Fermionic Admissibility, Pauli Exclusion, and Creation–Annihilation Operators in the Einstein-Locked OT/GKSL Source–Readout Framework

Gwenolé Bocquet[1]

[1] *0009-0008-0215-8844*

(Dated: June 12, 2026)

We formulate the role of Pauli exclusion and creation–annihilation operators inside the Einstein-locked OT/GKSL source–readout framework. In the standard fermionic formalism, Pauli exclusion is encoded either by antisymmetry of many-fermion states or by the canonical anticommutation relations of the fermionic creation and annihilation operators [1–4]. In the OT/GKSL framework, however, classical spacetime, local particle language, and relativistic causal semantics are not primitive structures. The native level is an open-system density-matrix dynamics on an effective finite support, equipped in the detailed-balance sector with quantum optimal-transport geometry and entropic ordering [5–8]. Classical spacetime geometry, Dirac dynamics, and particle-level descriptions appear only as certified readouts on controlled windows [9–11].

The purpose of this paper is to identify the precise structural location of the fermionic Fock/CAR layer in that hierarchy. We show that Pauli exclusion should be formulated as a native fermionic admissibility condition on the source sector, prior to certified spacetime readout, prior to the recovered Dirac sector, and prior to reduced constitutive–holonomic branch classification. Creation and annihilation operators are retained, but their status is reclassified: they are CAR generators representing admissible source-sector transitions, not primitive spacetime events. At the native level, the fundamental object remains the density matrix and its completely positive GKSL evolution; creation and annihilation language becomes a representation of fermionic source transitions when a CAR-compatible sector has been selected.

This formulation preserves the standard fermionic content in its certified domain while aligning it with the source–readout ontology of the OT/GKSL corpus. It also clarifies how Pauli compatibility constrains the construction of Dirac-type endomorphisms, HS/NJL bilinears, reduced chiral variables, effective mass branches, vacuum-like residual source slots, and CDM-like intermediate branches [12–17], without modifying the Einstein–Hilbert kinetic block or introducing a new spacetime ontology. The resulting interpretation is that standard Fock and Dirac particle language is recovered as a certified representation of an admissible fermionic source sector, rather than postulated as the primitive ontology of the framework.

I. INTRODUCTION AND SCOPE

The Pauli exclusion principle and the associated creation–annihilation operator formalism are among the central structural ingredients of modern quantum physics. In ordinary many-body quantum mechanics and quantum field theory, identical fermions are described by antisymmetric states, or equivalently by operators satisfying the canonical anticommutation relations

$$\{c_i, c_j^\dagger\} = \delta_{ij}, \qquad \{c_i, c_j\} = 0. \tag{1}$$

These relations imply

$$(c_i^\dagger)^2 = 0, \tag{2}$$

and hence exclude the double occupation of a single fermionic mode. In the usual particle-language presentation, this is often summarized by saying that two identical fermions cannot occupy the same quantum state [1–3]. In a density-matrix formulation, the same algebraic structure also implies the occupation bound

$$0 \le \mathrm{Tr}(\rho n_i) \le 1, \qquad n_i = c_i^\dagger c_i, \tag{3}$$

while still allowing mixed states and fractional mean occupations.

The aim of the present paper is not to revise this standard result. Rather, it is to determine where that result belongs inside the Einstein-locked OT/GKSL source–readout framework. This question is nontrivial because the ontology of the framework is not the usual one in which particles and fields are first placed on a primitive classical spacetime manifold. In the OT/GKSL corpus, the native level is an open-system dynamics of density operators on an effective support. Classical spacetime geometry, local causal semantics, and standard low-energy matter equations are not assumed at the outset; they appear only as certified readout structures on controlled domains [8, 10, 11].

