Cosmological Branches and Event Classes from Spectral-Affine Selection

Andrea Ceccotti

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Abstract

This paper develops the cosmological layer of the premetric affine sequence only after the prior construction of the physical quotient, relational clock, Lorentzian readout, metric-affine effective dynamics, spectral matter kernels, and gauge normal forms. Its purpose is not to fit a cosmological history and not to introduce a multiverse narrative. It proves instead a controlled mathematical framework in which cosmological alternatives are represented by branch chambers in a finite or locally finite semialgebraic gate space, and physically meaningful cosmological events are represented by equivalence classes of wall crossings and branch transitions. We define branch coordinates, gate margins, chamber sign vectors, event windows, transition graphs, continuous-clock Markov generators, terminal classes, and absorption probabilities. The main results include a chamber stability theorem, a wall transversality theorem, event-class invariance under reparametrization, positivity and conservation for the branch master equation, the absorption formula for terminal branch classes, and a falsification theorem showing which failures invalidate the branch readout. A worked four-branch calculation derives absorption probabilities and expected hitting times explicitly, while a linearized event-window calculation gives the crossing probability $\Phi(-m/\sigma)$ for a one-gate Gaussian perturbation and its multigate union bound. Cosmology is therefore treated here as a consequence of branch selection and event-class readout, not as an assumed background spacetime history. The paper deliberately avoids numerical cosmological claims, particle phenomenology, and speculative terminal-universe interpretations. Those require additional observational maps and source terms not supplied by the branch formalism alone.

Contents

1 Introduction

Cosmology is usually introduced by assuming a spacetime geometry and then imposing large-scale symmetry, matter content, and dynamical equations. This order is natural inside general relativity and its effective-field-theoretic extensions, but it is not natural in a construction whose earlier layers deliberately avoid assuming spacetime primitives. Once a relational quotient, a clock functional, a Lorentzian readout, a metric-affine effective geometry, spectral matter kernels, and gauge normal forms have been constructed, the next question is not which Friedmann model to choose. The next question is how a mathematically defined branch of effective physics is selected, how transitions between branches are classified, and under what conditions those transitions can be interpreted as cosmological events.

The present paper develops this branch layer. The word cosmological is used in a restricted sense. A cosmological branch is not yet a full universe with a fitted expansion history, a baryon content, a dark sector, or a reheating model. It is a stable readout chamber of the effective affine-spectral structure after the previously established gates have been passed. A cosmological event is not an arbitrarily named occurrence in time. It is an equivalence class of transitions between such chambers, controlled by wall crossings, gate-margin loss, source activation, or terminal absorption. This vocabulary is deliberately austere because the branch layer would otherwise become a repository for speculation. The aim is to build a formalism that is hard to misuse.

The mathematical object underlying the construction is a gate space. Its coordinates collect the effective quantities whose signs, ranks, gaps, and positivity conditions were shown in earlier papers to decide whether a readout is admissible. Some gates are inequalities, such as positivity of a spatial block or nonnegativity of a kinetic form. Some gates are rank constraints, such as the persistence of a spectral sector. Some gates are gap constraints, such as isolation of a Riesz projector or stability of a Lorentzian chamber. Once these gates are assembled, their sign and rank pattern divides the space of effective parameters into chambers. A branch is a connected component of the admissible region. A wall is a codimension-one failure of one or more gates. An event class is the invariant data of moving from one branch chamber to another through a wall, modulo harmless reparametrizations and descriptive choices.

This point of view changes the role of cosmological calculation. Instead of starting with a scale factor and then asking which model it obeys, one first calculates the chamber graph, transition rates or weights, terminal classes, event windows, and absorption probabilities. Only after this calculation is done may one attach observational readout maps such as expansion, curvature, relic density, or late-time effective sources. The order matters. If expansion history is introduced too early, the branch structure is no longer derived; it is fitted. If terminal events are named before the gate algebra is known, the classification is narrative rather than mathematical. If transition probabilities are written without a generator satisfying positivity and conservation, they are not probabilities.

The main results of the paper are therefore structural but computational. We prove that finite polynomial gate families define semialgebraic chamber decompositions. We prove a stability theorem showing that branch membership is unchanged under perturbations smaller than the gate margin. We prove a wall transversality theorem for single-gate crossings and explain how multiple-gate collisions define higher-codimension event classes. We define transition generators with respect to the relational clock and prove positivity and total-probability conservation for the branch master equation. We then derive the continuous-time absorption formula for terminal branch classes, together with expected hitting times. Finally, we calculate a minimal four-branch example and a linearized event-window crossing probability. These are not toy decorations: they are the calculations required before cosmology can be read as branch dynamics.

The scope remains limited. This paper does not identify our universe with a specific chamber, does not predict observed cosmological parameters, does not infer black-hole-born branches, does not discuss multiverse ontology, and does not use empirical data. It establishes the mathematical layer that such claims would have to pass through. In this sense it is both conservative and necessary. A later cosmological specialization may add observational maps and source models; it may not bypass the branch and event-class gates developed here.

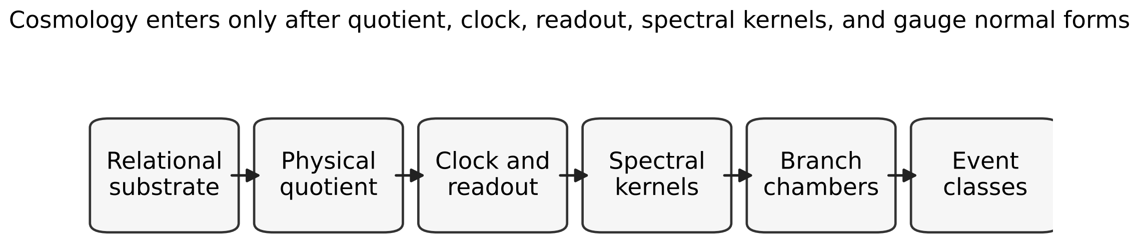

Figure 1: Position of the branch layer in the sequence. Cosmology appears only after the quotient, clock, readout, spectral kernel, and gauge normal-form layers are available. The diagram is intentionally ordered to prevent branch language from being used as an early replacement for missing dynamics.

2 Mathematical contract and inherited structures

The branch formalism uses the previous layers only through explicit mathematical inputs. This section records those inputs without restating the full construction. Let $\mathcal{Q} = \mathcal{S}/\!\sim$ denote the physical quotient, let $\tau : \mathcal{Q} \to \mathbb{R}$ be the relational clock on the admissible part of the quotient, let H_{eff} be the effective Hessian obtained after projection and Schur reduction, let $\mathcal{K}_V(z)$ be the spectral matter kernel associated with a compact carrier sector when such a sector exists, and let $K_{\mathfrak{g}}$ denote the positive gauge kinetic form after normal-form reduction. These objects enter the present paper only through their gate functions. The branch layer does not assume their existence; it assumes that prior papers have supplied them and that the relevant gates can be evaluated.

A branch coordinate is any scalar or finite-dimensional effective quantity built from the preceding objects that has an invariant meaning under admissible re-description. Examples include a Hessian inertia margin, a spectral gap, a determinant of a positive block, a Schur complement margin, a kinetic positivity margin, a source-activation parameter, or a compactness index. The actual list is theory-dependent. What matters here is that each coordinate descends to the quotient and is not changed by a harmless choice of presentation.

Definition 2.1 (Branch coordinate)**.** *Let $\mathcal{Q}_{\text{adm}} \subset \mathcal{Q}$ be the part of the physical quotient on which the readout, spectral, and kinetic structures needed for the branch layer are defined. A branch coordinate is a map*

$$\theta^a : \mathcal{Q}_{\text{adm}} \longrightarrow \mathbb{R} \tag{1}$$

that is invariant under physical equivalence and admissible re-description. A finite family $\theta = (\theta^1, \ldots, \theta^N)$ defines a branch-coordinate map

$$\theta : \mathcal{Q}_{\text{adm}} \longrightarrow \Theta \subseteq \mathbb{R}^N. \tag{2}$$

The passage from the quotient to branch coordinates is already a lossy readout. It keeps only those effective quantities relevant to chamber membership and event classification. This is analogous to keeping order parameters in phase-transition theory, but the analogy should not be

$$
\begin{tikzcd}
\mathcal{Q}_{\mathrm{adm}} \arrow[r, "\theta"] \arrow[d, "\tau"'] & \Theta \arrow[r, "\mathcal{G}"] & \{g_i\}_{i=1}^m \arrow[d, "\text{sign/rank/gap}"] \\
\mathbb{R} \arrow[r, "\text{transition order}"'] & \text{branch graph} \arrow[r, "\text{events}"'] & \text{event classes}
\end{tikzcd}
$$

Figure 2: Mathematical contract of the branch layer. No external time, background cosmology, or observed expansion model is inserted at this stage. The relational clock orders transitions, while the branch gates are inherited from earlier effective structures.

overextended. Here the order parameters are not thermodynamic assumptions; they are margins and invariants computed from the affine-spectral construction.

Definition 2.2 (Gate family)**.** *A finite branch gate family on* Θ *is a collection*

$$\mathcal{G} = \{g_i : \Theta \to \mathbb{R}\}_{i=1}^m \tag{3}$$

with the interpretation that a sign, rank, or positivity condition is satisfied when $g_i(\theta) > 0$*, is critical when* $g_i(\theta) = 0$*, and fails when* $g_i(\theta) < 0$*. When a rank or spectral condition is naturally expressed by a determinant, discriminant, spectral gap, or principal minor, it is represented by the corresponding gate function.*

The finite gate-family assumption is not a claim that reality has finitely many possible diagnostics. It is a local statement: every finite article, computation, or experimental specialization evaluates only a finite set of gates. Infinite gate families can be treated by compactness or local finiteness assumptions, but the finite case is the correct baseline for an auditable paper.

Assumption 2.3 (Polynomial or analytic gates)**.** *Unless explicitly stated otherwise, the gates are polynomial or real analytic functions of the branch coordinates on the domain under consideration. Polynomial gates give semialgebraic chambers; analytic gates give locally finite stratifications under standard regularity hypotheses. The proofs below use polynomial gates when a finite global chamber decomposition is required.*

Definition 2.4 (Admissible branch region)**.** *The admissible branch region is*

$$\Theta_+ = \{\theta \in \Theta \,:\, g_i(\theta) > 0 \textit{ for every mandatory gate } i\}. \tag{4}$$

Optional gates and terminal gates may be tracked separately; the mandatory set consists of those conditions without which the effective readout used by the paper is not defined.

