CAS QFT Lift I:
Spectral Fock Quantization, Quantum Feshbach Reduction, and Readout Renormalization

Andrea Ceccotti

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Abstract

This version constructs a theorem-led quantum-field-theoretic lift of the spectral-affine effective framework. The construction does not attach an ordinary spacetime-first quantum field theory to the theory from the outside. Instead it starts from isolated spectral sectors, promotes their Riesz ranges to one-particle Hilbert spaces, applies the Fock functor, represents the resulting operator algebra through the Lorentzian readout map, and derives effective Green functions by quantum Feshbach-Schur reduction. The central outputs are an explicit spectral-Fock QFT lift theorem, a Gaussian path-integral derivation of quantum Schur reduction, a quantum Schur action theorem, a Feshbach Green-function theorem, a pole/self-energy equivalence, a Kato stability theorem for radiative perturbations, a readout-scale matching prescription for renormalization-group flow, and a gate ledger for gauge fixing, BRST consistency, anomalies, and observable amplitudes. The result is a genuine effective QFT readout of the spectral-affine construction. It is not presented as a completed microscopic quantum theory of the full pre-readout substrate; it is the operational quantum field theory generated by the isolated sectors that survive the readout and pass the stated gates.

Contents

1 Purpose and public claim

The purpose of this document is to turn the spectral-affine observable chain into a quantum-field-theoretic interface that is mathematically explicit enough to support propagators, loop corrections, renormalization, pole masses, amplitudes, widths, and branching ratios. The construction is deliberately not spacetime-first. In an emergent-readout theory, the usual procedure of postulating fields on a fixed Lorentzian manifold and quantizing them at equal time cannot be the foundational starting point. The correct starting point is the spectral sector that the theory already isolates. Only after such a sector has been promoted to a Hilbert/Fock representation should it be read as a local field on the effective Lorentzian branch.

The public claim is therefore precise. Given an isolated physical spectral sector, a positive physical inner product, a stable readout map, gauge/anomaly consistency, and a controlled matching scale, the spectral-affine construction produces an effective quantum field theory. The chain is

$$P_\Gamma(K) \xrightarrow{\mathcal{F}} \mathcal{F}_\pm(\operatorname{Ran} P_\Gamma) \xrightarrow{\mathcal{R}} \widehat{\Phi}(x) \xrightarrow{\Gamma_{\text{eff}}} G^{(n)} \xrightarrow{\text{LSZ/readout}} \mathcal{S}. \tag{1}$$

This is not a decorative diagram. Each arrow is constructed below as a theorem, a gate, or an explicit calculation.

1.1 What changes compared with the purely classical reduction

The earlier reduction based on Schur complements was classically correct but incomplete as a quantum interface. At the classical or stationary level, eliminating a hidden sector gives a Schur complement in the Hessian. In the path integral, the same elimination gives the Schur term plus a functional determinant. Thus the classical effective Hessian becomes the tree part of the quantum effective action. This is the first major upgrade of the present document:

$$K_{LL} - K_{LH}K_{HH}^{-1}K_{HL} \quad \rightsquigarrow \quad \Gamma^{(1)}_{\text{eff}} = S_{\text{Schur}} + \frac{i}{2}\operatorname{Tr}\log K_{HH} \tag{2}$$

for a bosonic Gaussian hidden sector, with the corresponding sign change for fermionic determinants.

The second major upgrade is that spectral carriers are not called particles by declaration. A compact spectral carrier becomes a quantum object only when its Riesz sector supports a Fock representation, its projected Green function has stable poles, and those poles admit a Lorentzian readout into amplitudes. The Feshbach pole condition is therefore identified with the ordinary QFT on-shell pole condition.

2 Version 2 hard core: the QFT lift theorem

The v2 revision promotes the construction from an interface proposal to a theorem-led effective QFT lift. The basic issue is not whether one can write ordinary fields after the Lorentzian branch has appeared. One can. The issue is whether those fields are generated from the intrinsic spectral data of the theory rather than imported as external degrees of freedom. The answer is affirmative under explicit gates.

Theorem 2.1 (Spectral-Fock QFT lift theorem)**.** *Let K be a closed effective spectral-affine kernel on a state space $\mathcal{H}$. Suppose that a contour $\Gamma \subset \rho(K)$ isolates a physical spectral sector, that $\mathfrak{h}_\Gamma = \operatorname{Ran} P_\Gamma(K)$ carries a positive physical inner product after quotienting unphysical directions, that the statistics gate assigns CAR or CCR, and that a Lorentzian readout map $\mathcal{R}_\Gamma$ represents the resulting modes as sections over an effective branch. If the gauge sector passes the BRST gate, the chiral sector passes the anomaly gate, the hidden sector admits quantum Feshbach-Schur reduction, and the readout scale is treated as a matching scale, then* *the sector defines an effective QFT readout with Fock states, Green functions, pole conditions, renormalization push-forward, and observable-amplitude gates.*

Proof. The proof is constructive. The Riesz projector

$$P_\Gamma(K) = \frac{1}{2\pi i}\oint_\Gamma (z-K)^{-1}\,dz \tag{3}$$

selects an invariant sector. Positivity turns its range into a one-particle Hilbert space. The Fock functor then constructs $\mathcal{F}_\pm(\mathfrak{h}_\Gamma)$ with the appropriate CAR or CCR algebra. The readout map represents basis vectors as local modes, hence turns the abstract creation and annihilation algebra into field operators. The quadratic part of the action gives inverse Green functions. Hidden-sector integration gives the quantum Schur kernel plus determinant terms. The Feshbach identity identifies the projected resolvent with the inverse of the effective Feshbach operator, so zeros of that operator are poles of the Green function. Kato stability controls the persistence of the spectral sector under loop-induced perturbations. Gauge and anomaly gates ensure that the quantum measure is not inconsistent. RG matching then transports readout boundary data to comparator scales, and LSZ/readout extracts amplitudes when finite residues and external states exist. □

2.1 Gaussian path integral derivation of quantum Schur

For retained variables ϕ and hidden variables χ, take the quadratic action

$$S^{(2)}[\phi,\chi] = \frac{1}{2}\phi K_{LL}\phi + \phi K_{LH}\chi + \frac{1}{2}\chi K_{HH}\chi. \tag{4}$$

Completing the square gives

$$S^{(2)}[\phi,\chi] = \frac{1}{2}\phi\left(K_{LL} - K_{LH}K_{HH}^{-1}K_{HL}\right)\phi \tag{5}$$

$$+\frac{1}{2}\left(\chi + K_{HH}^{-1}K_{HL}\phi\right)K_{HH}\left(\chi + K_{HH}^{-1}K_{HL}\phi\right). \tag{6}$$

Therefore

$$\exp\{i\Gamma_{\text{eff}}^{(1)}[\phi]\} = \int \mathcal{D}\chi\,\exp\{iS^{(2)}[\phi,\chi]\} \tag{7}$$

$$\propto (\det K_{HH})^{-1/2}\exp\left\{\frac{i}{2}\phi\left(K_{LL} - K_{LH}K_{HH}^{-1}K_{HL}\right)\phi\right\}. \tag{8}$$

Thus the one-loop bosonic effective action is

$$\Gamma_{\text{eff}}^{(1)}[\phi] = \frac{1}{2}\phi K_{\text{Schur}}\phi + \frac{i}{2}\operatorname{Tr}\log K_{HH}, \qquad K_{\text{Schur}} = K_{LL} - K_{LH}K_{HH}^{-1}K_{HL}. \tag{9}$$

For Grassmann hidden variables the determinant enters with the opposite loop sign:

$$\Gamma_{\text{1loop}} = \frac{i}{2}\operatorname{Tr}\log K_B - i\operatorname{Tr}\log D_F + \cdots. \tag{10}$$

This is the precise point where the classical Schur complement becomes the tree part of a quantum effective action.

