Physical Non-Nullity and the Affine Substrate
A Premetric Foundation for the CAS Programme

CAS Theory Research Series

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Abstract

We develop a premetric foundation for a physical theory in which metric spacetime is not assumed as primitive. The starting point is a distinction between mathematical zero and physical nullity: physical content is admissible only insofar as it supports distinguishability, relation and comparison. We formalize physical presentations, relation grammars, re-description equivalence, local data spaces and comparison transport. We prove that any nontrivial comparison of local data requires a transport grammar. Under explicit affine-readability, affine-preservation and composition hypotheses, this transport grammar admits an affine minimal carrier, unique up to the declared re-description equivalences. The resulting structure does not yet derive spacetime, matter or cosmology; rather, it defines the premetric substrate on which later clock, signature, metric and field-readout gates must be built. The paper is intentionally narrow: it supplies a controlled first layer for the CAS programme and a ledger of what remains open before stronger geometric or physical claims can be made.

Keywords: premetric foundations; affine transport; relation grammar; metric-affine geometry; spacetime emergence; physical nullity.

Contents

1 Introduction

Most physical theories begin after a large amount of structure has already been granted: a differentiable manifold, a metric or causal order, fields over that manifold, and rules for comparing measurements. Even many approaches called "metric-free" still start with a smooth carrier, a causal order, a bundle, or a connection-like object whose comparison role is already implicit. The present strategy is stricter. It asks what must be in place before those objects are even well typed as physical structures. A genuinely premetric theory must first answer a more primitive question:

> *What counts as physically non-null content, and what must be present before two pieces of such content can be compared?*

This paper isolates that question. Its purpose is not to derive the full CAS programme, nor to claim a completed unification. The aim is more surgical: define the minimal conceptual and mathematical layer before metric spacetime, while making explicit which assumptions are definitions, which are conditional theorems and which are open obligations. The proposed order is

$$\text{non-nullity} \longrightarrow \text{distinguishability} \longrightarrow \text{relation} \longrightarrow \text{comparison} \longrightarrow \text{transport}.$$

Only after that chain is explicit can one ask whether the transport grammar has an affine realization, and only after later gates can one speak of clocks, signature, frame fields, effective metrics, matter sectors or observations.

1.1 Relation to Existing Geometric Programmes

The idea that spacetime geometry may be reconstructed from more primitive structures is not new. Classical and modern examples include affine approaches associated with Eddington and Schrödinger, Weyl's gauge-geometric programme, reconstruction of geometry from light propagation and free fall in the Ehlers–Pirani–Schild tradition, causal determination results, causal-set approaches, and the mature metric-affine gauge literature [1, 2, 3, 4, 5, 6, 8]. The present paper differs in its first move. It does not begin with causal order, conformal structure, projective structure, differentiability or a