The branch layer can therefore be summarized by the diagram in figure 2. The physical quotient supplies invariant states. The clock supplies the ordering parameter used for transition dynamics. The effective Hessian, spectral kernels, and kinetic forms supply gate functions. The gate functions define chambers, walls, and event classes.

3 Branch chambers and semialgebraic event walls

The first mathematical task is to make precise what a branch is. A branch is not simply a point in parameter space, because finite perturbations and continuous evolution should not change the branch unless a gate is crossed. It is therefore a chamber: a connected component of a sign-defined admissible region.

Definition 3.1 (Sign vector). *Given a gate family $\mathcal{G} = \{g_i\}_{i=1}^m$, the sign vector at a noncritical point θ is*

$$\sigma(\theta) = \big(\operatorname{sgn} g_1(\theta), \ldots, \operatorname{sgn} g_m(\theta)\big) \in \{-1, +1\}^m. \tag{5}$$

When some $g_i(\theta) = 0$, the point belongs to a wall stratum and its sign vector is partially undefined. The active set is

$$I(\theta) = \{i : g_i(\theta) = 0\}. \tag{6}$$

Definition 3.2 (Branch chamber). *A branch chamber is a connected component of a set of the form*

$$C_\sigma = \{\theta \in \Theta : \operatorname{sgn} g_i(\theta) = \sigma_i \text{ for all } i\}, \tag{7}$$

restricted to the admissible region when mandatory gates are specified. A cosmological branch is an admissible branch chamber together with the inherited readout data that remain valid throughout that chamber.

This definition is intentionally topological and algebraic rather than interpretive. A chamber may later be mapped to an expanding branch, a contracting branch, a terminal branch, or an observationally viable branch, but those names require extra readout maps. At the present level the branch is the connected region in which the defining gates do not change sign or rank.

Theorem 3.3 (Semialgebraic chamber decomposition). *Let $\Theta \subseteq \mathbb{R}^N$ be semialgebraic and let $g_1, \ldots, g_m$ be polynomial functions. Then the noncritical sets obtained by fixing the signs of the g_i are semialgebraic, and each has finitely many connected components when restricted to a compact semialgebraic subset of Θ. Consequently, a compact gate domain has finitely many branch chambers.*

Proof. For each sign vector $\sigma \in \{-1, +1\}^m$, the set C_σ is described by a finite Boolean combination of polynomial inequalities, namely $\sigma_i g_i(\theta) > 0$ for $i = 1, \ldots, m$, together with the polynomial equalities and inequalities defining Θ. Hence C_σ is semialgebraic. A standard finiteness theorem for semialgebraic sets states that a semialgebraic set has finitely many connected components on a compact semialgebraic domain. Since there are only finitely many sign vectors, the total number of chambers is finite on such a domain. The result is local in noncompact settings by restricting to compact windows in branch-coordinate space. □

Definition 3.4 (Wall stratum and event support). *For an index set $I \subseteq \{1, \ldots, m\}$, the wall stratum supported on I is*

$$W_I = \{\theta \in \Theta : g_i(\theta) = 0 \text{ for } i \in I, \quad g_j(\theta) \neq 0 \text{ for } j \notin I\}. \tag{8}$$

A transition whose path intersects W_I and changes the signs of precisely the gates in I has event support I.

Proposition 3.5 (Transversal wall crossing). *Let $\gamma : (-\delta, \delta) \to \Theta$ be a C^1 path and let g_i be C^1. Suppose $g_i(\gamma(0)) = 0$ and*

$$\left.\frac{\mathrm{d}}{\mathrm{d}s} g_i(\gamma(s))\right|_{s=0} = \nabla g_i(\gamma(0)) \cdot \dot{\gamma}(0) \neq 0. \tag{9}$$

Then $g_i(\gamma(s))$ changes sign at $s = 0$, and the crossing is stable under sufficiently small C^1 perturbations of γ and g_i.

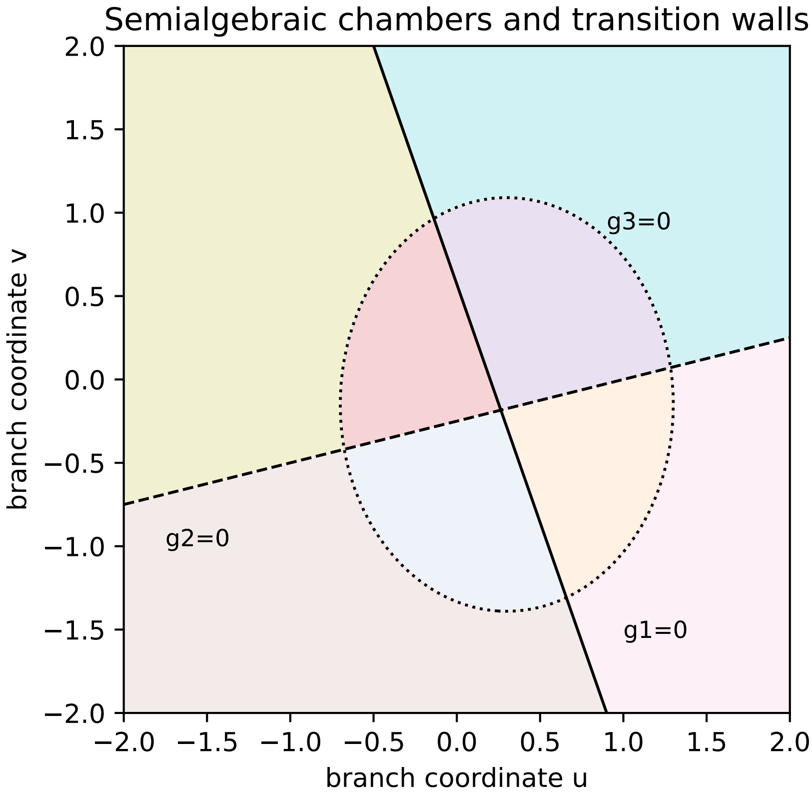

Figure 3: A schematic two-dimensional branch-coordinate slice with three gates. The solid, dashed, and dotted curves represent $g_1 = 0$, $g_2 = 0$, and $g_3 = 0$. Branches are connected sign chambers. Event classes live on wall crossings, not on arbitrary labels assigned to the regions.

Proof. The derivative condition implies that $h(s) = g_i(\gamma(s))$ has a simple zero at $s = 0$. By Taylor expansion,

$$h(s) = h'(0)s + o(s). \tag{10}$$

For sufficiently small positive and negative s, the sign of $h(s)$ is therefore the sign of $h'(0)s$, so the gate changes sign. Simplicity of the zero is an open condition in the C^1 topology: if the perturbed derivative remains nonzero and the perturbation is small on a compact neighborhood, the implicit-function theorem supplies a nearby unique zero, and the sign change persists. □

4 Event classes and re-description invariance

A cosmological event must be invariant under the same kind of descriptive freedom that earlier papers removed by quotienting. Otherwise the event classification depends on coordinates used to draw branch space rather than on physical gate data. The correct object is therefore not a coordinate crossing but an equivalence class of crossings with the same gate support, orientation, and admissible before-and-after chambers.

Definition 4.1 (Elementary event)**.** *An elementary event is a tuple*

$$e = (C_-, C_+, I, o) \tag{11}$$

where C_- and C_+ are branch chambers, I is the active wall support crossed by a path from C_- to C_+, and o records the orientation of the sign change on the active gates. For a single gate i, the *orientation is $o_i = - \to +$ or $+ \to -$ according to the sign of g_i before and after the crossing.*

Definition 4.2 (Event-class equivalence). *Two elementary events are equivalent if there exists an admissible re-description of branch coordinates that maps the before chamber to the before chamber, the after chamber to the after chamber, preserves the active gate ideal generated by $\{g_i : i \in I\}$ up to multiplication by nonvanishing positive functions, and preserves the orientation of the sign change. An event class is an equivalence class of elementary events.*

This definition allows the same event to be represented in different coordinate charts or by different positive normalizations of the gates. It disallows changes that flip a gate by multiplying it by a negative function, because that would reverse the meaning of admissibility. It also disallows collapsing two distinct active gates into one unless the gate ideal and rank of the active differential system are preserved.

Theorem 4.3 (Event-class invariance). *Let $\phi : U \to U'$ be a C^1 diffeomorphism between branch-coordinate charts and suppose the transformed gates satisfy*

$$g_i'(\phi(\theta)) = a_i(\theta) g_i(\theta), \qquad a_i(\theta) > 0 \tag{12}$$

on a neighborhood of the crossing. Then chamber membership, wall support, and crossing orientation are preserved. Hence the elementary event determines the same event class in the two charts.

Proof. Because $a_i > 0$, g_i and $g_i' \circ \phi$ have the same sign at corresponding points and vanish on the same set. Therefore the sign vector away from the wall is preserved. The active set is also preserved because $g_i(\theta) = 0$ if and only if $g_i'(\phi(\theta)) = 0$. Along a crossing path γ, the signs of $g_i(\gamma(s))$ and $g_i'(\phi(\gamma(s)))$ are identical for $s \neq 0$ sufficiently small, so the orientation of sign change is identical. Thus the tuple (C_-, C_+, I, o) is unchanged up to the prescribed re-description. $\square$

Remark 4.4. *The positivity of the multiplier is essential. Multiplication by a negative non-vanishing function leaves the zero set fixed but reverses the admissible side of the gate. Such a transformation is not a harmless re-description of the branch event; it changes the physical meaning of the gate.*

5 Branch dynamics in relational clock time

Once chambers and event classes are defined, the next task is to describe transitions. The relational clock from the earlier construction supplies the ordering parameter. The branch formalism does not introduce an external Newtonian time. It writes a coarse-grained transition dynamics with respect to τ on the quotient-readout domain. This is a crucial distinction: the Markov generator below is not fundamental time evolution on a hidden background. It is an effective generator for event-class transitions ordered by the relational clock.

Definition 5.1 (Branch transition graph). *The branch transition graph is a directed graph*

$$\Gamma_B = (\mathcal{B}, E) \tag{13}$$

whose vertices are admissible branch chambers and whose directed edges are event classes. An edge $\alpha \to \beta$ exists when there is an admissible crossing path from chamber C_α to chamber C_β with nonzero transition weight or rate.