2.2 Feshbach Green theorem and self-energy

Let P be the retained sector and $Q = 1 - P$. If $Q(K - z)Q$ is invertible, the Feshbach operator is

$$F_P(z) = P(K - z)P - PKQ[Q(K - z)Q]^{-1}QKP. \tag{11}$$

Block inversion gives

$$P(K - z)^{-1}P = F_P(z)^{-1}. \tag{12}$$

Consequently the projected Green function has poles when

$$\det F_P(z) = 0. \tag{13}$$

In a local readout chart this is the same condition as

$$p^2 - m_0^2 - \Sigma(p^2) = 0, \tag{14}$$

with the Feshbach term playing the role of self-energy. This establishes the promised bridge between compact spectral carriers and QFT on-shell conditions: a carrier is not a particle by name, but its Feshbach-resolvent pole is exactly the mathematical object that can become a particle pole after readout.

2.3 Kato stability under radiative perturbations

Let $K(\lambda) = K_0 + \lambda V$ and suppose Γ isolates a spectral cluster of K_0. For $z \in \Gamma$,

$$z - K(\lambda) = (z - K_0)\left[1 - \lambda(z - K_0)^{-1}V\right]. \tag{15}$$

If $\|\lambda(z-K_0)^{-1}V\| < 1$ uniformly on Γ, the Neumann series gives an analytic resolvent. Therefore

$$P_\Gamma(\lambda) = \frac{1}{2\pi i}\oint_\Gamma (z - K(\lambda))^{-1}\, dz \tag{16}$$

exists, varies analytically, and has locally constant rank. Simple poles then move analytically until a gap closes or a threshold collision occurs. This is the stability theorem required for radiative corrections.

2.4 BRST and anomaly gates as quantum-measure gates

The gauge-fixed sector must be controlled by a nilpotent BRST differential,

$$sA_\mu^a = D_\mu^{ab}c^b, \qquad sc^a = -\frac{1}{2}f^a{}_{bc}c^bc^c, \qquad s\bar{c}^a = B^a, \qquad sB^a = 0. \tag{17}$$

Nilpotency follows from closure of the projected algebra and the Jacobi identity. The chiral sector must also pass the anomaly gate. In the all-left convention, Yukawa and anomaly conditions give

$$q = \frac{1}{6}, \quad u = -\frac{2}{3}, \quad d = \frac{1}{3}, \quad l = -\frac{1}{2}, \quad e = 1, \tag{18}$$

with

$$2q + u + d = 0, \qquad 3q + l = 0, \tag{19}$$

$$6q + 3u + 3d + 2l + e = 0, \qquad 6q^3 + 3u^3 + 3d^3 + 2l^3 + e^3 = 0. \tag{20}$$

The Witten parity count gives $N_2 = 3 + 1 = 4 = 0 \mod 2$. Hence the minimal chiral package is admissible as a QFT matter sector.

2.5 Readout matching and observable extraction

The readout scale is a matching scale, not a vague cutoff:

$$g_a(\Lambda_{\mathrm{read}}), \quad y_f(\Lambda_{\mathrm{read}}), \quad \lambda_H(\Lambda_{\mathrm{read}}), \quad C_i(\Lambda_{\mathrm{read}}). \tag{21}$$

Below that scale, ordinary effective-QFT running applies:

$$\mu\frac{dg_a}{d\mu} = \beta_{g_a}, \qquad \mu\frac{dC_i}{d\mu} = \gamma_i{}^j C_j. \tag{22}$$

Only after RG push-forward and finite pole residues does the LSZ/readout gate produce amplitudes,

$$\mathcal{A}(i \to f) = \prod_r Z_r^{-1/2} \lim_{p_r^2 \to m_r^2} (p_r^2 - m_r^2) G^{(n)}. \tag{23}$$

This closes the operational QFT chain from isolated spectral sector to observable amplitudes.

3 Input contract

Definition 3.1 (Effective spectral-affine input)**.** *The QFT lift assumes an effective operator or kernel K acting on a separable auxiliary state space $\mathcal{H}$ after the first physical quotient and readout gates have isolated the branch of interest. The operator may be a Hessian, a Dirac-type operator, a kinetic operator, or a block kernel obtained from a spectral-affine effective action. Its physical sectors are selected by contours in the resolvent set.*

The field content is not postulated as a Standard-Model list. It is reconstructed from five classes of spectral objects:

Sector	Spectral object	Quantum lift	Readout field
Gauge	projected affine connection sector	bosonic Fock representation after gauge fixing	$A^a_\mu(x)$
Scalar	isolated coherent scalar sector	bosonic Fock representation	$H(x)$ or $h(x)$
Fermionic	chiral zero modes of an affine-Dirac operator	antisymmetric Fock representation	$\psi_f(x)$
Vorton carrier	compact Riesz cluster	sector-dependent Fock representation	$\chi_V(x)$ only if the readout localizes it
Hidden/heavy	complementary sector	integrated or retained by Feshbach	Wilson coefficients, self-energies, or hidden fields

Gate 3.1 (No premature particle identification)**.** *A compact spectral carrier is not identified with a particle until it has: an isolated Riesz sector, a physical Fock representation, a readout-localized field or external state, a stable pole or mass shell, and at least one computable amplitude into visible or specified hidden final states.*

4 Riesz projectors as one-particle sectors

Let K be a closed operator with non-empty resolvent set. Suppose that a contour Γ encloses a compact part of $\mathrm{spec}(K)$ and no other spectral point. The Riesz projector is

$$P_\Gamma(K) = \frac{1}{2\pi i}\oint_\Gamma (z-K)^{-1}\, dz. \tag{24}$$

The physical one-particle candidate space is

$$\mathfrak{h}_\Gamma = \operatorname{Ran} P_\Gamma(K). \tag{25}$$

This definition is independent of any coordinate chart on the emergent spacetime. It is a spectral definition.

Assumption 4.1 (Physical inner product)**.** *The range $\mathfrak{h}_\Gamma$ carries a positive physical inner product $\langle \cdot, \cdot \rangle_\Gamma$ after the removal of null, gauge, or unphysical directions. If the preliminary form is indefinite, the QFT lift is applied only after the physical quotient or Krein-to-Hilbert reduction.*

Theorem 4.1 (Riesz sector as one-particle space)**.** *Under the isolation and positivity assumptions, $\mathfrak{h}_\Gamma$ is a well-defined one-particle Hilbert space for the spectral sector selected by Γ.*

Proof. The Riesz projector is idempotent by the standard resolvent identity. Its range is invariant under K and contains precisely the spectral component enclosed by Γ. The positivity assumption supplies the Hilbert completion of this range. No spacetime coordinate has been used, so the result is a spectral one-particle construction rather than a spacetime-first postulate. □

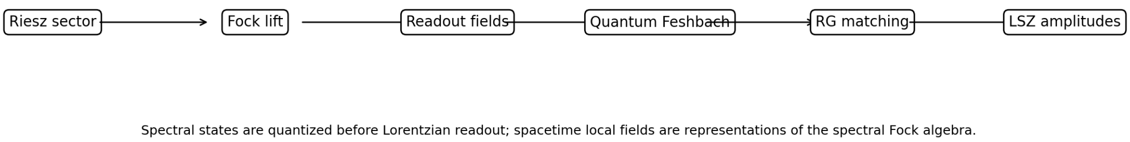

Figure 1: The operational QFT lift starts from isolated spectral sectors, not from a pre-existing field list on spacetime.