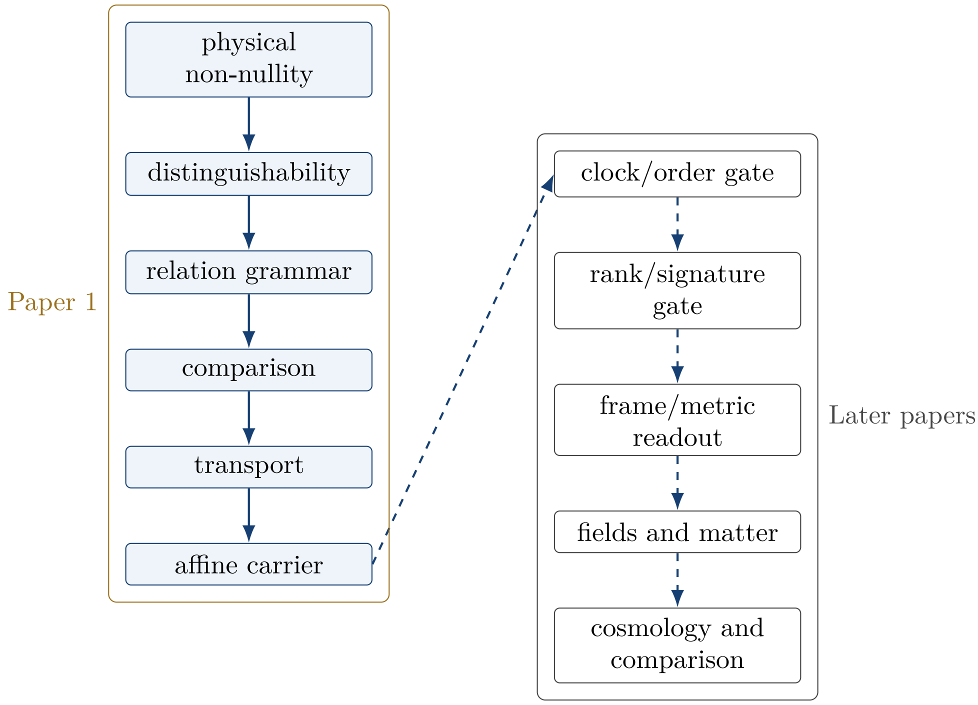

Figure 1: Dependency architecture of the first CAS preprint. The solid chain is the scope of this paper. Dashed arrows are downstream gates and are not used as evidence for the foundation.

metric-affine field theory. It begins with physical non-nullity and asks what must be available for comparison to be meaningful at all.

1.2 Claim Boundary

The following claims are in scope:

- physical nullity is not the same notion as a mathematical zero element;
- physically non-null content requires admissible distinguishability;
- nontrivial comparison of local data requires a transport grammar;
- under affine-readability and affine-preservation hypotheses, affine transport is a minimal premetric carrier;
- metric geometry, observer time, matter, cosmology and numerical comparison are downstream layers.

The following claims are deliberately out of scope: a unique derivation of spacetime, rank four, Lorentzian signature, particle spectra, coupling constants, dark-sector ontology, cosmological parameters, or observational validation.

2 Dependency Architecture

Figure 1 is a dependency graph, not a historical timeline. The methodological rule is simple: no arrow may be reversed. Metric geometry may later clarify the meaning of a branch, but it cannot be used to justify the premetric assumptions from which the branch is supposed to be read out.

Guardrail 2.1 (No backflow). No result from metric readout, particle phenomenology, cosmology or likelihood comparison may be used as a premise in the proof of physical non-nullity, relation grammar or affine transport.

3 Physical Nullity and Mathematical Zero

Definition 3.1 (Mathematical zero). *Let $\mathcal{S}$ be a declared mathematical structure. A mathematical zero is a distinguished element $0_{\mathcal{S}} \in \mathcal{S}$ satisfying the zero laws specified by that structure.*

Definition 3.2 (Physical nullity). *Physical nullity is the absence of admissible physically distinguishable content in a presentation. It is not an element of a vector space, algebra or field unless an additional representation has been supplied.*

The distinction matters. A mathematical zero can be manipulated inside a formal structure; physical nullity is the absence of the very content on which physical predicates would act. Confusing the two smuggles representation into ontology.

Definition 3.3 (Physical presentation). *A physical presentation is a pair*

$$\mathfrak{P} = (E, \mathcal{A}),$$

where E is a domain of candidate contents and $\mathcal{A}$ is a declared class of admissible predicates, relations, comparisons or readouts.

Definition 3.4 (Physically non-null domain). *The physically non-null domain of $\mathfrak{P}$ is*

$$E_{\text{phys}} = \{x \in E : \exists A \in \mathcal{A} \text{ such that } A(x) \text{ acts nontrivially}\}.$$

Postulate 3.1 (Admissibility of non-null content). *A physical theory may begin only from content in E_{phys}. Elements outside E_{phys} may be useful formal placeholders, but they are not physical content until an admissible predicate acts nontrivially on them.*

Remark 3.1. This is the first genuine assumption of the paper. It may eventually be derived from a deeper consistency principle, but Paper 1 treats it as the minimal admissibility condition.

4 Re-Description and Physical Equivalence

Any premetric foundation must separate physical content from the presentation used to describe it. Without such a separation, a later change of notation, coordinate, fiber chart or local encoding could be mistaken for a new physical effect.