The primitive dynamical object is a density matrix

$$\rho(\lambda) \in D^{\circ}(H_{\text{eff}}), \tag{4}$$

evolving under a GKSL/Lindblad generator,

$$\frac{d\rho}{d\lambda} = -i[H,\rho] + \sum_j \left(L_j \rho L_j^{\dagger} - \frac{1}{2}\{L_j^{\dagger} L_j, \rho\} \right). \tag{5}$$

The GKSL structure supplies the standard completely positive Markovian open-system discipline [5, 6]. In detailed-balance sectors, the dissipative part of the dynamics admits a quantum optimal-transport gradient-flow interpretation for relative entropy, providing an intrinsic entropic ordering on state space [7]. The corresponding classical readout is asserted only on certified windows, where stable record channels, temporal readability, coframe nondegeneracy, bridge control, and spectral support remain jointly admissible.

This hierarchy changes the interpretation of Pauli exclusion. If classical spacetime locality is itself a certified readout property, then Pauli exclusion cannot be fundamentally grounded in the statement that two particles cannot be located in the same classical state of an already given spacetime. In the OT/GKSL framework, Pauli exclusion must instead be placed at the native fermionic source level. It is a constraint on the admissible fermionic structure of the effective support, the density matrices, the allowed source transitions, and the fermionic bilinear sectors from which later Dirac, HS/NJL, and reduced branch descriptions descend.

The central proposal of this paper is therefore to introduce a fermionic admissibility lock. A fermionic OT/GKSL sector is admissible only if its effective support, density states, GKSL channels, endomorphism sector, bilinear reductions, and certified fermionic readouts preserve a CAR-compatible structure. In schematic form, the relevant chain is

$$\mathcal{A}_{\text{CAR}} \longrightarrow H_{\text{eff}}^{F} \longrightarrow D_F^{\circ}(H_{\text{eff}}^{F}) \longrightarrow \rho_F(\lambda) \longrightarrow Y(\rho) \longrightarrow D(\rho) \longrightarrow \text{certified fermionic readout.} \tag{6}$$

Here $\mathcal{A}_{\text{CAR}}$ denotes the canonical anticommutation algebra, H_{eff}^{F} the fermionic effective support, D_F° the corresponding admissible density-state manifold, and $Y(\rho)$ the state-dependent endomorphism entering the fermionic readout sector.

This placement is consistent with the certified recovery of Dirac dynamics developed in the OT/GKSL corpus. The Dirac equation is recovered not from the final scalar collective sector, but from a partially reduced fermionic sector in which the Dirac-type operator remains explicit [9]. In that sector one has, schematically,

$$D(\rho) = i\gamma^{\mu}(\nabla_{\mu}^{\text{ro}} - ie_{\text{eff}} A_{\mu}) - Y(\rho), \tag{7}$$

with $Y(\rho)$ inherited from the fermionic source structure. The recovered Dirac equation is therefore a certified low-energy representation of an underlying admissible fermionic source sector, not the primitive starting point of the full framework.

The same logic applies to creation and annihilation operators. In ordinary Fock language, $c_i^{\dagger}$ and c_i create and annihilate fermionic quanta in specified modes. In the OT/GKSL framework, they are retained as the algebraic generators of a CAR representation, but they are not interpreted as primitive events occurring in a pre-given classical spacetime. The native dynamical object is the transformation of ρ under admissible completely positive evolution. Creation and annihilation are therefore understood more fundamentally as source-sector transition generators:

$$\rho \longmapsto \mathcal{E}_i^{\pm}(\rho), \qquad \mathcal{E}_i^{\pm} : D_F^{\circ}(H_{\text{eff}}^{F}) \longrightarrow D_F^{\circ}(H_{\text{eff}}^{F}), \tag{8}$$

where the channel $\mathcal{E}_i^{\pm}$ must remain compatible with the CAR structure, trace preservation or controlled exchange, positivity, spectral support, and any relevant parity or superselection rule.