5 The Fock functor over spectral sectors

Given a one-particle space $\mathfrak{h}_\Gamma$, define

$$\mathcal{F}_+(\mathfrak{h}_\Gamma) = \bigoplus_{n=0}^{\infty} \operatorname{Sym}^n \mathfrak{h}_\Gamma, \qquad \mathcal{F}_-(\mathfrak{h}_\Gamma) = \bigoplus_{n=0}^{\dim \mathfrak{h}_\Gamma} \wedge^n \mathfrak{h}_\Gamma. \tag{26}$$

The choice of sign is not arbitrary: it is determined by the spin/statistics gate after Lorentzian readout, or by the chiral/spinorial origin of the sector when that origin is already known.

For fermionic sectors, define creation and annihilation operators by the CAR

$$\{a(u), a^\dagger(v)\} = \langle u, v \rangle_\Gamma, \qquad \{a(u), a(v)\} = 0. \tag{27}$$

For bosonic sectors, define them by the CCR

$$[a(u), a^\dagger(v)] = \langle u, v \rangle_\Gamma, \qquad [a(u), a(v)] = 0. \tag{28}$$

Theorem 5.1 (Spectral Fock lift)**.** *Every isolated physical spectral sector satisfying the positivity gate admits a canonical Fock lift, bosonic or fermionic according to the statistics gate.*

Proof. The theorem is the application of the Fock functor to the one-particle Hilbert space $\mathfrak{h}_\Gamma$. The construction is functorial: an isometry of one-particle spaces induces a unitary transformation of the Fock representation. Therefore the quantization is attached to the spectral sector itself and not to a coordinate-dependent choice of basis. □

6 Readout maps and emergent local fields

The Lorentzian readout is a representation map

$$\mathcal{R}_\Gamma : \mathfrak{h}_\Gamma \longrightarrow \Gamma(E_\Gamma \to M_{\text{eff}}), \qquad u \mapsto u_\nu(x), \tag{29}$$

where M_{eff} is the emergent Lorentzian readout branch and E_Γ is the effective bundle in which the sector appears. In an orthonormal basis $\{e_j\}$ of $\mathfrak{h}_\Gamma$, the local field operator is

$$\widehat{\Phi}(x) = \sum_j \left[a_j u_j(x) + a_j^\dagger u_j^*(x)\right] \tag{30}$$

for a real bosonic sector, with the usual particle/antiparticle form for complex or fermionic sectors,

$$\widehat{\psi}(x) = \sum_j \left[a_j u_j(x) + b_j^\dagger v_j(x)\right]. \tag{31}$$

This is not the starting point. It is the representation of the spectral Fock algebra after readout.

Gate 6.1 (Readout localization)**.** *A spectral Fock sector becomes a local QFT sector only if the readout map supplies distributions or modes whose two-point function has controlled support, causal propagation in the effective Lorentzian metric, and finite residues at the physical poles.*

7 Quantum Schur complement

Consider a real bosonic quadratic action decomposed into light/readout variables ϕ and heavy/hidden variables χ:

$$S^{(2)}[\phi,\chi] = \frac{1}{2}\phi K_{LL}\phi + \phi K_{LH}\chi + \frac{1}{2}\chi K_{HH}\chi. \tag{32}$$

Completing the square gives

$$\chi K_{HH}\chi + 2\phi K_{LH}\chi = (\chi + K_{HH}^{-1}K_{HL}\phi)K_{HH}(\chi + K_{HH}^{-1}K_{HL}\phi) - \phi K_{LH}K_{HH}^{-1}K_{HL}\phi. \tag{33}$$

Therefore the functional integral over χ gives

$$\exp\{i\Gamma_{\text{eff}}^{(1)}[\phi]\} = \int \mathcal{D}\chi\, \exp\{iS^{(2)}[\phi,\chi]\} \tag{34}$$

$$\propto \exp\left\{\frac{i}{2}\phi(K_{LL} - K_{LH}K_{HH}^{-1}K_{HL})\phi\right\}(\det K_{HH})^{-1/2}. \tag{35}$$

Writing the determinant as an exponential, one obtains

$$\Gamma_{\text{eff}}^{(1)}[\phi] = \frac{1}{2}\phi K_{\text{Schur}}\phi + \frac{i}{2}\operatorname{Tr}\log K_{HH}, \tag{36}$$

where

$$K_{\text{Schur}} = K_{LL} - K_{LH}K_{HH}^{-1}K_{HL}. \tag{37}$$

For fermionic hidden variables the determinant appears in the numerator and the trace-log sign changes accordingly.

Theorem 7.1 (Quantum Schur action theorem)**.** *The classical Schur complement is the tree part of the one-loop quantum effective action obtained by integrating out a Gaussian hidden sector. The quantum correction is the functional determinant of the hidden-sector kinetic operator.*

Proof. The proof is exactly the completion of the square above. The stationary solution $\chi_* = -K_{HH}^{-1}K_{HL}\phi$ gives the Schur complement, while the integration over fluctuations around χ_* gives the determinant. Hence the classical hidden-sector elimination and the one-loop functional integral are the same construction at two levels of approximation. □

8 Feshbach reduction of Green functions

Let P be a spectral projector and $Q = 1 - P$. For an operator K and spectral parameter z, define

$$F_P(z) = P(K - z)P - PKQ[Q(K - z)Q]^{-1}QKP. \tag{38}$$

Whenever the inverse on the Q sector exists, the projected resolvent satisfies

$$P(K - z)^{-1}P = F_P(z)^{-1}. \tag{39}$$

This is the Green-function form of the QFT lift. It does not merely eliminate hidden variables from the action; it gives the exact projected propagator.

Theorem 8.1 (Feshbach Green-function theorem)**.** *For a closed decomposition $P + Q = 1$ and z such that $Q(K - z)Q$ is invertible, the projected full Green function on* $\operatorname{Ran} P$ *is the inverse of the Feshbach operator $F_P(z)$.*

Proof. Write $K - z$ in block form. The inverse of a block matrix has PP block equal to the inverse of the Schur complement of the QQ block. This Schur complement is precisely $F_P(z)$. □

9 Pole equation and self-energy

In ordinary QFT, the physical mass or resonance is read from a pole of a full propagator. In a scalar toy sector,

$$G(p^2) = \frac{i}{p^2 - m_0^2 - \Sigma(p^2) + i0}, \tag{40}$$

so the pole condition is

$$p^2 - m_0^2 - \Sigma(p^2) = 0. \tag{41}$$

In the Feshbach language, the same statement is

$$\det F_P(z) = 0. \tag{42}$$

The self-energy is the Feshbach correction

$$\Sigma_P(z) = PKQ[Q(K - z)Q]^{-1}QKP. \tag{43}$$

Thus

$$F_P(z) = P(K - z)P - \Sigma_P(z). \tag{44}$$

Theorem 9.1 (Feshbach pole/self-energy equivalence)**.** *For a spectral sector selected by P, the zeros of* $\det F_P(z)$ *are precisely the poles of the projected Green function. In a Lorentzian readout, these are the candidate on-shell masses or resonances of the sector.*

Proof. By the Feshbach Green-function theorem, the projected propagator is $F_P(z)^{-1}$. It has a pole exactly when $F_P(z)$ fails to be invertible, which is expressed by $\det F_P(z) = 0$ in finite rank or by the corresponding Fredholm determinant in the trace-class setting. □