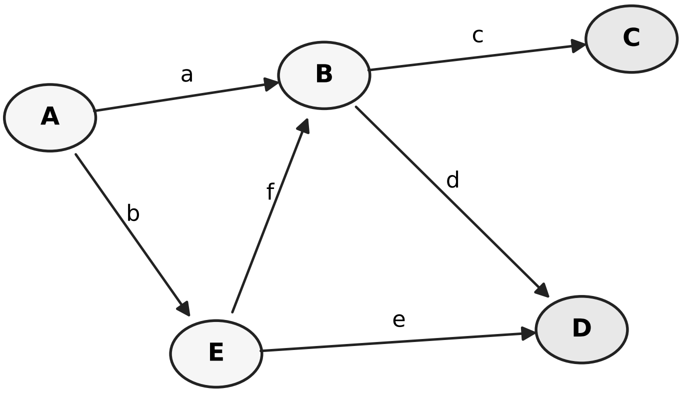

Figure 4: Directed branch-event graph. Vertices are chambers and arrows are event classes. Terminal chambers are represented as absorbing nodes. The labels on arrows are rates or weights only after a transition model has been supplied; the graph itself is purely structural.

Definition 5.2 (Continuous-clock generator). *For a finite branch set $\mathcal{B} = \{1, \ldots, n\}$, a continuous-clock transition generator is a matrix $L = (L_{\alpha\beta})$ satisfying*

$$L_{\alpha\beta} \geq 0 \quad (\alpha \neq \beta), \qquad L_{\alpha\alpha} = -\sum_{\beta \neq \alpha} L_{\alpha\beta}. \tag{14}$$

The branch probabilities $p_\alpha(\tau)$ evolve by

$$\frac{\mathrm{d}p}{\mathrm{d}\tau} = pL, \tag{15}$$

where p is a row vector.

Theorem 5.3 (Positivity and conservation). *Let L be a finite continuous-clock generator. If $p(0)$ is a probability vector, then $p(\tau) = p(0)e^{\tau L}$ is a probability vector for every $\tau \geq 0$.*

Proof. The row sums of L vanish by definition, so $L\mathbf{1} = 0$. Hence

$$\frac{\mathrm{d}}{\mathrm{d}\tau} p(\tau)\mathbf{1} = p(\tau)L\mathbf{1} = 0, \tag{16}$$

which proves conservation of total probability. Positivity follows from the standard generator property. For completeness one may choose $\lambda \geq \max_\alpha(-L_{\alpha\alpha})$ and write

$$e^{\tau L} = e^{-\lambda\tau} e^{\tau(L+\lambda I)}. \tag{17}$$

The matrix $L + \lambda I$ has nonnegative entries, so its exponential has nonnegative entries. Multiplication by the positive scalar $e^{-\lambda\tau}$ preserves nonnegativity. Therefore a nonnegative initial vector remains nonnegative and its sum remains one. □

The generator may be derived from microscopic source terms, stochastic coarse graining, tunneling estimates, deterministic uncertainty under unresolved variables, or empirical transition weights in an applied model. This paper does not choose among these mechanisms. It requires only that any proposed generator pass the positivity and conservation test. If it does not, its entries cannot be interpreted as branch-event rates.

6 Terminal branches, absorption, and expected hitting times

Some branches may be terminal relative to the selected gate set: once entered, the effective readout used for transitions does not leave them. Terminality does not mean metaphysical finality. It means absorbing behavior within the chosen transition graph and gate resolution. Different observational maps or additional degrees of freedom may refine a terminal node into subbranches, but the finite calculation at hand treats it as absorbing.

Definition 6.1 (Terminal branch class)**.** *A branch chamber C_α is terminal for a transition generator L if*

$$L_{\alpha\beta} = 0 \qquad \text{for all } \beta \neq \alpha. \tag{18}$$

A terminal event class is an event class whose target branch is terminal.

Suppose the finite branch set is ordered so that transient branches come first and terminal branches come last. Then the generator has block form

$$L = \begin{pmatrix} Q & R \\ 0 & 0 \end{pmatrix}, \tag{19}$$

where Q acts on transient branches and R contains transition rates from transient to terminal branches.

Theorem 6.2 (Continuous-time absorption formula)**.** *Assume every transient branch is absorbed almost surely into one of the terminal branches and all eigenvalues of Q have strictly negative real parts. Then the matrix of absorption probabilities is*

$$B = (-Q)^{-1} R. \tag{20}$$

The expected hitting-time vector from transient branches is

$$t = (-Q)^{-1}\mathbf{1}. \tag{21}$$

Proof. For an initial transient branch, the transient probability density at clock value τ is given by $e^{\tau Q}$. The instantaneous probability flux into terminal branches is therefore $e^{\tau Q} R$. Integrating this flux over all future clock values gives

$$B = \int_0^\infty e^{\tau Q} R \, \mathrm{d}\tau. \tag{22}$$

Since all eigenvalues of Q have negative real parts, $e^{\tau Q} \to 0$ as $\tau \to \infty$, and

$$\int_0^\infty e^{\tau Q} \, \mathrm{d}\tau = (-Q)^{-1}. \tag{23}$$

This proves $B = (-Q)^{-1} R$. The survival probability in the transient sector is $e^{\tau Q}\mathbf{1}$, so the

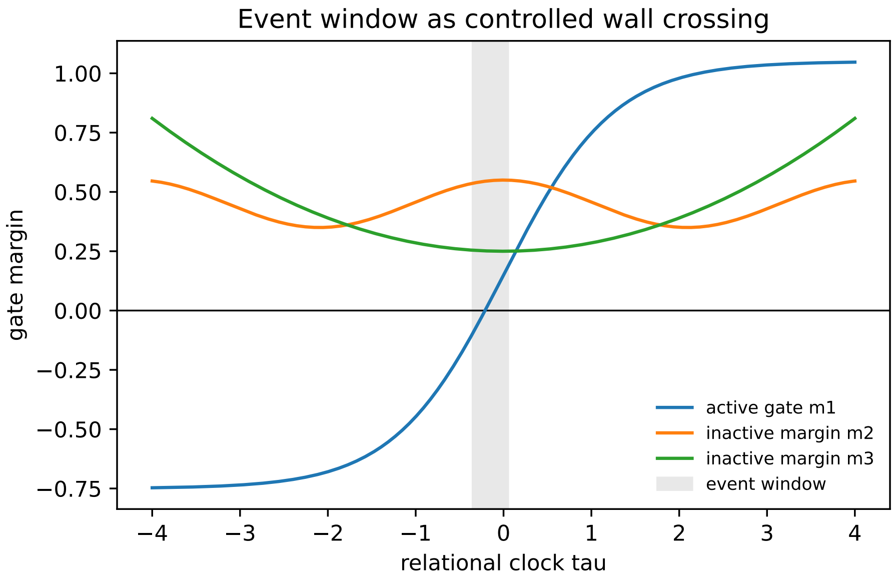

Figure 5: Gate margins along the relational clock. The active event window is the region in which one margin crosses zero while other mandatory margins remain positive. This is the controlled case; simultaneous loss of several margins is a higher-codimension event.

expected hitting time is

$$t = \int_0^\infty e^{\tau Q}\mathbf{1}\,\mathrm{d}\tau = (-Q)^{-1}\mathbf{1}. \tag{24}$$

□

This theorem is one of the basic computational results of the paper. It turns a branch-event graph with rates into concrete predictions about terminal outcomes and expected relational duration before terminal entry. These quantities are not yet observations of our universe. They are branch-level outputs that become observational only after a readout map is supplied.

7 Event windows and crossing probabilities

A wall crossing is rarely resolved as an exact instant in a coarse-grained effective theory. What one sees is a window in relational clock time during which a gate margin becomes small enough that unresolved perturbations, hidden-sector fluctuations, or source terms can change the chamber assignment. The event-window formalism quantifies this statement.

Definition 7.1 (Gate margin and event window)**.** *For a path $\theta(\tau)$ and a gate g_i, the signed margin is*

$$m_i(\tau) = g_i(\theta(\tau)). \tag{25}$$

Given a tolerance $\eta > 0$, the i-th event window is

$$W_i(\eta) = \{\tau : |m_i(\tau)| \le \eta\}. \tag{26}$$

A multigate event window for an active set I is $W_I(\eta) = \cap_{i\in I} W_i(\eta)$.

Proposition 7.2 (Linearized one-gate crossing probability)**.** *Let $m > 0$ be the deterministic margin to a gate wall and suppose the unresolved normal perturbation is a centered Gaussian*

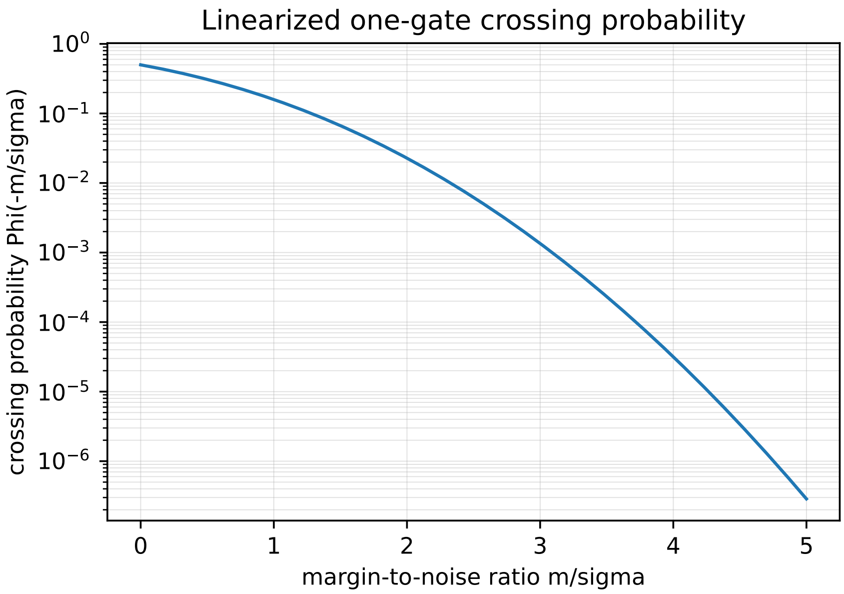

Figure 6: Linearized one-gate crossing probability as a function of the margin-to-noise ratio. The exponential decay of the tail shows why large gate margins are stable and why near-wall windows dominate event statistics.