This paper has three main objectives. First, it recalls the standard formulation of Pauli exclusion and creation–annihilation operators in fermionic quantum theory. Second, it places the corresponding CAR/Fock structure inside the native layer of the OT/GKSL source–readout hierarchy. Third, it clarifies the consequences of that placement for Dirac recovery, HS/NJL bilinears, reduced chiral variables, effective mass generation, vacuum-like residual sourcing, and CDM-like branch behavior.

The scope is deliberately precise. We do not claim to derive the CAR algebra from the OT/GKSL dynamics alone. We do not claim a global equivalence between the OT/GKSL framework and textbook QED. We also do not treat the standard Fock representation as invalid. The claim is instead structural: once a fermionic sector is present in the

OT/GKSL framework, its admissibility must be governed by a CAR-compatible Pauli lock at the native source level; the standard Fock and Dirac languages are then recovered as certified representations on appropriate windows.

This placement preserves the Einstein lock. The two-derivative Einstein–Hilbert kinetic sector remains universal, with no state-dependent prefactor multiplying $R[g]$. Preparation-dependent or state-dependent fermionic effects enter through the source, response, endomorphism, holonomic, or certification sectors, rather than through a deformation of the gravitational kinetic block. The same source-side placement underlies both low-energy laboratory testability and cosmological stress-test implementations of the framework [18].

The resulting interpretation is simple. Pauli exclusion is not a rule imposed on already localized particles in an already classical spacetime. It is a native admissibility condition on fermionic source states. Creation and annihilation operators are not removed; they are reinterpreted as CAR representation generators for admissible source transitions. Particle language, Dirac dynamics, and Fock occupation become certified readout descriptions of a deeper open-system fermionic source structure.

II. STANDARD FERMIONIC BASELINE

A. Antisymmetry and the exclusion principle

The standard fermionic formalism starts from the indistinguishability of identical particles and the antisymmetry of the many-fermion wave function. For two identical fermions, exchange of the two arguments gives

$$\Psi(x_1, x_2) = -\Psi(x_2, x_1). \tag{9}$$

If both particles are assigned to the same one-particle state, the two arguments become identical in the relevant quantum labels and the wave function must satisfy

$$\Psi(x, x) = -\Psi(x, x), \tag{10}$$

hence

$$\Psi(x, x) = 0. \tag{11}$$

This is the elementary wave-function form of Pauli exclusion. It states that a fully antisymmetric fermionic state cannot contain two identical fermions occupying the same one-particle mode.

In the usual physical interpretation, this is expressed by saying that two identical fermions cannot occupy the same quantum state. This statement is correct in the standard setting, but it is important to note what it already assumes. It assumes that the relevant one-particle modes have been defined, that an antisymmetric many-body sector has been selected, and that the particle-state language is meaningful in the regime under consideration. In a framework where classical spacetime and particle language are readout-level structures, these assumptions must themselves be located inside the hierarchy of the theory.

B. CAR algebra and number operators

The second-quantized form of the same structure is the canonical anticommutation relation algebra. For fermionic modes labelled by i, j, one introduces creation and annihilation operators $c_i^\dagger$ and c_i satisfying

$$\{c_i, c_j^\dagger\} = \delta_{ij}, \qquad \{c_i, c_j\} = 0, \qquad \{c_i^\dagger, c_j^\dagger\} = 0. \tag{12}$$

Taking $i = j$ in the last relation gives

$$2(c_i^\dagger)^2 = 0, \tag{13}$$

and therefore

$$(c_i^\dagger)^2 = 0. \tag{14}$$

This is the algebraic form of Pauli exclusion: applying the same creation operator twice gives zero.

