parallel their scales will be parallel, and the intervals of graduation will be the same.

Let (*a*, 0)—(*b*, 6) be the plane, and (*c*, 0)—(*d*, 1)—(*e*, 2)—(*k*, 6), the surface.

Draw *l m* parallel to *a b*, and tangent to the projections of the horizontal sections of the surface, draw lines perpendicular to *l m*, meeting it in the points *m*, *p*, *q*, &c.

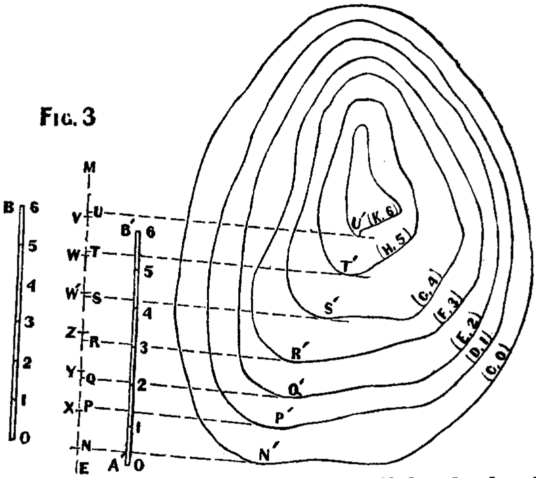
Beginning at the lowest of these points, lay off towards *m* distances equal to one of the equal parts of *a b*, and let *x*, *y*, *z*, &c., be the points of division.

These points will be in the scale of a plane drawn through *n n'* parallel to the given plane, and will be at the heights 1, 2, 3, 4, &c., respectively.

But since this plane in rising from l to m, has a height (2) at the point y, it must pass below q, and therefore below $q\ q'$, and must cut the curve $(e, 2)$, in some point. Hence it cannot be the required plane.

Commencing at p, lay off in like manner divisions equal to those of $a\ b$, and repeat the construction with respect to each of the points n, p, q, &c., until one is found from which the divisions being laid off all the points lie nearer to l than the corresponding points n, p, q, &c., at the same elevation. These will be the required points of graduation of the scale of the required plane. In the diagram, r is the point from which the divisions must be laid off.

FIG. 3

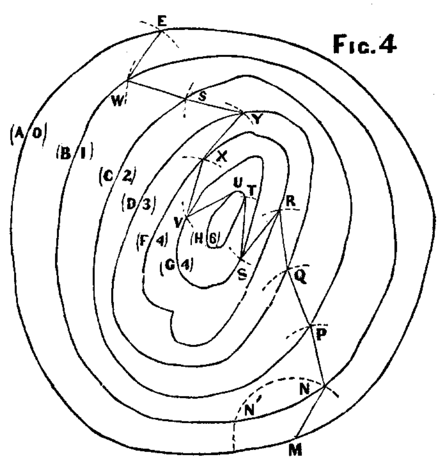


To avoid confusion in the diagram, a parallel to $l\ m$ has been drawn, and the scale is constructed on it. r' is the point of contact, and $(a', 0)$—b', 6 is the tangent plane.

PROBLEM V. Fig. 4.—To ascend a given hill with a given slope.

Let $(a, 0)$—$(h, 6)$ be the hill, and $\frac{m}{n}$ the given slope. The secant planes being at the unit's distance apart, $1 \div \frac{m}{n} = \frac{n}{m}$ will be the length of the projection of that portion of the required line which joins any two adjacent contour lines, m being the point at which the ascent is to begin, with m as a centre, a radius equal to $\frac{n}{m}$ describe the arc of a circle cutting $(b, 1)$ in m; in the same manner with n as a centre find p; then q, r, s, t, and $m\ n\ p\ q\ r\ s\ t$ will

FIG. 4



be the projection of the required line. If it is required to descend the hill so as to reach the bottom at a point as nearly opposite to m as possible, trace $(h, 6)$ to u, and with u as a centre and the same radius as before describe an arc cutting $(g, 5)$ in v; then find x, y, z, w, e, and $u\ v\ x\ y\ z\ w\ e$ will be the projection of the line of descent.

If the slope of descent should differ from that of ascent, the radius will be found by dividing 1 by the slope. The contour lines are supposed to be so near to each other that the surface between them may be generated by the motion of a right line resting upon both and remaining constantly perpendicular to one, or making equal angles with them both. The arc described from m as a centre, and a radius equal to $m\ n$ cuts $(b, 1)$ in two places, m' and n, and there may be two lines joining $(a, 0)$ and $(b, 1)$ which will fulfil the given condition. In the same manner it can be shown, that having found n' and n, four lines may join them with the next curve, all of which will fulfil the given condition. Hence quite a number of lines, all beginning at m, may be drawn which will fulfil the required condition. Although these lines all begin at the same point, they will not end at the same. In a case of practice, then, these lines, or several of them, should be examined in detail, and that one selected which actual examination proves to be best.

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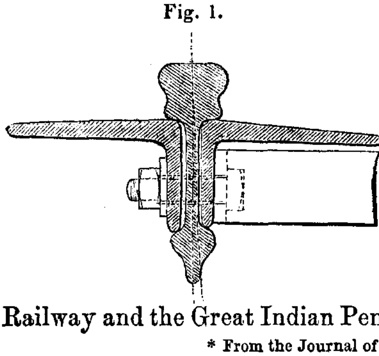
*Adams's System of Permanent Way.**

In February, 1856, an account was given in the pages of this *Journal* of a system of wrought iron Permanent Way for Railways, by Mr. W. Bridges Adams, denominated the Suspended Girder Rail. The testimony in its favor by several eminent railway engineers, and the practical experience since gained confirmatory of their opinions, together with its adoption for more than one important line, make it probable that this class of way will be ultimately preferred, wherever iron is considered imperative and timber inadmissible. Experiments are still going on to ascertain the minimum of bearing surface which will suffice, in order to keep down the weight and cost to the absolutely requisite quantity, and it is probable that a considerable reduction will be the result. For the purposes of export to India this is of great importance, as every ton weight averages thirty shillings for freight—about one-sixth added to the original cost of the materials. And wrought iron is in all structures a very considerable reduction of dead weight, with equal strength and greater security as compared with cast iron. The two sections hitherto applied are as follows:—Single-headed (Fig. 1) and Double-headed (Fig. 2).

Fig. 1.



Meanwhile two of the highest Indian authorities—the East Indian Railway and the Great Indian Peninsular Railway—appear to consider

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