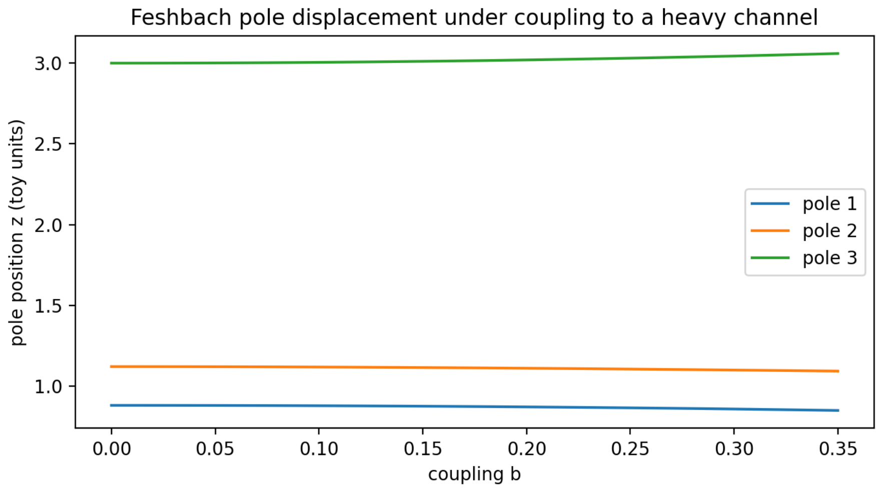

Figure 2: Toy Feshbach pole displacement as a compact carrier is coupled to a heavy complement. The plot is diagnostic, not a fit.

10 Kato stability and radiative perturbations

Let

$$K(\lambda) = K_0 + \lambda V \tag{45}$$

with V relatively bounded with respect to K_0. If the contour Γ remains inside the resolvent set of $K(\lambda)$, then

$$P_\Gamma(\lambda) = \frac{1}{2\pi i} \oint_\Gamma (z - K(\lambda))^{-1} \, dz \tag{46}$$

varies analytically with λ. In physical language, if loop corrections do not close the spectral gap, the isolated sector survives quantization.

Theorem 10.1 (Radiative stability of an isolated sector)**.** *Assume a spectral cluster of K_0 is separated from its complement by a gap $g > 0$. If the radiative perturbation λV is relatively bounded with norm below the gap threshold, then the rank of the Riesz projector is stable and simple poles move analytically.*

Proof. This is the Kato analytic perturbation theorem for isolated spectral subspaces. The resolvent remains analytic on the contour while the contour does not intersect the spectrum. The Riesz integral therefore defines an analytic projector with constant rank. A simple eigenvalue or pole enclosed by the contour has an analytic perturbative expansion until collision with another spectral point or a continuum threshold. □

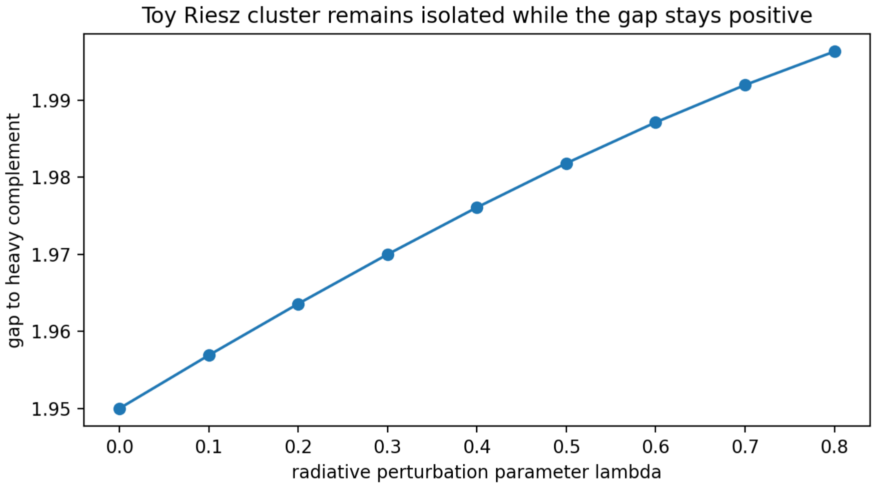

Figure 3: Toy gap stability check. The Riesz sector remains well-defined while the gap to the heavy complement remains positive.

11 Gauge fields, gauge fixing, and BRST

The gauge layer after readout has the leading form

$$\mathcal{L}_{\mathrm{gauge}} = -\frac{1}{4}\sum_a \kappa_a F^a_{\mu\nu} F^{\mu\nu}_a, \qquad g_a = \kappa_a^{-1/2}. \tag{47}$$

For QFT, this is not enough. The gauge orbit must be fixed. Choose a gauge functional $G^a[A]$. The gauge-fixing and ghost terms are

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{2\xi} G^a G^a, \qquad \mathcal{L}_{\mathrm{gh}} = \bar{c}^a \frac{\delta G^a}{\delta \alpha^b} c^b. \tag{48}$$

In covariant Yang-Mills gauge, $G^a = \partial^\mu A^a_\mu$ and

$$\mathcal{L}_{\mathrm{gh}} = \bar{c}^a(-\partial^\mu D^{ab}_\mu) c^b. \tag{49}$$

The BRST differential is

$$sA^a_\mu = D_\mu c^a, \tag{50}$$

$$sc^a = -\frac{1}{2} f^a{}_{bc} c^b c^c, \tag{51}$$

$$s\bar{c}^a = B^a, \tag{52}$$

$$sB^a = 0. \tag{53}$$

The gate is

$$s^2 = 0, \qquad sS_q = 0. \tag{54}$$

The nilpotency follows from the Lie algebra Jacobi identity. Thus the QFT lift requires algebraic closure of the projected gauge sector, not just a numerical value of a coupling.

Gate 11.1 (Gauge QFT gate)**.** *A projected gauge sector is a quantizable QFT sector only if its algebra closes to the required accuracy, its gauge-fixed quadratic kernel is invertible, the ghost complex is defined, and the BRST differential is nilpotent.*

12 Chiral anomaly gate

The fermionic lift is quantizable only if the chiral gauge package is anomaly-safe. In left-handed Weyl convention, a minimal chiral package has charges

$$Q = (3,2)_q, \quad u^c \text{ optional}, \quad u^c = (1,1)_0 \text{ if sterile}, \tag{55}$$

with

$$u = -q - h, \qquad d = -q + h, \qquad e = h - l, \qquad h = \frac{1}{2}. \tag{56}$$

The anomaly equations are

$$\begin{aligned} SU(3)^2U(1): &\quad 2q + u + d = 0, &(57)\\ SU(2)^2U(1): &\quad 3q + l = 0, &(58)\\ \text{grav}^2U(1): &\quad 6q + 3u + 3d + 2l + e = 0, &(59)\\ U(1)^3: &\quad 6q^3 + 3u^3 + 3d^3 + 2l^3 + e^3 = 0. &(60) \end{aligned}$$

Solving gives

$$q = \frac{1}{6}, \quad u = -\frac{2}{3}, \quad d = \frac{1}{3}, \quad l = -\frac{1}{2}, \quad e = 1. \tag{61}$$

The Witten global anomaly requires an even number of weak doublets. With three colored quark doublets and one lepton doublet,

$$N_2 = 3 + 1 = 4 = 0 \mod 2. \tag{62}$$

Therefore the chiral package passes the anomaly gate once the spectral zero-mode package is supplied.