Definition 4.1 (Re-description map). *Let $\mathfrak{P} = (E, \mathcal{A})$ and $\mathfrak{P}' = (E', \mathcal{A}')$ be physical presentations. A re-description map is a map*

$$\phi : E \supset U \longrightarrow U' \subset E'$$

together with a pullback/pushforward rule for admissible predicates such that, for every predicate $A' \in \mathcal{A}'$ in the declared comparison domain, there is an $A \in \mathcal{A}$ satisfying

$$A(x) = A'(\phi(x))$$

whenever both sides are defined.

Definition 4.2 (Physical equivalence of presentations). *Two presentations $\mathfrak{P}$ and $\mathfrak{P}'$ are physically equivalent on domains $U \subset E$ and $U' \subset E'$ if there exists an invertible re-description map $\phi : U \to U'$ whose inverse is also a re-description map and which preserves all admissible distinguishability and relation judgments.*

Proposition 4.1 (Equivalence does not create content). *If $\mathfrak{P}$ and $\mathfrak{P}'$ are physically equivalent on U and U', then no physically non-null item, distinction or admissible relation is created merely by passing from one presentation to the other.*

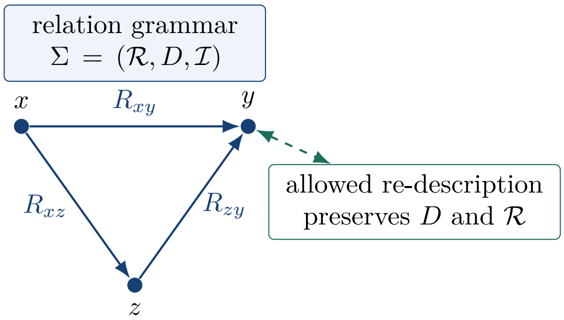

Figure 2: A relation grammar declares which distinctions, relations and re-descriptions are physically admissible before metric distance or observer time has been introduced.

Proof. Physical equivalence requires preservation of admissible predicates, distinguishability and relations in both directions. Therefore any presentation-internal physical statement in U has a corresponding statement in U', and conversely. A difference not detected by the admissible predicate/relation class is a difference of representation, not a new physical content item inside the declared theory. □

Guardrail 4.1 (No coordinate ontology). An object introduced only by a re-description convention cannot be promoted to physical ontology unless it changes an admissible predicate, relation, comparison or later readout.

5 Distinguishability and Relation Grammar

Definition 5.1 (Distinguishability predicate)**.** *A distinguishability predicate on E_{phys} is a relation*

$$D : E_{\mathrm{phys}} \times E_{\mathrm{phys}} \to \{0, 1\}$$

such that $D(x, y) = 1$ means that x and y can be separated by at least one admissible physical criterion.

Definition 5.2 (Relation grammar)**.** *A relation grammar is a triple*

$$\Sigma = (\mathcal{R}, D, \mathcal{I}),$$

where $\mathcal{R}$ is a class of admissible relations on E_{phys}, D is a distinguishability predicate and $\mathcal{I}$ is a class of invariance or equivalence rules preserving physical content under re-description.

Lemma 5.1 (Relation requires distinguishability)**.** *If no admissible predicate can distinguish x from y, then any relation between x and y is physically unobservable inside the presentation $\mathfrak{P}$.*

Proof. Inside $\mathfrak{P}$, physical statements are mediated by $\mathcal{A}$. If all admissible predicates act identically on x and y, then replacing x by y changes no admissible statement. A claimed relation between them has no presentation-internal physical effect unless a further predicate is supplied. □

6 Comparison Requires Transport

Relation alone is not comparison. To compare local data attached to different contents, one must first say how data at one location are represented at the other.