The number operator of mode i is

$$n_i = c_i^\dagger c_i. \tag{15}$$

Using the CAR relations, one obtains

$$n_i^2 = n_i. \tag{16}$$

Therefore the spectrum of n_i is contained in

$$\mathrm{Spec}(n_i) = \{0, 1\}. \tag{17}$$

For a density matrix ρ on the fermionic Fock space, the expected occupation obeys

$$0 \le \mathrm{Tr}(\rho n_i) \le 1. \tag{18}$$

This last inequality is often the most useful form in an open-system or statistical setting. It also prevents a common misunderstanding: Pauli exclusion does not forbid mixed states, reduced states, thermal states, or fractional mean occupations. It forbids double occupation of the same fermionic mode at the level of the CAR algebra.

C. Creation and annihilation operators

In the standard Fock representation, the vacuum state $|0\rangle$ satisfies

$$c_i|0\rangle = 0, \tag{19}$$

and a one-particle state in mode i is obtained by

$$|i\rangle = c_i^\dagger|0\rangle. \tag{20}$$

A general finite fermionic basis state is built by applying distinct creation operators,

$$|i_1, \ldots, i_N\rangle = c_{i_1}^\dagger \cdots c_{i_N}^\dagger|0\rangle, \tag{21}$$

with all indices distinct. The sign of the state changes under exchange of two creation operators because

$$c_i^\dagger c_j^\dagger = -c_j^\dagger c_i^\dagger. \tag{22}$$

Thus antisymmetry, Fock occupation, and Pauli exclusion are three equivalent descriptions of the same fermionic structure.

This standard construction remains fully valid in the domains where a fermionic Fock representation is the correct description. The question addressed in this paper is not whether this construction is correct. The question is how it should be embedded into a framework where the primitive object is not a particle state on a fixed spacetime background, but a density-matrix source dynamics whose classical particle and spacetime descriptions are recovered only as certified readouts.

III. NATIVE SOURCE DYNAMICS AND CERTIFIED READOUT

A. Density matrices as native objects

The OT/GKSL framework starts from a native open-system state rather than from a classical spacetime field configuration. The primitive object is a density matrix

$$\rho(\lambda) \in D^\circ(H_{\mathrm{eff}}), \tag{23}$$

where $D^\circ(H_{\rm eff})$ denotes faithful density operators on an effective support $H_{\rm eff}$. The native evolution is of GKSL form,

$$\frac{d\rho}{d\lambda} = -i[H,\rho] + \sum_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right). \tag{24}$$

Here λ is the native evolution parameter. It is not assumed to be a classical time coordinate. The generator in Eq. (24) is trace-preserving and completely positive under the standard GKSL assumptions [5, 6].

In detailed-balance sectors, the dissipative part of the dynamics admits an optimal-transport gradient-flow interpretation for relative entropy [7]. One may write schematically

$$\frac{d\rho}{d\lambda} = -\nabla_{\rm OT} D(\rho\|\pi), \tag{25}$$

where π is a faithful stationary state and $D(\rho\|\pi)$ is the relative entropy. The corresponding entropy production is

$$\sigma(\rho_\lambda) = -\frac{d}{d\lambda} D(\rho_\lambda\|\pi) \geq 0. \tag{26}$$

This supplies a native entropic ordering, but not yet a classical tick.

B. Readout as a certified structure

Classical spacetime quantities are not introduced as primitive objects. They arise from record channels and readout maps on certified domains. A local readout map has the schematic form

$$\Pi = (X^0, X^1, X^2, X^3), \tag{27}$$

where the $X^a(\rho)$ are operational record functionals. In a minimal linear realization one may write

$$X^a(\rho) = \mathrm{Tr}(\rho R_a), \qquad R_a = R_a^\dagger. \tag{28}$$

The associated operational coframe is

$$e^a_{\rm op} = dX^a. \tag{29}$$

A spacetime readout is certified only when the temporal leg, the coframe, and the OT-to-readout bridge remain jointly controlled. This defines a certified spacetime window

$$W_{\rm st} \subseteq W_{\rm acc}. \tag{30}$$

Within this hierarchy, classical spacetime, local causal semantics, and relativistic locality are readout-level structures, not native axioms [10, 11].