scalar $\xi \sim N(0, \sigma^2)$. *Then the probability that the perturbation carries the system across the wall is*

$$\mathbb{P}(m + \xi < 0) = \Phi\left(-\frac{m}{\sigma}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{m}{\sqrt{2}\sigma}\right). \tag{27}$$

Proof. The event $m + \xi < 0$ is the event $\xi < -m$. Dividing by σ gives a standard normal variable $Z = \xi/\sigma$, hence

$$\mathbb{P}(m + \xi < 0) = \mathbb{P}\left(Z < -\frac{m}{\sigma}\right) = \Phi\left(-\frac{m}{\sigma}\right). \tag{28}$$

The complementary-error-function form is the standard identity $\Phi(-x) = \frac{1}{2}\operatorname{erfc}(x/\sqrt{2})$ for $x \geq 0$. □

For several gates, one should not multiply one-dimensional probabilities unless the perturbations are independent in the gate-normal basis. The generally valid first control is a union bound. Let $m_i > 0$ be margins and let ξ_i be gate-normal perturbations with variances σ_i^2. Then

$$\mathbb{P}\left(\exists i \in I : m_i + \xi_i < 0\right) \leq \sum_{i \in I} \Phi\left(-\frac{m_i}{\sigma_i}\right). \tag{29}$$

If the covariance matrix is known, sharper multivariate normal integrals may be used, but the union bound is already enough for a falsification gate: a proposed stable branch with large quoted margins cannot also claim frequent spontaneous wall crossings unless it supplies a source term that invalidates the Gaussian small-noise estimate.

8 Stability of branch membership

A branch chamber is meaningful only if small re-description errors, numerical errors, or perturbations do not immediately change branch membership. Stability is controlled by margins. The next theorem is elementary but indispensable, because it turns gate values into quantitative robustness bounds.

Definition 8.1 (Minimum mandatory margin). *For a compact set $K \subset \Theta_+$, the minimum mandatory margin is*

$$\Delta_K = \min_{\theta \in K} \min_{i \in \mathcal{G}_{\mathrm{mand}}} g_i(\theta). \tag{30}$$

If $\Delta_K > 0$, the set is uniformly inside the admissible region.

Theorem 8.2 (Chamber stability under bounded perturbations). *Let K be a compact subset of a branch chamber and suppose each gate g_i is Lipschitz on a neighborhood of K with constant L_i. If*

$$\|\delta\theta\| < \min_i \frac{|g_i(\theta)|}{L_i} \tag{31}$$

for every $\theta \in K$ and every active gate in the chamber definition, then $\theta + \delta\theta$ remains in the same sign chamber. In particular, if $\Delta_K > 0$ and $L = \max_i L_i$, then every perturbation satisfying

$$\|\delta\theta\| < \Delta_K / L \tag{32}$$

preserves mandatory admissibility.

Proof. By the Lipschitz condition,

$$|g_i(\theta + \delta\theta) - g_i(\theta)| \le L_i \|\delta\theta\|. \tag{33}$$

If $L_i\|\delta\theta\| < |g_i(\theta)|$, then $g_i(\theta + \delta\theta)$ cannot have the opposite sign from $g_i(\theta)$ and cannot vanish. Applying this to every gate defining the chamber proves sign preservation. The uniform version follows by replacing $|g_i(\theta)|$ with the compact minimum Δ_K and L_i with L. □

Spectral branches require an analogous gap stability. If a branch coordinate is a spectral gap separating a Riesz sector from the rest of the spectrum, Kato-type perturbation estimates show that the sector persists as long as the perturbation norm remains below a fixed fraction of the gap. This is why the spectral matter paper had to precede the cosmological branch paper: without gap control, branch labels built from spectral sectors would be unstable.

Proposition 8.3 (Spectral isolation margin). *Let K be a closed operator with an isolated spectral cluster enclosed by a contour Γ at distance $\delta > 0$ from the rest of the spectrum. If $\|\Delta K\| < \delta$, then the perturbed resolvent exists on Γ and the Riesz projector*

$$P_\Gamma(K + \Delta K) = \frac{1}{2\pi i} \int_\Gamma (z - K - \Delta K)^{-1} \, \mathrm{d}z \tag{34}$$

has the same rank as $P_\Gamma(K)$.

Proof. For $z \in \Gamma$, the resolvent identity gives

$$z - K - \Delta K = (z - K)\left(I - (z - K)^{-1}\Delta K\right). \tag{35}$$

Since $\|(z - K)^{-1}\| \le 1/\delta$ and $\|\Delta K\| < \delta$, the Neumann series for $\left(I - (z - K)^{-1}\Delta K\right)^{-1}$ converges. Thus the perturbed resolvent exists on Γ. The projector depends continuously on the perturbation, and rank is integer-valued and locally constant under norm-continuous projector deformations. Therefore the rank is unchanged. □

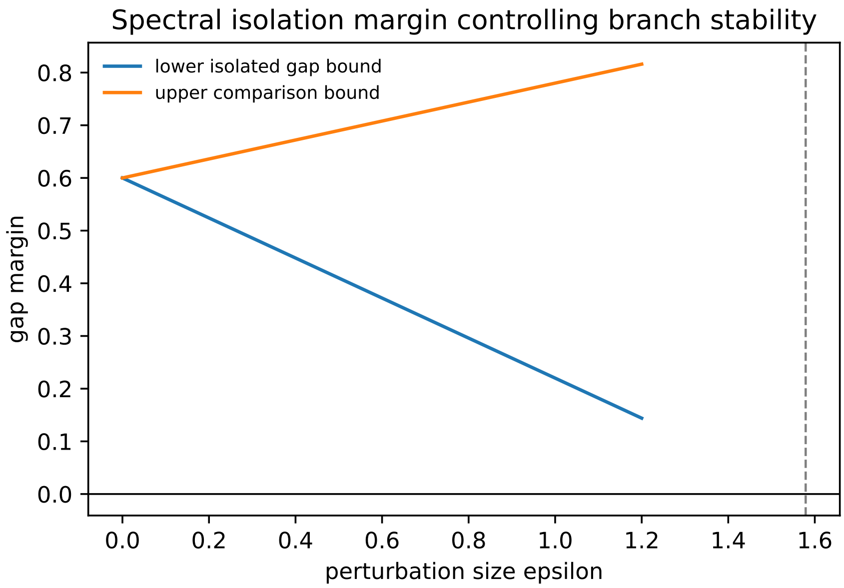

Figure 7: Schematic spectral isolation margin. The branch label tied to a spectral sector is stable only while the perturbation remains below the gap margin. If the margin is lost, the branch classification must be recomputed.

9 Worked branch calculation

We now calculate a finite branch model explicitly. The example is not meant to describe our universe. It is a minimal transparent calculation showing how terminal probabilities and hitting times are obtained once a branch graph and rates have been supplied.

Consider two transient branches A, B and two terminal branches C, D. Let the nonzero rates be

$$A \to B : a, \qquad A \to C : b, \qquad B \to C : c, \qquad B \to D : d, \tag{36}$$

with all rates positive. Ordering transient branches as (A, B) and terminal branches as (C, D) gives

$$Q = \begin{pmatrix} -(a+b) & a \\ 0 & -(c+d) \end{pmatrix}, \qquad R = \begin{pmatrix} b & 0 \\ c & d \end{pmatrix}. \tag{37}$$

The inverse of $-Q$ is

$$(-Q)^{-1} = \begin{pmatrix} \dfrac{1}{a+b} & \dfrac{a}{(a+b)(c+d)} \\ 0 & \dfrac{1}{c+d} \end{pmatrix}. \tag{38}$$

Therefore the absorption matrix is

$$B = (-Q)^{-1}R = \begin{pmatrix} \dfrac{b}{a+b} + \dfrac{ac}{(a+b)(c+d)} & \dfrac{ad}{(a+b)(c+d)} \\ \dfrac{c}{c+d} & \dfrac{d}{c+d} \end{pmatrix}. \tag{39}$$

The expected hitting times are

$$t_A = \frac{1}{a+b} + \frac{a}{(a+b)(c+d)}, \qquad t_B = \frac{1}{c+d}. \tag{40}$$

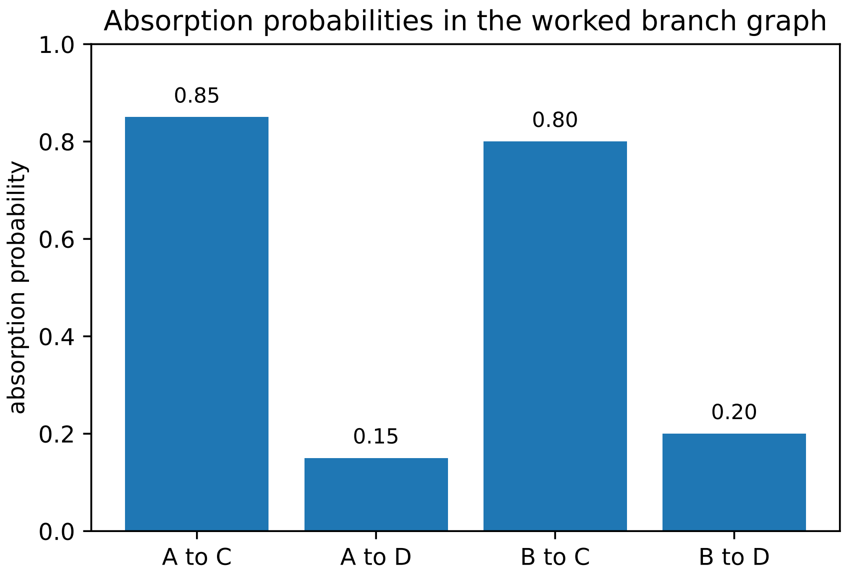

Figure 8: Absorption probabilities for the four-branch calculation. The values are not fitted to data; they are the exact result of the displayed generator with $a = 0.30$, $b = 0.10$, $c = 0.20$, and $d = 0.05$.

These formulas are easy to audit. Starting at A, the process either jumps directly to C with probability $b/(a + b)$ or first jumps to B with probability $a/(a + b)$ and is later absorbed into C or D with probabilities $c/(c + d)$ and $d/(c + d)$. The matrix formula reproduces exactly this path decomposition.