Definition 6.1 (Local data space)**.** *For each $x \in E_{\mathrm{phys}}$, let F_x denote the local data space associated with x. At this stage F_x is not assumed to be a tangent space, a Hilbert space or a field fiber. Its type must be declared by the presentation.*

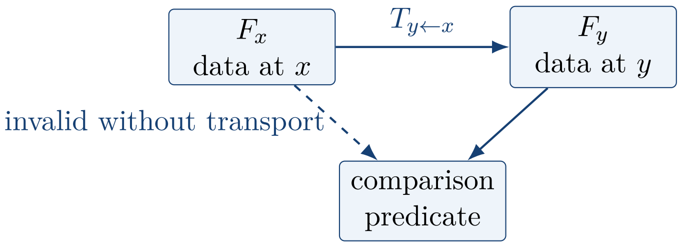

Figure 3: Comparison of local data is not defined until a transport map identifies how data from one local space are represented in another.

Definition 6.2 (Transport grammar)**.** *A transport grammar is a family of maps*

$$T_{y\leftarrow x} : F_x \to F_y$$

defined for admissibly related pairs (x, y), together with identity, composition and re-description rules whenever those operations are physically meaningful.

Theorem 6.1 (Comparison requires transport)**.** *Let F_x and F_y be local data spaces associated with distinguishable contents $x, y \in E_{\mathrm{phys}}$. A nontrivial comparison predicate between data in F_x and data in F_y requires either a transport map $T_{y\leftarrow x} : F_x \to F_y$, a transport map $T_{x\leftarrow y} : F_y \to F_x$, or a third comparison space receiving maps from both.*

Proof. A comparison predicate must accept arguments of a common declared type. If F_x and F_y are not already identified, an expression comparing $u_x \in F_x$ and $v_y \in F_y$ is not well typed. A map from one local space to the other, or maps from both to a common comparison space, supplies the missing typing data. Without such a map the comparison is a formal juxtaposition, not a presentation-internal physical statement. □

7 Affine Transport as Conditional Minimal Carrier

The previous theorem establishes the need for transport. It does not yet say that transport must be affine. Affine structure enters only after a further hypothesis: the local data preserve convex or affine combinations and the admissible comparisons respect that structure.

Definition 7.1 (Affine-readable local data)**.** *Local data are affine-readable if each F_x is modeled as an affine space over a declared vector space V_x, and physically meaningful interpolation or difference statements are invariant under affine re-description.*

Definition 7.2 (Affine-preserving comparison)**.** *A comparison rule is affine-preserving if the maps used to compare local data preserve affine combinations:*

$$T_{y\leftarrow x}((1-\lambda)u + \lambda v) = (1-\lambda)T_{y\leftarrow x}(u) + \lambda T_{y\leftarrow x}(v)$$

whenever the expression is admissible.

Theorem 7.1 (Conditional affine carrier theorem)**.** *Assume:*

1. *local data are affine-readable;*
2. *admissible comparisons are affine-preserving;*
3. *identity and composition are required for repeated comparison along admissible relation chains;*
4. *re-description maps preserve the affine comparison structure;*
5. *no metric, norm, angle, clock or Lorentzian signature is available as a primitive.*

$$
\begin{tikzcd}
F_x \arrow[r, "T_{y\leftarrow x}"] \arrow[d, dashed, "\phi_x"'] \arrow[dr, phantom, "\text{equivalent transport when the square commutes}"] & F_y \arrow[d, dashed, "\phi_y"] \\
F'_{\phi(x)} \arrow[r, "T'_{\phi(y)\leftarrow\phi(x)}"'] & F'_{\phi(y)}
\end{tikzcd}
$$

Figure 4: Re-description equivalence for affine transport. The physical comparison is unchanged when transport commutes with admissible re-description.

Then the minimal premetric carrier of comparison is an affine transport package

$$T_{\text{aff}} = (M_{\text{pre}}, \{F_x\}_{x\in E_{\text{phys}}}, \{T_{y\leftarrow x}\}),$$

where M_{pre} denotes the premetric carrier domain and the maps $T_{y\leftarrow x}$ are affine maps compatible with the declared identity and composition rules.