13 Readout scale as matching scale

The readout scale should not be presented as an arbitrary replacement for the renormalization group. It is the matching scale at which spectral-affine data are converted into QFT boundary data:

$$\Lambda_{\text{read}}: \qquad \{\kappa_a, y_f, \lambda_H, C_i, z_V\}_{\text{CAS}} \longmapsto \{g_a, y_f, \lambda_H, C_i, m_V\}_{\text{QFT}}. \tag{63}$$

From that scale to an experimental scale μ, the standard QFT push-forward is governed by beta functions:

$$\mu\frac{dg_a}{d\mu} = \beta_{g_a}, \qquad \mu\frac{dy_f}{d\mu} = \beta_{y_f}, \qquad \mu\frac{d\lambda_H}{d\mu} = \beta_\lambda, \qquad \mu\frac{dC_i}{d\mu} = \gamma_{ij}C_j. \tag{64}$$

For a one-loop gauge toy model,

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_a^2(\Lambda_{\text{read}})} - \frac{b_a}{8\pi^2}\log\frac{\mu}{\Lambda_{\text{read}}}. \tag{65}$$

This explains why several forward-ledger quantities are scheme-sensitive: CAS supplies boundary data, while QFT supplies scale transport.

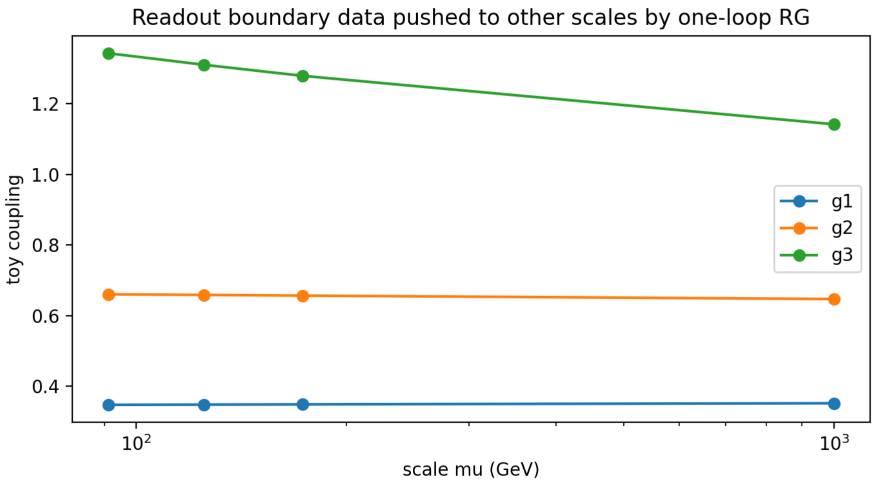

Figure 4: Toy readout-scale matching and one-loop running. The figure illustrates the interface, not a fitted prediction.

14 LSZ/readout gate

The QFT lift reaches observables only after Green functions have external poles with finite residues. For a field Φ, suppose

$$G_2(p) = \frac{iZ}{p^2 - m_{\text{phys}}^2 + i0} + \text{regular terms.} \tag{66}$$

Then the external readout state is normalized by $Z^{-1/2}$ and amplitudes are obtained by the usual reduction, adapted to the readout map:

$$\mathcal{A}(i \to f) = \prod_{r \in \text{ext}} Z_r^{-1/2} \left[\prod_r (p_r^2 - m_r^2) G^{(n)}(p_1, \dots, p_n) \right]_{p_r^2 \to m_r^2} . \tag{67}$$

The decay and scattering layer then uses

$$\Gamma(i \to f) = \frac{1}{2m_i} \int \overline{|\mathcal{A}(i \to f)|^2} \, d\Phi_n, \qquad BR(i \to f) = \frac{\Gamma(i \to f)}{\Gamma_{\text{tot}}}. \tag{68}$$

This recovers the existing observable pipeline as the downstream consequence of the QFT lift.

15 Vorton carrier as quantum spectral sector

A vorton carrier is represented by a compact Riesz sector

$$P_V = P_{\Gamma_V}(K), \qquad \mathfrak{h}_V = \operatorname{Ran} P_V. \tag{69}$$

If $\mathfrak{h}_V$ is finite rank, the quantum state space is immediately constructed as $\mathcal{F}_\pm(\mathfrak{h}_V)$. Coupling to a complement is governed by the Feshbach operator

$$F_V(z) = K_{VV} - z - K_{VE}(K_{EE} - z)^{-1} K_{EV}. \tag{70}$$

A simple two-level carrier coupled to one heavy channel has the pole equation

$$(M - z)((m - z)^2 - \omega^2) - |b|^2(m - z) = 0. \tag{71}$$

For small b, the light poles shift as

$$z_{\pm} = m \pm \omega - \frac{|b|^2}{2(M - m \mp \omega)} + O(|b|^4). \tag{72}$$

This is the same object that QFT calls a self-energy correction. The carrier becomes a physical particle candidate only after its poles receive a readout, residues, charges, and amplitudes.

16 Connection to the forward observable ledger

The forward observable ledger supplies the downstream numerical targets of this QFT lift. The relation is structural:

$$\text{spectral sector} \to \text{Fock/QFT sector} \to \text{matching data} \to \text{RG push-forward} \to \text{observable ledger row}. \tag{73}$$

The status categories are now sharpened:

Status	Meaning in the QFT lift
source-derived	The formula exists in the canonical externalized source chain and has a forward computation.
forward-computed	The quantity is computed from the seed/readout chain, but a deeper parent-operator proof may still be pending.
kernel-conditional	The formula is valid once a stated kernel, projector, or overlap is supplied.
microscopic-parent-open	The effective QFT object exists, but its derivation from the full pre-readout parent operator is not complete.
benchmark-only	External data are used for comparison or calibration, not as a blind output.
scheme-sensitive	The quantity depends on QFT scheme, scale, threshold, or non-perturbative matching.

The QFT lift does not weaken the forward ledger. It explains how ledger quantities become QFT quantities: through boundary data at Λ_{read}, Green-function poles, Wilson coefficients, amplitudes, and RG transport.

17 Failure modes

The construction is useful because it is falsifiable at the interface level. The main failure modes are:

1. The spectral sector is not isolated; then no stable Riesz projector exists.

2. The physical inner product is not positive after quotient; then no Hilbert/Fock representation exists.

3. The readout map does not produce controlled local modes; then no local QFT representation is obtained.

4. The hidden kernel is not invertible or not perturbatively controllable; then quantum Schur/Feshbach fails.

5. Gauge algebra does not close sufficiently; then BRST quantization is not available.

6. Chiral anomalies do not cancel; then the chiral gauge QFT is inconsistent.
7. Radiative corrections close the spectral gap; then Kato stability is lost.
8. RG matching scale is ambiguous beyond tolerances; then numerical ledger rows become scheme-dependent.
9. No external pole/residue exists; then there is no LSZ/readout amplitude.

18 Constructive algorithm of the QFT lift

The lift is not a metaphor. It is an algorithm that can be executed on any sector once the required spectral data are available. The algorithm has two layers. The first layer is intrinsic and spectral. The second layer is the external readout by which the spectral quantities become the objects normally called quantum fields, propagators, masses, widths, and cross sections.

1. Choose the effective kernel or operator K delivered by the variational or spectral construction.
2. Identify a candidate physical sector by a contour Γ that encloses a stable part of $\mathrm{spec}(K)$.
3. Form $P_\Gamma = (2\pi i)^{-1} \oint_\Gamma (z - K)^{-1} dz$.
4. Test idempotence, rank stability, and compatibility with the physical inner product.
5. Promote $\mathrm{Ran}\, P_\Gamma$ to a one-particle Hilbert space $\mathfrak{h}_\Gamma$.
6. Apply the Fock functor to obtain $\mathcal{F}_\pm(\mathfrak{h}_\Gamma)$.
7. Use the Lorentzian readout map $\mathcal{R}_\Gamma$ to represent basis vectors by modes on the effective branch.
8. Build the quadratic readout kernel and invert it after gauge fixing if necessary.
9. Integrate hidden or heavy complements by quantum Schur or by Feshbach reduction.
10. Define Green functions and locate their projected poles.
11. Match the readout-scale data to Wilsonian QFT boundary data.
12. Apply RG push-forward to the scale at which an observable is defined.
13. Use the LSZ/readout gate to convert pole residues and Green functions into amplitudes.
14. Insert amplitudes into the phase-space formulas for rates and cross sections.