For the numerical benchmark

$$a = 0.30, \qquad b = 0.10, \qquad c = 0.20, \qquad d = 0.05, \tag{41}$$

we obtain

$$B = \begin{pmatrix} 0.85 & 0.15 \\ 0.80 & 0.20 \end{pmatrix}, \qquad t_A = 5.5, \qquad t_B = 4.0. \tag{42}$$

The calculation is included in the source bundle as a reproducible CSV and JSON ledger. The numbers have no direct cosmological interpretation until an observational readout map is attached to the terminal classes.

10 Readout maps and the boundary of cosmological interpretation

Branch dynamics becomes cosmology only after branch variables are mapped to effective observables. Let $\mathcal{O}_{\mathrm{cos}}$ denote a chosen family of cosmological readout functionals. Examples might include an effective scale observable, a curvature scalar, a relic-density proxy, a source density, or a late-time acceleration diagnostic. The branch paper does not construct these maps explicitly. It states the condition under which they may be attached without corrupting the branch formalism.

Definition 10.1 (Cosmological readout map)**.** *A cosmological readout map is a collection of quotient-invariant functions*

$$\mathcal{O}_{\mathrm{cos}} : \mathcal{Q}_{\mathrm{adm}} \longrightarrow \mathbb{R}^r \tag{43}$$

that factor through the branch-coordinate map on the branch layer, possibly after adding controlled source variables. Equivalently, there exists a map $\bar{\mathcal{O}}_{\cos}$ *such that*

$$\mathcal{O}_{\cos} = \bar{\mathcal{O}}_{\cos} \circ \theta \tag{44}$$

within the resolution of the branch model.

The factorization condition is essential. If the purported cosmological observable depends on data not contained in the branch coordinates or controlled source variables, then the branch model is incomplete for that observable. This is not a defect; it is the point of an auditable theory. It tells the reader exactly what must be added before empirical comparison is legitimate.

Proposition 10.2 (No cosmological history without a readout map)**.** *A branch graph with transition generator determines event-class probabilities and expected hitting times, but it does not by itself determine a scale factor, redshift relation, curvature history, or matter-density history. Such histories require a cosmological readout map and source model.*

Proof. The branch generator L acts on probability vectors over chambers. Its outputs are chamber probabilities, absorption probabilities, and clock-ordered transition statistics. A scale factor or curvature history is a function with values in a different observable space. Unless a map from chambers and source variables to that observable space is specified, there is no mathematical operation that converts $p(\tau)$ into such a history. Therefore the branch model alone cannot determine the history. □

This proposition is a guardrail against overinterpretation. It prevents branch theory from being used as a poetic description of cosmology before it has supplied the maps needed for observational statements. It also clarifies the role of later work: later papers or applications may construct explicit readout maps, but they must not retroactively pretend that the branch graph already contained them.

11 Falsification gates for branch cosmology

A branch cosmology is falsifiable at several levels before it reaches data. Some failures are purely mathematical. If the gate functions do not descend to the quotient, the branch coordinates are not physical. If the chamber decomposition changes under harmless re-description, event classes are not invariant. If transition rates fail positivity or conservation, they are not probabilities. If terminal probabilities do not sum to one under an absorbing hypothesis, the generator is inconsistent. If a claimed stable branch has margins smaller than the perturbations used in the same calculation, the branch label is unstable.

Theorem 11.1 (Branch-readout falsification theorem)**.** *A proposed branch-cosmology model fails at the structural level if any of the following conditions holds: (i) at least one mandatory gate is not invariant under physical equivalence; (ii) branch chambers are not stable under admissible re-description; (iii) the transition generator has negative off-diagonal entries or nonzero row sums; (iv) a claimed terminal class has outgoing transition rate in the same model; (v) a claimed stable chamber has perturbations larger than its gate margins without a supplied transition event; or (vi) a claimed cosmological observable does not factor through the declared branch coordinates and source variables. These failures are independent of empirical data.*

Proof. Condition (i) means that chamber assignment depends on presentation rather than physical state, so the branch is not physical. Condition (ii) means event classes are coordinate artifacts. Condition (iii) violates the generator theorem and therefore invalidates probability interpretation. Condition (iv) contradicts the definition of terminality. Condition (v) contradicts the stability theorem: if perturbations exceed the margin, the chamber cannot be claimed stable unless a transition is included. Condition (vi) contradicts the definition of a cosmological readout map. Each failure invalidates the branch-cosmology model before observational comparison is attempted. □

The falsification theorem is deliberately severe. Its role is to keep the series scientifically useful. A model that passes these structural gates may still be empirically wrong, but a model that fails them is not yet a coherent mathematical object.

Table 1: Structural falsification ledger for branch cosmology.

Gate	Required check	Failure mode
Quotient descent	$g_i(x) = g_i(y)$ for $x \sim y$	Branch label is descriptive, not physical
Chamber stability	Positive margin under perturbations	Branch assignment is not robust
Event invariance	Active wall support preserved by re-description	Event class is coordinate-dependent
Generator positivity	$L_{\alpha\beta} \geq 0$ for $\alpha \neq \beta$	Rates cannot be probabilities
Conservation	$L\mathbf{1} = 0$	Total branch probability drifts
Terminality	No outgoing rate from terminal class	Claimed terminal branch is not terminal
Absorption	$B\mathbf{1} = \mathbf{1}$ under absorbing hypothesis	Missing branch or inconsistent rates
Readout factorization	$\mathcal{O}_{\cos} = \bar{\mathcal{O}}_{\cos} \circ \theta$	Cosmological observable not contained in model

12 Adjacency, incidence, and the event category

The finite chamber decomposition gives more than a list of possible branches. It gives an incidence structure: chambers meet along walls, walls meet along higher-codimension strata, and paths through the decomposition compose event classes. This is the correct replacement for an informal cosmological history at the branch level. A history is not a sequence of names; it is a path in the incidence graph together with its active wall data.

Definition 12.1 (Chamber adjacency)**.** *Two branch chambers C_α and C_β are adjacent along a wall support I if the closures $\overline{C}_\alpha$ and $\overline{C}_\beta$ meet in a nonempty subset of W_I and if their sign vectors differ exactly on the gates in I. The adjacency is regular if the differentials $\{\mathrm{d}g_i : i \in I\}$ have rank $|I|$ on a dense open subset of the shared wall.*

Regular adjacency is the generic case for clean event classification. When the active differentials are linearly independent, the local wall has codimension $|I|$, and crossing several gates at once is a higher-codimension event. Nonregular adjacency is not forbidden, but it must be audited separately because the apparent coincidence of walls may be caused by a hidden algebraic dependence among gates.

Lemma 12.2 (Local normal form of a regular wall)**.** *Let $\theta_0 \in W_I$ and suppose the differentials $\{\mathrm{d}g_i(\theta_0) : i \in I\}$ are linearly independent. Then there are local coordinates $(x_1, \dots, x_N)$ centered at θ_0 such that, for $i \in I$, the active gates are locally represented by $g_i = x_i$ after multiplication by an invertible Jacobian transformation. In these coordinates the wall stratum is locally $x_i = 0$ for all $i \in I$.*

Proof. The map $G_I = (g_i)_{i \in I}$ has differential of rank $|I|$ at θ_0. By the constant-rank theorem, G_I can be completed to a local coordinate system in which its components are the first $|I|$ coordinates. Thus the equations defining W_I become $x_i = 0$ for $i \in I$ in a neighborhood of the point. The remaining gates are nonzero on the stratum by definition of W_I, so their signs remain locally fixed after shrinking the neighborhood. □

Definition 12.3 (Event path and event category)**.** *An event path is a finite sequence of adjacent chamber transitions*

$$C_{\alpha_0} \xrightarrow{e_1} C_{\alpha_1} \xrightarrow{e_2} \cdots \xrightarrow{e_k} C_{\alpha_k}. \tag{45}$$

The event category has branch chambers as objects and event paths modulo insertion or removal of identity transitions as morphisms. Composition is concatenation of paths when the target chamber of the first path equals the source chamber of the second.

Proposition 12.4 (Associativity of event histories)**.** *The event category is a category. In particular, event-path composition is associative and each chamber has an identity morphism.*

Proof. The identity morphism at a chamber is the empty event path that begins and ends at that chamber. If p, q, and r are composable event paths, then $(p \circ q) \circ r$ and $p \circ (q \circ r)$ are the same finite concatenated sequence of elementary events with parentheses placed differently. Hence composition is associative. Insertion or removal of empty paths does not change the sequence, so identities act as identities. This proves the category axioms. □

The categorical language is not decorative. It prevents a common mistake: treating a branch history as a coordinate curve rather than as a composable sequence of invariant event classes. Coordinate curves may be reparametrized, smoothed, or drawn differently. The event path records the chamber incidence and active gates, which are the invariant data relevant at this level.

13 Coarse graining and lumpability of branch classes

A realistic branch complex may contain many chambers. An external paper or observational application may want to group them into coarse cosmological classes. Such grouping is dangerous unless transition probabilities are preserved. The correct condition is lumpability: the total rate from any two fine states in the same macrostate to another macrostate must be the same. Otherwise the macro-dynamics depends on hidden fine-state information and is not closed.

Definition 13.1 (Branch coarse graining)**.** *Let $\mathcal{B}$ be a finite branch set and let $\pi : \mathcal{B} \to \widehat{\mathcal{B}}$ be a surjective map onto a set of coarse branch labels. The fiber $\pi^{-1}(\hat\alpha)$ is the fine branch class represented by the coarse branch $\hat\alpha$.*

Definition 13.2 (Strong lumpability)**.** *A generator L on $\mathcal{B}$ is strongly lumpable with respect to π if for every pair of coarse labels $\hat\alpha, \hat\beta$ and every pair of fine states $x, y \in \pi^{-1}(\hat\alpha)$,*

$$\sum_{z \in \pi^{-1}(\hat\beta)} L_{xz} = \sum_{z \in \pi^{-1}(\hat\beta)} L_{yz}. \tag{46}$$

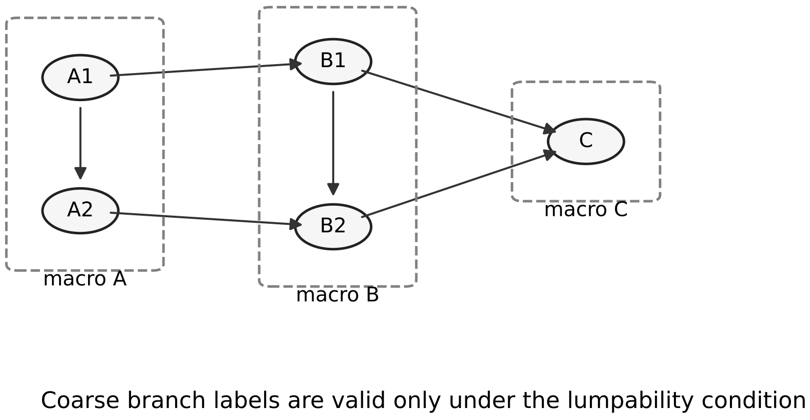

Figure 9: Coarse graining of a branch graph. Grouping fine chambers into macroscopic branch labels is legitimate only if the outgoing total rates to every other macro-class are independent of the fine representative. Otherwise the coarse label hides dynamical information.