Proof. By the comparison theorem, comparison requires transport or a common comparison space. By affine-readability, the local data spaces carry affine structure. By affine preservation, admissible comparison maps must preserve that structure. Identity and composition require these maps to form a stable transport grammar over admissible relation chains. Because no metric, norm, angle or clock is primitive, no stronger geometric carrier is available. The remaining structure is precisely affine transport: maps preserving affine combinations without requiring metric measurement. □

Corollary 7.1 (Normal form up to re-description)**.** *Under the hypotheses of Theorem 7.1, any two affine transport packages implementing the same admissible comparisons are equivalent whenever they are related by invertible affine re-description maps on the local data spaces.*

Proof. Let $T_{y\leftarrow x}$ and $T'_{\phi(y)\leftarrow\phi(x)}$ be two transport families and let $\phi_x : F_x \to F'_{\phi(x)}$ be invertible affine re-description maps. If

$$T'_{\phi(y)\leftarrow\phi(x)} \circ \phi_x = \phi_y \circ T_{y\leftarrow x}$$

for all admissible comparison pairs, then every transported datum in one presentation corresponds to exactly one transported datum in the other. Since the admissible predicates and relations are preserved by re-description, the two packages encode the same physical comparisons. □

Guardrail 7.1 (Conditionality)*.* The theorem does not prove that all physical data are affine-readable. It proves that once affine-readable comparison is required before metric geometry, affine transport is the minimal compatible carrier.

8 A Schematic Development

The abstract definitions above can look austere, so we spell out the intended development in a model-independent way. Let $x, y, z \in E_{\text{phys}}$ be three distinguishable contents. Suppose each content carries local data F_x, F_y, F_z and suppose the relation grammar admits relation chains $x \to y \to z$ and $x \to z$. If data from x can be compared with data at z both directly and through y, then the transport grammar must decide whether

$$T_{z\leftarrow x} = T_{z\leftarrow y} \circ T_{y\leftarrow x}$$

holds exactly, approximately, or only after quotienting by an admissible re-description rule in $\mathcal{I}$.

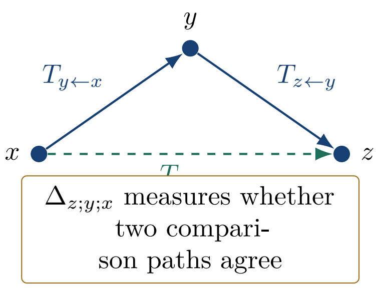

Figure 5: The first obstruction is not metric curvature; it is path-dependence of comparison transport.

Definition 8.1 (Transport defect)**.** *For an admissible triple (x, y, z), define the transport defect*

$$\Delta_{z;y;x} = T_{z\leftarrow x} - T_{z\leftarrow y} \circ T_{y\leftarrow x},$$

whenever subtraction is meaningful in the affine model of the target data space. If subtraction is not yet meaningful, the defect is replaced by the statement that the two comparison paths are inequivalent under $\mathcal{I}$.

Proposition 8.1 (First curvature-like datum)**.** *Transport defects are the earliest curvature-like data available in the premetric layer. They are not Riemannian curvature and do not require a metric.*

Proof. The defect compares two transport paths. It refers only to relation chains, local data spaces and transport maps. No length, angle, clock or metric connection is used. It is therefore curvature-like in the minimal sense of path-dependence of transport, but it is not yet curvature of a smooth spacetime connection. □

9 Consistency Checks

The premetric layer is useful only if it passes several negative tests. These tests are not aesthetic; they prevent hidden structure from entering the foundation.

9.1 Type Consistency

Every comparison must be well typed. If $u_x \in F_x$ and $v_y \in F_y$, then a symbolic expression such as $u_x - v_y$ is meaningless until a transport map or a common comparison space has been supplied. This is the technical core of the "comparison requires transport" theorem.

9.2 Metric Independence

No step in Sections 3–6 uses a distance function, inner product, Lorentzian signature, volume form, observer clock or stress-energy tensor. The only allowed structure is admissibility, relation, local data type and transport.

9.3 Dimensional Discipline

At the level of Paper 1, most quantities are typed rather than dimensioned. Dimensions enter only after a comparison scheme or scale readout is supplied. This is why no numerical constants, mass scales or cosmological parameters appear in the foundation.