This algorithm also defines what cannot be skipped. A sector with no isolated contour cannot be quantized as a stable spectral sector. A sector with no positive physical inner product cannot be promoted to a Hilbert/Fock representation. A pole with no readout residue cannot be called a particle mass. A channel with no amplitude and phase-space measure cannot be assigned a branching ratio.

19 Finite-rank and continuous sectors

The most transparent case is finite rank. If $\dim \operatorname{Ran} P_\Gamma = N < \infty$, the fermionic Fock space has dimension 2^N and the bosonic Fock space is infinite dimensional unless an occupation cutoff is imposed by dynamics. The finite-rank fermionic case is especially important for chiral matter sectors and compact carriers:

$$\mathcal{F}_-(\mathfrak{h}_\Gamma) = \mathbb{C} \oplus \mathfrak{h}_\Gamma \oplus \wedge^2 \mathfrak{h}_\Gamma \oplus \cdots \oplus \wedge^N \mathfrak{h}_\Gamma. \tag{74}$$

The algebra is generated by $a_i, a_i^\dagger$ with

$$\{a_i, a_j^\dagger\} = \delta_{ij}, \qquad \{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} = 0. \tag{75}$$

This makes the finite spectral rank physically meaningful: it controls the number of independent one-particle modes before any spacetime field expansion is written.

For continuous or infinite-rank sectors, Γ may isolate a band rather than a finite cluster. The one-particle space is then a Hilbert space of spectral wave packets. The Fock construction still works, but the readout map must control locality, support, and spectral measure. The correct object is no longer a finite matrix determinant, but a Fredholm determinant, zeta-regularized determinant, heat-kernel determinant, or Wilsonian regularization depending on the operator class.

Gate 19.1 (Finite-rank simplification). *A finite-rank Riesz sector may be quantized directly as a finite one-particle sector. An infinite-rank sector requires an additional spectral measure and regularization prescription before functional determinants and traces are meaningful.*

20 Spin, statistics, and the chiral package

The Fock sign is determined by the spinorial origin of the sector and by the Lorentzian readout. For zero modes of an affine Dirac operator, the sector is fermionic. For projected gauge and scalar sectors, it is bosonic. If a compact carrier is not yet assigned a Lorentz representation, the QFT lift keeps its Fock sign as a gate rather than assigning it by name.

For a chiral family sector, the one-particle space takes the schematic form

$$\mathfrak{h}_F = P_L P_{\text{int}} \operatorname{Ker} D_{\text{aff}}, \tag{76}$$

where P_L is a chirality projector and P_{int} selects internal quantum numbers. The local fermion field after readout is then

$$\widehat{\psi}_f(x) = \sum_n \left[a_{fn} u_{fn}(x) + b_{fn}^\dagger v_{fn}(x) \right]. \tag{77}$$

The chiral anomaly calculation is not optional. It is the statement that the fermionic measure can be consistently defined under the gauge symmetries of the readout. A chiral sector that fails the anomaly gate may still exist as a formal spectral sector, but it cannot define a consistent gauge QFT readout.

21 Quantum Schur beyond the Gaussian approximation

The Gaussian formula is the exact one-loop result for a quadratic hidden sector. If the hidden sector has interactions,

$$S[\phi, \chi] = S_0[\phi, \chi] + S_{\text{int}}[\phi, \chi], \tag{78}$$

then the effective action is

$$\exp\{i\Gamma_{\text{eff}}[\phi]\} = \exp\{iS_{\text{tree}}[\phi]\} \exp\left\{\frac{i}{2} \operatorname{Tr}\log K_{HH}\right\} \langle\exp\{iS_{\text{int}}[\phi,\eta]\}\rangle_{K_{HH}}\,, \tag{79}$$

where η is the shifted hidden fluctuation. Expanding the final expectation value gives the loop expansion. Thus the classical Schur term, the one-loop determinant, and higher loop diagrams are not separate ideas. They are successive orders of the same hidden-sector functional integral.

The practical implication is important. A classical Schur complement in the earlier theory should not be thrown away. It should be promoted to the tree component of Γ_{eff}. The quantum correction ledger then asks which determinants, traces, and loop insertions must be added for the precision required by a given observable.

22 Dyson equation as Feshbach equation

The Feshbach correction has the form of a self-energy. Write

$$F_P(z) = K_P^{(0)} - z - \Sigma_P(z), \qquad \Sigma_P(z) = PKQ[Q(K-z)Q]^{-1}QKP. \tag{80}$$

The projected Green function is

$$G_P(z) = F_P(z)^{-1}. \tag{81}$$

This is the same algebraic structure as the Dyson-resummed propagator

$$G(p) = \Big(G_0(p)^{-1} - \Sigma(p)\Big)^{-1}. \tag{82}$$

Therefore the QFT lift identifies the Feshbach complement with the geometric origin of the self-energy. Loop corrections are then perturbations of the Feshbach kernel, not external decorations of a classical pole equation.

Proposition 22.1 (Dyson-Feshbach dictionary)**.** *For every projected sector P, the following dictionary holds:*

QFT object	*Spectral-affine/Feshbach object*
free inverse propagator	$P(K-z)P$
self-energy	$PKQ[Q(K-z)Q]^{-1}QKP$
full projected propagator	$F_P(z)^{-1}$
pole mass/resonance	*zero of* $\det F_P(z)$
wave-function residue	*inverse derivative of F_P at the pole*
threshold correction	*singularity or branch structure of $Q(K-z)Q$*

23 Residues and wave-function normalization

A pole is not enough. It must have a residue. For a scalar projected sector with simple pole z_*, write

$$F_P(z) = F_P'(z_*)(z-z_*) + O((z-z_*)^2). \tag{83}$$

Then

$$G_P(z) = \frac{1}{F_P'(z_*)}\frac{1}{z-z_*} + \cdots\,. \tag{84}$$

The residue is

$$Z_P = \left(F_P'(z_*)\right)^{-1}. \tag{85}$$

For matrix-valued F_P, one projects onto the left and right null vectors of $F_P(z_*)$. If $F_P(z_*)r = 0$ and $\ell^\dagger F_P(z_*) = 0$, then the pole residue along that mode is controlled by

$$Z_*^{-1} = \ell^\dagger F_P'(z_*)r. \tag{86}$$

This supplies the normalization entering LSZ/readout reduction. Hence a spectral carrier requires not just a pole equation, but a residue calculation.

24 Readout-scale matching of masses and couplings

The matching scale is where spectral-affine quantities are translated into QFT boundary data. A mass-like spectral pole z_* may be read as

$$m_{\mathrm{read}}^2 = z_* \tag{87}$$

when the readout variable is a relativistic p^2 coordinate. If the spectral parameter is energy-like, the map is instead $m_{\mathrm{read}} = z_*$. The QFT lift therefore requires a declared spectral-to-Lorentzian coordinate convention for each sector.

The same applies to couplings. A kinetic normalization κ_a gives

$$g_a(\Lambda_{\mathrm{read}}) = \kappa_a^{-1/2}. \tag{88}$$

But this is a boundary condition, not necessarily the value measured at another scale. Experimental comparison requires

$$g_a(\mu) = U_a(\mu, \Lambda_{\mathrm{read}})g_a(\Lambda_{\mathrm{read}}), \tag{89}$$

where U_a is the RG evolution operator in the effective QFT.