Theorem 13.3 (Closed coarse branch generator)**.** *If L is strongly lumpable with respect to π, then there is a unique generator $\widehat{L}$ on $\widehat{\mathcal{B}}$ such that coarse probabilities evolve autonomously by*

$$\frac{\mathrm{d}\hat{p}}{\mathrm{d}\tau} = \hat{p}\widehat{L} \tag{47}$$

whenever the fine probabilities evolve by $\mathrm{d}p/\mathrm{d}\tau = pL$. The entries are

$$\widehat{L}_{\hat{\alpha}\hat{\beta}} = \sum_{z\in\pi^{-1}(\hat{\beta})} L_{xz}, \qquad x \in \pi^{-1}(\hat{\alpha}), \tag{48}$$

for $\hat{\alpha} \neq \hat{\beta}$, with diagonal entries fixed by zero row sums.

Proof. Strong lumpability says that the displayed sum is independent of the choice of x inside the fine class $\pi^{-1}(\hat{\alpha})$, so $\widehat{L}$ is well defined. Let $\hat{p}_{\hat{\alpha}} = \sum_{x\in\pi^{-1}(\hat{\alpha})} p_x$. Then

$$\frac{\mathrm{d}\hat{p}_{\hat{\beta}}}{\mathrm{d}\tau} = \sum_{z\in\pi^{-1}(\hat{\beta})} \sum_{x\in\mathcal{B}} p_x L_{xz} \tag{49}$$

$$= \sum_{\hat{\alpha}} \sum_{x\in\pi^{-1}(\hat{\alpha})} p_x \sum_{z\in\pi^{-1}(\hat{\beta})} L_{xz} \tag{50}$$

$$= \sum_{\hat{\alpha}} \hat{p}_{\hat{\alpha}} \widehat{L}_{\hat{\alpha}\hat{\beta}}. \tag{51}$$

Thus the coarse probabilities satisfy the closed equation. Nonnegative off-diagonal entries and zero row sums are inherited from L, so $\widehat{L}$ is a generator. □

The lumpability theorem is especially important for cosmology because observational language is almost always coarse. Words such as expanding, terminal, metastable, or source-dominated may represent large families of fine chambers. If the fine generator is not lumpable, these words do not support a closed transition law. They may remain useful descriptions, but they cannot be used as autonomous states in a branch master equation.

14 Selection functionals and deterministic branch flow

Not every branch transition model is stochastic. In some applications the branch coordinates may follow an effective deterministic flow except near walls, where unresolved variables produce transition probabilities. A useful baseline is a gradient-like selection flow driven by a branch functional. This is not assumed to be fundamental; it is a calculational bridge between continuous chamber motion and discrete event classes.

Definition 14.1 (Selection functional)**.** *A selection functional is a differentiable function*

$$U : \Theta \to \mathbb{R} \tag{52}$$

whose decrease along branch-coordinate paths represents relaxation of the effective branch readout. Given a positive definite mobility matrix $G(\theta)$, the associated gradient selection flow is

$$\dot{\theta} = -G(\theta)\nabla U(\theta), \tag{53}$$

where the dot denotes derivative with respect to the relational clock.

Proposition 14.2 (Lyapunov decrease)**.** *Along a gradient selection flow with $G(\theta)$ symmetric positive definite,*

$$\frac{\mathrm{d}U}{\mathrm{d}\tau} = -\nabla U(\theta)^T G(\theta)\nabla U(\theta) \leq 0. \tag{54}$$

Equality holds precisely at critical points of U when G is positive definite.

Proof. By the chain rule,

$$\frac{\mathrm{d}U}{\mathrm{d}\tau} = \nabla U(\theta)^T \dot{\theta} = -\nabla U(\theta)^T G(\theta)\nabla U(\theta). \tag{55}$$

Positive definiteness of G makes the quadratic form nonnegative and zero only when $\nabla U = 0$. □

The selection functional supplies one way of generating candidate paths through the chamber complex. It does not replace event classes. A deterministic trajectory that crosses a wall must still be classified by active gates and sign changes. A trajectory that approaches a wall asymptotically may instead generate a long event window with noise-sensitive crossing probability. A trajectory trapped in a chamber minimum generates a metastable branch, but metastability is meaningful only relative to margins, sources, and perturbation scales.

15 Detailed balance, stationary branch measures, and entropy production

If transition rates are supplied, one may ask whether the branch dynamics has an equilibrium measure or whether it contains directed non-equilibrium currents. This is again a mathematical question before it is a cosmological interpretation. Detailed balance is a strong condition; failure of detailed balance indicates irreversible branch flow relative to the chosen coarse graining.

Definition 15.1 (Stationary measure and detailed balance)**.** *A probability vector π is stationary for a finite generator L if*

$$\pi L = 0. \tag{56}$$

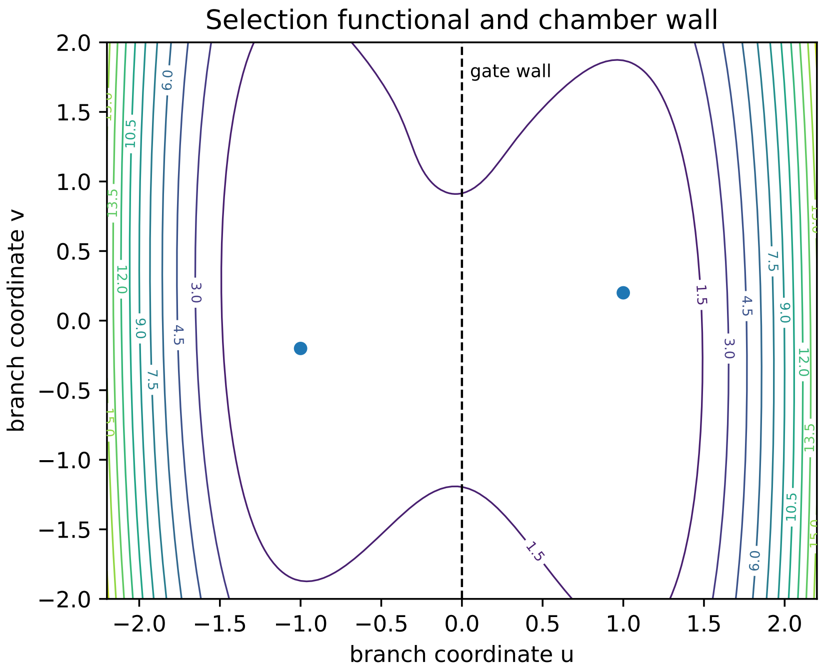

Figure 10: A selection functional on a two-dimensional branch slice. Deterministic relaxation may move the branch coordinates inside a chamber or toward a wall. Crossing the wall still requires the event-class analysis; the potential picture alone is not a cosmological history.

It satisfies detailed balance if

$$\pi_\alpha L_{\alpha\beta} = \pi_\beta L_{\beta\alpha} \tag{57}$$

for all distinct branches α, β.

Proposition 15.2 (Detailed balance implies stationarity)**.** *If π satisfies detailed balance, then $\pi L = 0$.*

Proof. For each β,

$$(\pi L)_\beta = \sum_{\alpha \neq \beta} \pi_\alpha L_{\alpha\beta} + \pi_\beta L_{\beta\beta} \tag{58}$$

$$= \sum_{\alpha \neq \beta} \pi_\beta L_{\beta\alpha} - \pi_\beta \sum_{\alpha \neq \beta} L_{\beta\alpha} = 0, \tag{59}$$

where the second equality uses detailed balance and the generator row-sum condition. □

When detailed balance fails, the antisymmetric edge current

$$J_{\alpha\beta} = \pi_\alpha L_{\alpha\beta} - \pi_\beta L_{\beta\alpha} \tag{60}$$

measures stationary circulation on the branch graph. A standard entropy-production diagnostic is

$$\Sigma = \frac{1}{2} \sum_{\alpha \neq \beta} (\pi_\alpha L_{\alpha\beta} - \pi_\beta L_{\beta\alpha}) \log \frac{\pi_\alpha L_{\alpha\beta}}{\pi_\beta L_{\beta\alpha}}, \tag{61}$$

with the convention that terms with equal flux vanish and transitions with zero reverse flux require separate limiting treatment.

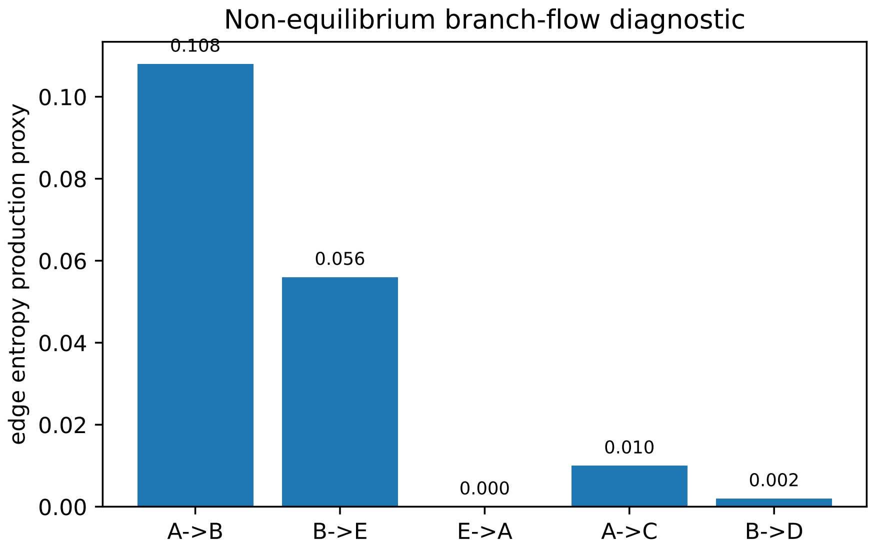

Figure 11: A branch-flow diagnostic. Nonzero edge entropy production indicates irreversible circulation in the chosen coarse branch graph. It is a diagnostic of the transition model, not an empirical cosmological claim by itself.