9.4 Dependency Discipline

The paper is invalidated if one argues that the affine substrate is correct because later sectors look phenomenologically promising. Later success can motivate interest, but cannot serve as proof of the premetric premises.

10 Comparison With Adjacent Programmes

The construction is adjacent to several known programmes but should not be collapsed into any of them.

10.1 Affine and Metric-Affine Geometry

Affine and metric-affine theories usually begin with a smooth manifold and connection, or with gauge fields valued in affine or linear groups [2, 3, 8]. The present paper is upstream: it asks why a transport grammar is needed before such a smooth connection is available.

10.2 Ehlers–Pirani–Schild Reconstruction

The Ehlers–Pirani–Schild line reconstructs spacetime geometry from light propagation and free fall [4]. CAS Paper 1 does not yet assume light rays, freely falling particles or a four-dimensional manifold. Those objects belong to later readout gates.

10.3 Causal Structure and Causal Sets

Causal approaches start with order as a primitive or near-primitive structure [5, 6]. CAS Paper 1 is more agnostic: relation grammar can include order-like structure, but order is not assumed to be the only admissible relation.

10.4 Gauge-Theoretic Language

Gauge theory shows how comparison between local descriptions becomes a connection and parallel transport. Paper 1 shares that instinct, but strips it back to the premetric level: before a gauge group is declared, one must already know what is being compared and what counts as an admissible re-description.

11 Likely Objections and Precise Replies

11.1 Is this just metric-affine geometry in disguise?

No. Metric-affine geometry normally begins with a differentiable manifold, an independent affine connection and often a metric or coframe sector whose relation to the connection is varied dynamically. This paper does not assume a smooth manifold, metric, coframe, torsion tensor or non-metricity tensor. It derives only the need for a transport grammar, and then only conditionally identifies an affine carrier when local data and comparisons are affine-readable. A metric-affine field theory may be a later readout, not a premise of Paper 1.

11.2 Is affine structure being smuggled in through the data spaces?

Only conditionally. The paper does not claim that arbitrary physical data are affine-readable. It proves a controlled implication:

$$\text{affine-readable data} + \text{affine-preserving comparison} \Longrightarrow \text{affine transport carrier.}$$

If a future CAS sector uses non-affine data, that sector must either provide a different transport grammar or show how affine readability emerges.

11.3 Where is physics, if there is no metric yet?

At this layer, physics enters through admissibility, distinguishability, relation and comparison, not through measurement of length, time or energy. This is intentionally prior to metric physics. The claim is not that metric physics is unnecessary, but that metric physics should not be used to justify the premetric comparison grammar from which it is later supposed to emerge.

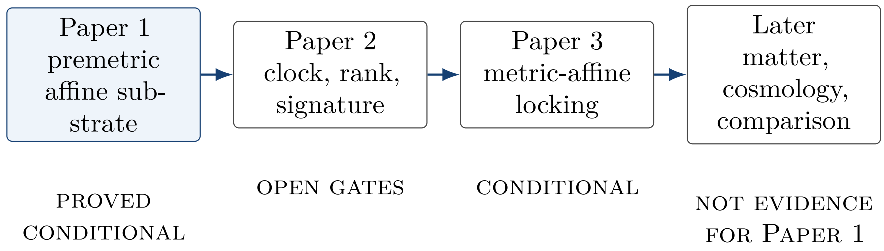

Figure 6: Publication ladder. Paper 1 is designed to be foundational, not maximal.

12 First Readable Functional

The premetric affine substrate is still not a field theory. To begin dynamics or stability analysis one needs a readable functional layer.

Definition 12.1 (Admissible candidate and variation module)**.** *Let Ω_0 be an admissible candidate presentation. The variation module $\mathcal{V}_{\Omega_0}$ consists of first-order variations preserving the domain, relation grammar and affine transport typing to first order.*

Definition 12.2 (First readable functional)**.** *A first readable functional is a map*

$$L_0 : \mathcal{C}_{\text{adm}} \to \mathbb{R}$$

on a declared class of admissible candidates. Its second variation at Ω_0 is

$$H_0 = \delta^2 L_0|_{\Omega_0}.$$

Guardrail 12.1. H_0 is not automatically a spacetime Hessian. It becomes a physical kernel only after pairing, quotienting, stability and later readout gates have been declared.