25 Wilson coefficients and operator basis

Once hidden and heavy sectors are integrated out, the effective QFT contains local and quasi-local operators:

$$S_{\mathrm{eff}} = S_{d\le 4} + \sum_{d>4}\sum_i \frac{C_i^{(d)}(\Lambda_{\mathrm{read}})}{\Lambda_{\mathrm{read}}^{d-4}} \int d^4x\, \mathcal{O}_i^{(d)}(x). \tag{90}$$

The coefficients are not arbitrary. In the lift they come from overlap integrals, Feshbach complements, functional determinants, or residual readout corrections. Their RG flow is

$$\mu\frac{d}{d\mu}C_i(\mu) = \gamma_{ij}(\mu)C_j(\mu). \tag{91}$$

This is the correct place for rare decays, hidden leakage, radiative channels, and threshold corrections. It also gives a clean explanation of why a forward observable may be robust in one scheme but require additional matching data in another.

26 BRST and anomaly checks as quantum-measure gates

Gauge fixing gives a propagator, but anomaly cancellation gives a measure. The generating functional has the schematic form

$$Z[J] = \int \frac{\mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}H\mathcal{D}c\mathcal{D}\bar{c}}{\mathrm{Vol}(\mathcal{G})} \exp\left(iS_q[A,\psi,H,c,\bar{c}] + iJ\cdot\Phi\right). \tag{92}$$

After gauge fixing, BRST invariance replaces naive gauge invariance. If the chiral fermion measure has an anomaly, the BRST Ward identities fail. Thus the anomaly gate is a quantum condition, not a bookkeeping check.

The anomaly gate can be phrased as

$$\mathcal{A}_{abc} = \mathrm{Tr}_R(T_a\{T_b, T_c\}) = 0 \tag{93}$$

for local gauge anomalies, together with the mixed and global checks appropriate to the effective gauge group. When this gate passes, the chiral spectral sector is eligible for QFT quantization.

27 Observable construction

The final output of the lift is not a formal path integral; it is a controlled way to compute observables. A typical decay channel is constructed by the chain

$$\Gamma\text{-sector} \to \mathcal{F} \to \widehat{\Phi}(x) \to C_i(\mu)\mathcal{O}_i(\mu) \to \mathcal{A}(i \to f) \to \Gamma(i \to f) \to BR. \tag{94}$$

A scattering process is similarly

$$G^{(n)} \to \mathcal{A} \to d\sigma. \tag{95}$$

The existing forward observable ledger becomes the downstream table in which each observable row records: spectral source, matching scale, QFT object, RG scheme, pole/residue status, amplitude status, and experimental comparison.

28 Promotion conditions

The following promotion ladder replaces vague language:

Promotion	Required condition
spectral candidate to quantum sector	isolated Riesz sector plus positive physical inner product
quantum sector to readout field	stable readout map into effective Lorentzian modes
readout field to propagating degree	invertible gauge-fixed quadratic kernel and pole prescription
propagating degree to particle candidate	isolated pole with finite residue and physical quantum numbers
particle candidate to observable state	LSZ/readout amplitude to at least one allowed channel
observable state to prediction	all coefficients fixed at matching scale and RG scheme declared
benchmark to prediction	external target removed from inputs and used only downstream

This table is the guardrail against overclaiming. It also allows genuine progress: a result can be strong at the quantum-sector level even if it is not yet a precision collider prediction.

29 Minimal symbolic test suite

The accompanying script implements a toy suite with four checks. First, it computes a Schur effective kernel and the associated trace-log determinant. Second, it solves a Feshbach pole equation for a compact two-level carrier coupled to a heavy complement. Third, it verifies that a toy Riesz cluster remains rank-stable under a bounded perturbation. Fourth, it pushes toy readout coupling data across scales using one-loop RG equations. These checks are not physical fits. Their role is to make the QFT lift reproducible at the level of algebraic identities.

30 Conclusion

The QFT lift is real in the following operational sense. Isolated spectral sectors define one-particle spaces by Riesz projection; those spaces define Fock representations; readout maps represent the Fock algebra as local fields; quantum Schur and Feshbach reduction define effective actions and Green functions; poles of the projected Green functions define mass/resonance candidates; Kato theory controls their stability under radiative perturbations; gauge fixing, BRST, and anomaly cancellation provide the consistency gates; the readout scale supplies QFT boundary data; RG flow transports them to experimental scales; LSZ/readout reduction produces amplitudes, widths, and cross sections.

This does not claim that the full pre-readout substrate has been quantized as a fundamental microscopic QFT. It does claim that the effective spectral-affine readout has a precise QFT lift. That is the interface needed to make the theory operational.

A Toy numerical checks

The accompanying script verifies the algebraic toys used in the text. The Schur effective kernel obtained from the toy matrices is

$$K_{\text{eff}} = \begin{pmatrix} 1.089754 & 0.077037 \\ 0.077037 & 1.345829 \end{pmatrix}. \tag{96}$$

The bosonic hidden determinant contributes a trace-log

$$\operatorname{Tr}\log K_{HH} = 2.991222. \tag{97}$$

The toy Feshbach poles for $b = 0.22$ are approximately

$$3.023997, 1.107890, 0.868113. \tag{98}$$

These numbers are included only to verify the identities and the data pipeline; they are not phenomenological fits.

B Formula registry

Object	Formula
Riesz projector	$P_\Gamma = (2\pi i)^{-1} \oint_\Gamma (z-K)^{-1} dz$
One-particle sector	$\mathfrak{h}_\Gamma = \operatorname{Ran} P_\Gamma$
Fock space	$\mathcal{F}_\pm(\mathfrak{h}_\Gamma) = \oplus_n \mathrm{Sym}/\mathrm{Alt}^n \mathfrak{h}_\Gamma$
Quantum Schur kernel	$K_{LL} - K_{LH} K_{HH}^{-1} K_{HL}$
Bosonic determinant	$(i/2) \operatorname{Tr}\log K_{HH}$
Feshbach operator	$F_P(z) = P(K-z)P - PKQ[Q(K-z)Q]^{-1}QKP$
Projected Green function	$P(K-z)^{-1}P = F_P(z)^{-1}$
Pole equation	$\det F_P(z) = 0$
RG push-forward	$\mu dg/d\mu = \beta_g$
LSZ/readout	external pole residue reduction of $G^{(n)}$

C Source integration note

The text deliberately avoids internal development labels. The integrated source content is represented publicly as six externalized layers: gauge-scalar projection, chiral consistency, spectral zero-mode carrier construction, decay-operator grammar, seed-to-observable forward map, and observable-ledger status taxonomy. The audit files in the bundle record the engineering provenance without requiring the public document to use internal labels.