Proposition 15.3 (Nonnegativity of entropy production)**.** *If all paired fluxes are positive, then* $\Sigma \geq 0$*, with equality if and only if detailed balance holds on every edge.*

Proof. For positive x and y, the expression $(x - y)\log(x/y)$ is nonnegative because log is increasing and has the same sign as $x - y$. Each unordered edge contribution in Σ is of this form. Therefore the sum is nonnegative. It vanishes exactly when each paired flux is equal, which is detailed balance. □

This diagnostic is useful for branch cosmology because some proposed event-class networks will be effectively irreversible. A terminal absorption graph is maximally non-equilibrium: probability flows into terminal classes and does not return. A cyclic branch graph may be reversible or irreversible depending on rates. The formalism distinguishes these cases without naming any of them as a universe.

16 Hazard rates from collapsing margins

Event windows can also be described by hazard rates. Suppose a gate margin $m(\tau) > 0$ decreases while the unresolved normal fluctuation scale σ remains approximately constant. A simple coarse hazard proxy is proportional to the instantaneous crossing probability $\Phi(-m(\tau)/\sigma)$ divided by a microscopic correlation time. The proportionality constant is model-dependent, but the margin dependence is fixed by the Gaussian calculation.

Definition 16.1 (Margin-controlled hazard proxy)**.** *Given a positive gate margin* $m(\tau)$*, noise scale* σ*, and correlation clock scale* $\ell_\tau > 0$*, define*

$$h(\tau) = \ell_\tau^{-1}\Phi\left(-\frac{m(\tau)}{\sigma}\right). \tag{62}$$

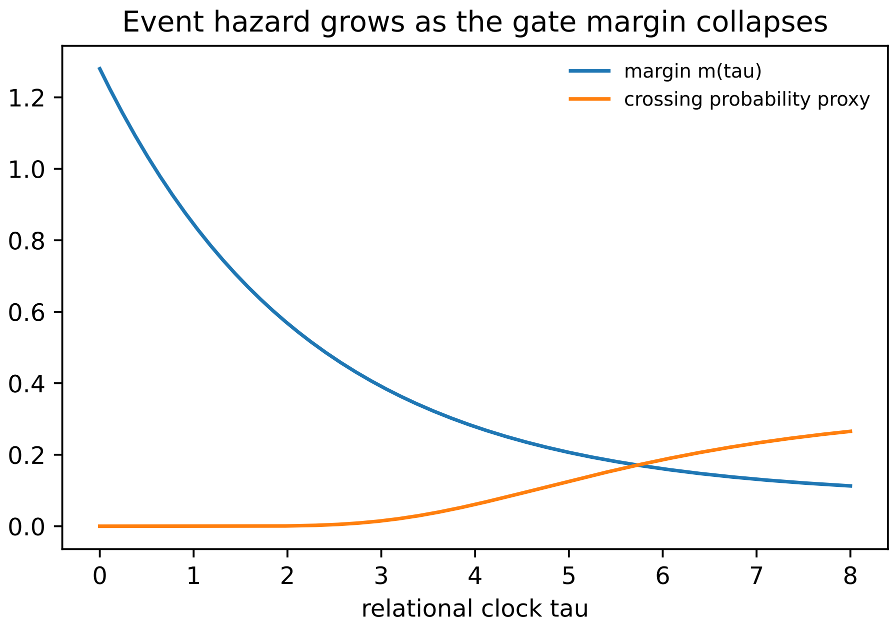

Figure 12: Hazard proxy produced by a collapsing margin. As the deterministic margin approaches the wall, the Gaussian crossing tail becomes appreciable. A model claiming both a collapsing margin and no events must supply a reason why the perturbation scale or correlation mechanism is absent.

The survival probability associated with this hazard proxy is

$$S(\tau) = \exp\left(-\int_{\tau_0}^{\tau} h(s)\,\mathrm{d}s\right). \tag{63}$$

Proposition 16.2 (Event probability from a hazard proxy)**.** *If h is a nonnegative integrable hazard rate on $[\tau_0, \tau_1]$, then the probability of at least one event in the interval is*

$$P_{\text{event}} = 1 - \exp\left(-\int_{\tau_0}^{\tau_1} h(s)\,\mathrm{d}s\right). \tag{64}$$

Proof. The survival probability S satisfies the differential equation $\dot{S} = -hS$ with $S(\tau_0) = 1$. Solving gives the displayed exponential. The probability of an event by τ_1 is one minus the survival probability at that endpoint. □

The hazard formulation makes event windows operational. It connects gate margins, perturbation scales, and transition rates. It also exposes another falsification route: if a model quotes small margins and sizable unresolved fluctuations but assigns zero transition rate, it is internally inconsistent unless a symmetry or conservation law forbids the crossing.

17 Relation to topology change and causal structure

The branch formalism is compatible with later topology-changing or causal-readout interpretations, but it does not assume them. This point is important because cosmological branch language can easily be confused with spacetime topology change. In general relativity, topology change has delicate constraints and often requires singular behavior or causality violations under classical assumptions. The present paper avoids importing those issues prematurely. A branch transition is a change in the effective gate chamber of the affine-spectral readout; it is not automatically a spacetime topology change.

If a later model supplies a Lorentzian spacetime readout on both sides of an event class, and if the readout maps the branch transition to a change in the topology of spatial slices, then topology-change theorems become relevant. Until that readout is supplied, the branch event lives in gate space, not in a spacetime manifold. Similarly, a terminal branch in the transition graph is an absorbing chamber of the selected effective model, not necessarily a singular endpoint of spacetime. This distinction keeps the paper within its mathematical contract.

Proposition 17.1 (No topology-change inference at branch level). *A branch transition $C_- \to C_+$ does not by itself imply a topology change of spacetime slices. Such an inference requires a spacetime readout map assigning manifolds or stratified spaces to the branches and a proof that the assigned topological type changes across the event.*

Proof. The branch transition is defined by sign, rank, gap, and gate data in Θ. Topological type of spacetime slices is not an element of this data unless a readout map explicitly assigns it. Without such a map, there is no mathematical object whose topology can be compared before and after the transition. Hence topology change cannot be inferred from branch transition alone. □

This proposition also protects the interpretation of terminal branches. A terminal branch may later be read as a sink, a freeze-out, an asymptotic phase, a failed readout, or a dynamically closed sector. The branch graph alone says only that the selected transition generator has no outgoing edge from that class.

18 Clock discretization and consistency with stochastic kernels

Numerical implementations and external applications often use discrete clock steps. A discrete kernel must be compatible with the continuous generator or must be declared as a separate model. The simplest consistency check is the small-step expansion

$$P_{\Delta\tau} = I + \Delta\tau L + O(\Delta\tau^2). \tag{65}$$

If this expansion is used outside its positivity range, the resulting matrix may fail to be stochastic even when L is a valid generator.

Proposition 18.1 (Euler stochasticity bound). *Let L be a finite continuous-clock generator and define*

$$P_\Delta = I + \Delta L. \tag{66}$$

If

$$0 \le \Delta \le \left(\max_\alpha \sum_{\beta \neq \alpha} L_{\alpha\beta} \right)^{-1}, \tag{67}$$

then P_Δ is a stochastic matrix.

Proof. For $\alpha \neq \beta$, $(P_\Delta)_{\alpha\beta} = \Delta L_{\alpha\beta} \ge 0$. The diagonal entry is

$$(P_\Delta)_{\alpha\alpha} = 1 - \Delta \sum_{\beta \neq \alpha} L_{\alpha\beta}, \tag{68}$$

which is nonnegative by the assumed bound. The row sum is

$$\sum_{\beta} (P_{\Delta})_{\alpha\beta} = 1 + \Delta \sum_{\beta} L_{\alpha\beta} = 1. \tag{69}$$

Thus P_{Δ} is stochastic. □

This proposition is operationally useful because it catches a common numerical error. One may have a correct generator but use a clock step too large for the explicit Euler kernel. The cure is not to alter rates by hand; it is to use the matrix exponential $e^{\Delta L}$ or reduce the step size. The matrix exponential is always stochastic for $\Delta \geq 0$ by the positivity theorem.

19 Event-path probabilities

Absorption probabilities summarize terminal outcomes, but a branch history may also require the probability density of a particular event path. For a continuous-time chain, the path probability includes both jump rates and exponential holding factors. This matters because two paths with the same ordered events may have different likelihoods depending on how long the system remains in intermediate chambers.

Let an event path visit chambers

$$\alpha_0 \to \alpha_1 \to \cdots \to \alpha_k \tag{70}$$

at ordered clock times

$$0 < t_1 < \cdots < t_k < T. \tag{71}$$

Let $\lambda_\alpha = \sum_{\beta \neq \alpha} L_{\alpha\beta}$ be the exit rate from chamber α. If no further event occurs between t_k and T, the path density is

$$\left[\prod_{j=0}^{k-1} e^{-\lambda_{\alpha_j}(t_{j+1}-t_j)} L_{\alpha_j \alpha_{j+1}} \right] e^{-\lambda_{\alpha_k}(T-t_k)}, \tag{72}$$

where $t_0 = 0$.

Proposition 19.1 (Path density normalization)**.** *For a finite generator L, summing and integrating the displayed density over all finite paths on $[0, T]$ gives total probability one.*

Proof. The construction is the standard jump-process expansion of the semigroup e^{TL}. The probability of no jump from a state α during an interval of length s is $e^{-\lambda_\alpha s}$, and the conditional density of a jump to β at the next jump is $L_{\alpha\beta}$. Iterating over all possible numbers of jumps and target states gives the Dyson expansion of the Markov semigroup. Since e^{TL} is stochastic, the sum of probabilities over all terminal states and all paths is one. □

This formula is useful when an event class is not only terminal but diagnostic. If a later observational readout maps a certain ordered sequence of walls to a recognizable cosmological signature, the probability of that signature is not simply an absorption probability. It is obtained by summing path densities over the event paths that realize the signature.