Proposition 12.1 (Readable quadratic object)**.** *If $\mathcal{V}_{\Omega_0}$ is equipped with a separating dual family $\mathcal{V}^\sharp_{\Omega_0}$ and an admissible premetric pairing $\langle \cdot, \cdot \rangle_0$ on the quotient by null re-description directions, then H_0 defines a readable quadratic object on that quotient.*

Proof. The quotient removes variations that change only presentation, not physical content. The separating dual family distinguishes remaining directions, and the pairing supplies a bilinear interpretation of the second variation. Thus H_0 becomes readable as a quadratic object on the declared quotient. □

13 What Has Not Been Derived

The strongest test of this preprint is whether it refuses false closure. The paper has not derived:

- smooth spacetime;
- an observed clock or proper time;
- rank four;
- Lorentzian signature;
- a nondegenerate frame field;
- an effective metric;
- matter spectra, gauge couplings or cosmology;
- any numerical observational validation.

14 Status Ledger

Item	Paper 1 treatment	Status
Physical nullity vs mathematical zero	Defined and used as the first conceptual distinction	definition
Physically non-null domain	Defined from admissible predicates	definition
Relation grammar	Defined as $(\mathcal{R}, D, \mathcal{I})$	definition
Re-description equivalence	Defined and shown not to create content	proposition
Comparison requires transport	Proved by type/common-domain argument	theorem
Affine transport	Derived under affine-readability and affine-preservation	conditional theorem
First readable quadratic object	Constructed after quotient and pairing	conditional construction
Clock/rank/signature	Deferred to later paper	open gate
Metric/matter/cosmology	Excluded from evidential role	downstream

15 Discussion

The contribution of this paper is deliberately modest in formal reach but large in positioning. It claims that before a theory can speak about geometry, fields or measurement, it must already possess a grammar for non-null content, distinguishability, relation and comparison. Once comparison is admitted, transport is not decorative; it is required for well-typed physical statements.

The affine theorem is the first point at which CAS acquires a recognizable geometric flavor. Yet that theorem is conditional. It does not say that affine transport is forced from nothing. It says that if local data are affine-readable and comparisons preserve affine structure before metric geometry is available, then affine transport is the minimal carrier. This is a useful theorem precisely because its hypotheses are visible.

16 Outlook

The next paper must address the first downstream gates:

1. existence and uniqueness conditions for a relational clock or order parameter;
2. physical projection and quotienting of null re-description directions;
3. rank selection;
4. Lorentzian signature gate;
5. nondegenerate frame readout;
6. effective metric construction.

Only after those gates can the CAS programme responsibly introduce metric-affine dynamics, coherent locking, matter sectors, gravity, cosmology or observational comparison.

A Notation Table

Symbol	Meaning
$\mathfrak{P} = (E, \mathcal{A})$	physical presentation
E	candidate content domain
$\mathcal{A}$	admissible predicates, relations, comparisons or readouts
E_{phys}	physically non-null domain
D	distinguishability predicate
$\Sigma = (\mathcal{R}, D, \mathcal{I})$	relation grammar
ϕ	re-description map between presentations
F_x	local data space at x
$T_{y \leftarrow x}$	transport map from F_x to F_y
T_{aff}	premetric affine transport package
Ω_0	admissible/readable candidate
$\mathcal{V}_{\Omega_0}$	admissible variation module
L_0	first readable functional
H_0	second variation of L_0 at Ω_0

B Open Obligations

1. Derive physical non-nullity from a deeper consistency principle, or keep it explicitly as the first admissibility postulate.
2. Develop equivalence criteria for relation grammars.
3. Extend re-description equivalence to richer comparison categories.
4. Specify when local data are affine-readable in concrete physical presentations.
5. Derive or classify the premetric pairing used to interpret H_0.
6. Prove clock, rank, signature and frame gates without importing observed spacetime as a premise.

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