D QFT gate ledger

Gate	Mathematical object	Pass condition	Output
Spectral isolation	contour Γ around a cluster of K	positive distance from the rest of the spectrum and finite or controlled rank	Riesz projector P_Γ
Hilbert positivity	inner product on $\operatorname{Ran} P_\Gamma$	positive after physical quotient or Krein reduction	one-particle space $\mathfrak{h}_\Gamma$
Fock promotion	$\mathfrak{h}_\Gamma$	CAR for fermions, CCR for bosons	$\mathcal{F}_\pm(\mathfrak{h}_\Gamma)$
Readout map	$\mathcal{R}_\Gamma$	stable Lorentzian representation and controlled localization	local modes $u_j(x)$
Quantum Schur	block kernel K_{LL}, K_{LH}, K_{HH}	K_{HH} invertible with prescription	tree Schur term plus trace-log
Quantum Feshbach	$P+Q=1$ and resolvent	$Q(K-z)Q$ invertible	projected Green function
Pole gate	$\det F_P(z)=0$	isolated simple pole and finite residue	mass or resonance readout
Gauge fixing	gauge orbit and gauge functional	invertible quadratic gauge-fixed kernel	propagators and ghosts
BRST	nilpotent differential s	$s^2=0$ and $sS_q=0$	physical gauge consistency
Anomaly gate	chiral trace sums	local and global anomalies vanish	consistent chiral measure
RG matching	Λ_{read}	boundary data specified at scale	running couplings and coefficients
LSZ/readout	external Green-function poles	residues finite and asymptotic/readout states defined	amplitudes and observables

E Theorem ledger

Theorem	Claim	Main hypotheses	Failure mode
Riesz-sector Fock lift	an isolated sector defines one-particle and Fock spaces	contour isolation, positivity, controlled rank	gap closure or indefinite metric
Quantum Schur action	hidden functional integration gives Schur plus determinant	Gaussian hidden kernel, invertibility	uncontrolled non-Gaussian sector
Feshbach Green function	projected resolvent equals inverse Feshbach operator	$P+Q=1$, invertible Q block	threshold or contour collision

Pole/self-energy equivalence	Feshbach zeros are QFT poles	isolated simple pole and analytic correction	branch cut or non-isolated pole
Kato stability	spectral ranks and simple poles persist under small perturbations	perturbation below spectral gap	loop correction closes gap
Readout matching	CAS boundary data can be RG-pushed to experimental scales	matching scale and beta functions declared	threshold or scheme ambiguity
Anomaly gate	chiral gauge sector is quantizable only when anomalies vanish	anomaly equations satisfied	local or global anomaly

F Field promotion map

Sector	Spectral object	One-particle space	Readout field	Statistics
Gauge	projected affine gauge sector	transverse/readout modes	$A^a_\mu(x)$	bosonic
Scalar	isolated coherent scalar mode	$\operatorname{Ran} P_H$	$H(x)$ or $h(x)$	bosonic
Fermion	zero modes of D_{aff}	chiral projected kernel	$\psi_f(x)$	fermionic
Carrier	compact Riesz cluster P_V	$\operatorname{Ran} P_V$	$\chi_V(x)$ if localized	gate-dependent
Hidden/heavy	complement Q	$\operatorname{Ran} Q$	Wilson/self-energy data	mixed
Metric-affine	readout perturbations	physical quotient modes	$h_{\mu\nu}, \delta\Gamma$	bosonic

G Renormalization matching ledger

Object	Boundary datum	QFT push-forward	Observable use
Gauge coupling	$g_a(\Lambda) = \kappa_a^{-1/2}$	β_{g_a}	scattering, widths, matching
Yukawa or mass operator	spectral matter kernel or $y_f(\Lambda)$	β_y and thresholds	fermion masses and Higgs decays
Scalar quartic	quartic overlap and scalar readout	β_λ	Higgs mass and stability
Wilson coefficient	Feshbach/overlap coefficient	anomalous dimensions	decay and rare-process channels
Carrier pole	$\det F_V(z) = 0$	self-energy/Kato displacement	candidate mass or resonance

H Public source integration map

The source integration used in this QFT lift is expressed here only in public conceptual terms. No development labels are needed for the scientific argument.

Externalized layer	Content integrated	Public wording
Gauge-scalar projection	projected connection, Yang-Mills kinetic form, electroweak and scalar masses	gauge-scalar readout layer
Chiral consistency	Yukawa charge equations, anomaly cancellation, Witten parity	chiral anomaly gate
Spectral carrier	affine-Dirac zero modes, Riesz projectors, compact carrier poles	spectral zero-mode carrier layer
Decay operators	operator, amplitude, phase-space, width, branching chain	decay operator grammar
Forward observables	seed-to-observable map and status taxonomy	observable forward ledger
QFT interface	Fock construction, Green functions, RG, LSZ	effective QFT readout

I Script excerpt

The complete script is included in the bundle. The excerpt below shows the core algebraic checks used to generate the toy data.

```
#!/usr/bin/env python3
"""CAS QFT Lift I v2.0 symbolic checks.
Checks: anomaly cancellation, two-sector quantum Feshbach poles,
Gaussian Schur determinant, Kato gap stability toy, RG matching demo,
and the su(2) Jacobi identity needed for BRST nilpotency.
"""
from fractions import Fraction
import json, math
import numpy as np

q=Fraction(1,6); u=Fraction(-2,3); d=Fraction(1,3); l=Fraction(-1,2); e=Fraction(1,1); h=Fraction(1,2)
anomalies={
 'SU3_SU3_U1': str(2*q+u+d),
 'SU2_SU2_U1': str(3*q+l),
 'grav_grav_U1': str(6*q+3*u+3*d+2*l+e),
 'U1_U1_U1': str(6*q**3+3*u**3+3*d**3+2*l**3+e**3),
 'Witten_SU2_doublets_mod2': str((3+1)%2)
}

def schur(KLL, KLH, KHH):
    KLL=np.array(KLL,dtype=float); KLH=np.array(KLH,dtype=float); KHH=np.array(KHH,dtype=float)
    return KLL-KLH@np.linalg.inv(KHH)@KLH.T

def two_sector_poles(m=1.0,M=4.0,g=0.2):
    roots=sorted(np.roots([1.0,-(m+M),m*M-g*g]).real.tolist())
    return {'exact':roots,'light_first_order':m-g*g/(M-m),'heavy_first_order':M+g*g/(M-m)}

def eps(a,b,c):
    if len({a,b,c})<3: return 0
    perm=[a,b,c]; inv=0
    for i in range(3):
        for j in range(i+1,3): inv += int(perm[i]>perm[j])
    return -1 if inv%2 else 1

def jacobi_su2():
    for a in range(3):
```

```
      for b in range(3):
       for c in range(3):
        for d in range(3):
         s=sum(eps(a,b,x)*eps(x,c,d)+eps(b,c,x)*eps(x,a,d)+eps(c,a,x)*eps(x,b,d) for x in range(3))
         if s!=0: return False
    return True

def rg_demo(g0=0.653,b=-19/6,mu0=357.62,mu=91.1876):
    inv=1/(g0*g0)-b/(8*math.pi*math.pi)*math.log(mu/mu0)
    return 1/math.sqrt(inv)

if __name__=='__main__':
    KLL=[[1.1,0.08],[0.08,1.35]]; KLH=[[0.2,0.05],[0.04,0.14]]; KHH=[[4.0,0.3],[0.3,5.0]]
    detlog=float(np.linalg.slogdet(np.array(KHH))[1])
    m=1.0; M=4.0; g=0.2; gap=abs(M-m)
    out={
      'anomalies': anomalies,
      'anomaly_pass': all(v=='0' for v in anomalies.values()),
      'schur_kernel': schur(KLL,KLH,KHH).tolist(),
      'bosonic_trace_log_hidden_block': detlog,
      'feshbach_two_sector': two_sector_poles(m,M,g),
      'kato_gap_bound': {'gap':gap,'perturb_norm':abs(g),'passes_gap_over_4': abs(g)<gap/4},
      'rg_one_loop_demo': {'g_mu0':0.653,'mu0':357.62,'mu':91.1876,'g_mu':rg_demo()},
      'su2_jacobi_for_brst_toy': jacobi_su2(),
      'status':'algebraic and numerical toy checks only; not a phenomenological fit'
    }
    print(json.dumps(out,indent=2))
```

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