20 Source-controlled rates and admissible parameter dependence

Transition rates may depend on source variables inherited from matter kernels, gauge sectors, or metric-affine modes. The branch layer can accept such dependence only if it preserves positivity, quotient descent, and clock consistency. Let $s \in \Sigma$ denote a vector of source controls, itself quotient-invariant or explicitly modeled as an external conditioning variable. A source-controlled generator is a family $L(s)$ satisfying the generator conditions for every admissible s.

Definition 20.1 (Source-controlled branch model)**.** *A source-controlled branch model consists of a source space Σ, a quotient-invariant source map $s : \mathcal{Q}_{\mathrm{adm}} \to \Sigma$, and a generator family $L(s)$ such that $L_{\alpha\beta}(s) \geq 0$ for $\alpha \neq \beta$ and $L(s)\mathbf{1} = 0$ for every $s \in \Sigma$.*

Proposition 20.2 (Conditioned conservation)**.** *For a source-controlled branch model with a prescribed source path $s(\tau)$, the nonautonomous master equation*

$$\frac{\mathrm{d}p}{\mathrm{d}\tau} = pL(s(\tau)) \tag{73}$$

conserves total probability and preserves positivity.

Proof. Conservation follows pointwise from $L(s(\tau))\mathbf{1} = 0$. Positivity follows by approximating the time-dependent evolution with products of short-time stochastic exponentials $\exp(\Delta\tau L(s_j))$, each of which is positive and row-stochastic. The limit of positive stochastic propagators is positive and stochastic. □

The source-controlled formalism is the correct place to connect later matter or gauge calculations to branch cosmology. It prevents source terms from being added as probability sinks or sources unless the model explicitly enlarges the state space. If probability appears to be lost, the missing sector must be added as a branch class or terminal class; otherwise the model is incomplete.

21 External-review proof obligations

The branch layer is intentionally positioned as the last paper of the first public sequence, but it should not be read as closure of the whole theory. It is a mathematical interface. For an external reviewer the correct question is not whether the paper has already described the observed universe, but whether every later cosmological claim is forced to pass through a finite list of explicit obligations. The following ledger records those obligations in a form that can be checked independently of the author's broader program.

Table 2: Proof obligations before branch cosmology can be specialized to an empirical model.

Layer	Required mathematical object	What must be shown
Quotient	$\mathcal{Q} = \mathcal{S}/\!\sim$	Branch coordinates are invariant under physical equivalence
Clock	τ	Transition ordering uses the relational clock and not an external parameter

Layer	Required mathematical object	What must be shown
Readout	H_{eff} and signature gates	Branch coordinates inherit valid Lorentzian readout margins where needed
Metric-affine	Distortion and source modes	Source-controlled rates are generated by quotient-descending fields
Spectral matter	Riesz/Feshbach kernels	Spectral branch labels have isolated gaps and stable ranks
Gauge	Normal-form kinetic factors	Gauge-sector source variables are normalized before entering rates
Branch chamber	Gate family $\mathcal{G}$	Chambers are semialgebraic or locally finite and margins are recorded
Event class	Wall supports I	Crossings are invariant under positive gate re-description
Transition model	Generator L or $L(s)$	Positivity, row-sum conservation, terminal classes, and absorption are checked
Observation	$\mathcal{O}_{\mathrm{cos}}$	Readout maps factor through branch coordinates and declared source variables

A future specialization that skips any row of table 2 is not a stronger cosmology; it is a less controlled one. Conversely, the ledger gives a precise path to empirical work. One may compute a gate family, derive or estimate a generator, attach an observational readout map, and then compare branch statistics with data. The present paper supplies the mathematical grammar for that process and the failure conditions that make it falsifiable.

22 Discussion

The branch layer is the first place in the sequence where cosmological language is allowed, but it is also the place where such language must be most tightly controlled. Earlier papers construct the quotient, clock, Lorentzian readout, metric-affine dynamics, spectral matter kernels, and gauge normal forms. The present paper asks what remains to be done before one can speak about cosmological alternatives and events. The answer is not a story about universes. It is a chamber decomposition, an event-class equivalence relation, a transition generator, a stability theory, and a set of readout maps that are not supplied for free.

This discipline is especially important because branch language is seductive. A wall crossing can be called a transition, a terminal class can be called an endpoint, and a chamber can be called a universe. Those words may eventually become appropriate, but only after additional readout maps and source models are supplied. The mathematical layer by itself gives branch states, event classes, transition probabilities, and hitting times. It does not give observed cosmological parameters. It does not identify our branch. It does not license an ontology of multiple universes. Those claims would be later, stronger, and much more exposed to empirical and mathematical failure.

The positive result is nevertheless significant. The paper shows that a branch theory can be constructed without importing a background cosmology. Given quotient-invariant gate functions inherited from the affine-spectral sequence, one obtains semialgebraic chambers, wall strata, event classes, and branch transition dynamics in relational clock time. Stability is controlled by explicit margins, event windows are quantified by crossing probabilities, and terminal outcomes are computed by a standard absorption formula. These are real calculations, not analogies. They provide the scaffolding on which a future empirical branch cosmology would have to be built.

The most important open problem is the derivation of the branch gate family from a fully specified parent operator rather than from a finite effective list. In applications one must compute the gate functions, determine their domains of validity, estimate perturbation margins, and derive transition generators from sources or unresolved variables. The second open problem is the construction of observational readout maps. Without them, the formalism cannot be compared with cosmological data. The third is the classification of higher-codimension event collisions, where several gates fail simultaneously and the simple transverse-crossing theorem no longer applies. These problems are not optional; they are exactly where the theory becomes testable.

23 Conclusion

This paper has developed a rigorous branch and event-class layer for a premetric affine sequence. Starting from quotient-invariant branch coordinates and finite gate families, we defined branch chambers, walls, event supports, event-class equivalence, transition graphs, continuous-clock generators, terminal classes, absorption probabilities, and event windows. We proved semialgebraic chamber finiteness on compact domains, transverse wall-crossing stability, event-class invariance under positive gate re-description, positivity and conservation of branch probabilities, the continuous-time absorption formula, margin-based chamber stability, spectral gap persistence, and structural falsification criteria. We also computed a four-branch absorbing example and a one-gate Gaussian crossing probability.

The result is deliberately narrower than a cosmological model. It does not fit data, does not identify a real cosmological branch, and does not introduce speculative event narratives. It proves the mathematical infrastructure required before such claims can be made. In this sense the branch layer is both a bridge and a filter: it connects the earlier affine-spectral construction to future cosmological applications, while filtering out claims that do not descend to the quotient, do not preserve event classes, do not conserve probability, or do not supply the necessary readout maps.

A Derivation of the absorption formula

For completeness we record the continuous-time absorption calculation in a form useful for audit. Let the state set be split into transient states T and absorbing states A, and let the generator be

$$L = \begin{pmatrix} Q & R \\ 0 & 0 \end{pmatrix}. \tag{74}$$

Starting in a transient state, the probability of still being transient at clock value τ is governed by $e^{\tau Q}$. The probability flux into absorbing state a during $[\tau, \tau + \mathrm{d}\tau]$ is the corresponding component of $e^{\tau Q} R \,\mathrm{d}\tau$. Therefore the total absorption matrix is

$$B = \int_0^\infty e^{\tau Q} R \,\mathrm{d}\tau. \tag{75}$$

If the transient block is stable, then $e^{\tau Q} \to 0$ and integration gives

$$\int_0^\infty e^{\tau Q}\,\mathrm{d}\tau = (-Q)^{-1}. \tag{76}$$

This recovers $B = (-Q)^{-1}R$. The same integral with R replaced by $\mathbf{1}$ gives the expected hitting time vector.

B Multigate Gaussian window estimate

Let $m \in \mathbb{R}^k$ be a vector of positive margins to k gate walls and let $\xi \sim N(0, \Sigma)$ be a centered Gaussian perturbation in the gate-normal coordinates. The exact probability of at least one wall crossing is

$$\mathbb{P}\left(\bigcup_{i=1}^{k}\{m_i + \xi_i < 0\}\right). \tag{77}$$

If the full covariance is known, this is a multivariate normal orthant integral over the complement of the translated positive orthant. In the absence of covariance data, the union bound gives

$$\mathbb{P}\left(\bigcup_{i=1}^{k}\{m_i + \xi_i < 0\}\right) \le \sum_{i=1}^{k}\mathbb{P}(m_i + \xi_i < 0) = \sum_{i=1}^{k}\Phi\left(-\frac{m_i}{\sqrt{\Sigma_{ii}}}\right). \tag{78}$$

This estimate is conservative but useful: it requires only the diagonal variances and is therefore available before a full fluctuation model is specified.

C Algorithmic branch audit

A finite branch model can be audited by the following deterministic procedure. First compute the branch coordinates from quotient-invariant data. Second evaluate every mandatory gate and record signs, ranks, gaps, and margins. Third decompose the admissible sign region into chambers on the compact domain of interest. Fourth identify wall supports and event classes by active gate sets and oriented sign changes. Fifth construct the transition graph and verify that all proposed rates are nonnegative. Sixth form the generator and check $L\mathbf{1} = 0$. Seventh identify terminal classes and compute the absorption matrix when the transient block is stable. Eighth attach observational readout maps only if they factor through the declared branch coordinates and source variables. Any failure in this algorithm must be reported before empirical interpretation.

D Reproducibility ledger for the worked example

The worked example uses

$$a = 0.30, \quad b = 0.10, \quad c = 0.20, \quad d = 0.05. \tag{79}$$

The transient and terminal blocks are

$$Q = \begin{pmatrix} -0.40 & 0.30 \\ 0 & -0.25 \end{pmatrix}, \qquad R = \begin{pmatrix} 0.10 & 0 \\ 0.20 & 0.05 \end{pmatrix}. \tag{80}$$

Then

$$(-Q)^{-1} = \begin{pmatrix} 2.5 & 3.0 \\ 0 & 4.0 \end{pmatrix}, \tag{81}$$

so that

$$B = (-Q)^{-1}R = \begin{pmatrix} 0.85 & 0.15 \\ 0.80 & 0.20 \end{pmatrix}, \qquad t = (-Q)^{-1}\mathbf{1} = \begin{pmatrix} 5.5 \\ 4.0 \end{pmatrix}. \tag{82}$$

The source bundle contains the same calculation in CSV and JSON form, together with the plotted absorption probabilities and crossing-probability curve.

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