C. Einstein lock and source-side placement

The metric readout sector is Einstein-locked. The two-derivative Einstein–Hilbert kinetic block is kept in standard form,

$$S_{\rm EH} = \frac{c^3}{16\pi G_0} \int d^4x\, \sqrt{-g}\, R[g], \tag{31}$$

with constant G_0. The corresponding placement rule is

$$\text{no state-dependent prefactor multiplying } R[g]. \tag{32}$$

State-dependent or preparation-dependent effects are therefore assigned to the source, response, holonomic, interface, or certification sectors, not to the gravitational kinetic block.

At the readout level one may write schematically

$$G_{\mu\nu}[g^{\rm ro}] = \frac{8\pi G_0}{c^4} \left(T^{\rm bg}_{\mu\nu} + T^{\rm src,eff}_{\mu\nu}[\rho] + T^{\rm resp}_{\mu\nu} \right) + \Delta^{\rm if}_{\mu\nu}. \tag{33}$$

The term $T^{\rm src,eff}_{\mu\nu}[\rho]$ represents the effective readable source generated from the native state, while $T^{\rm resp}_{\mu\nu}$ records the response/exchange contribution required by covariant closure. This is the same source-side placement used in low-energy tests and cosmological stress-test implementations of the framework [18].

IV. FERMIONIC SECTORS IN THE OT/GKSL HIERARCHY

A. Effective fermionic support

A fermionic sector requires more structure than a generic finite-dimensional support. Let $h_{\rm eff}$ denote the effective one-particle fermionic mode space selected on the working domain. The associated fermionic Fock space is

$$H_{\rm eff}^F = \mathcal{F}_-(h_{\rm eff}) = \bigoplus_{N=0}^{\dim h_{\rm eff}} \wedge^N h_{\rm eff}. \tag{34}$$

The minus sign indicates antisymmetrization. The algebra of fermionic mode operators on this support is the CAR algebra

$$\mathcal{A}_{\rm CAR}(h_{\rm eff}) = \langle c_i, c_i^\dagger \rangle. \tag{35}$$

This is the natural place of the Pauli structure in the OT/GKSL hierarchy.

If the support is selected spectrally, one may write schematically

$$h_{\rm eff} = h_{\rm eff}(\Lambda_\star; \rho), \tag{36}$$

where $\Lambda_\star$ is the effective cutoff and ρ denotes the state on the working domain. The detailed spectral construction may involve an internal Dirac-type operator and a cutoff projector. What matters for the present paper is that once a fermionic support is selected, its admissible state space is not an arbitrary bosonic density-state manifold. It must carry the CAR-compatible fermionic structure.

B. Admissible fermionic density states

The generic native state space $D^\circ(H_{\rm eff})$ must therefore be refined in fermionic sectors. We define the admissible fermionic density-state manifold by

$$D_F^\circ(H_{\rm eff}^F, \mathcal{A}_{\rm CAR}) \subset D^\circ(H_{\rm eff}^F). \tag{37}$$

Elements of D_F° are faithful density matrices on the fermionic effective support, interpreted with respect to the CAR algebra. They may be pure, mixed, thermal, reduced, coherent, or open-system states, but their fermionic content is constrained by the CAR structure.

This definition makes Pauli exclusion a structural property of the admissible source sector. It does not require the state to have a definite particle number. It does not require every GKSL channel to conserve particle number. It requires that the state and its admissible transformations remain representable within a fermionic CAR-compatible sector.

C. Fermionic endomorphisms and Dirac-type recovery

The certified Dirac recovery sector of the OT/GKSL corpus is naturally placed after the admissible fermionic source sector has been selected. The relevant Dirac-type operator has the schematic form

$$D(\rho) = i\gamma^\mu \left(\nabla_\mu^{\rm ro} - ie_{\rm eff} A_\mu\right) - Y(\rho), \tag{38}$$

where $Y(\rho)$ is a state-generated endomorphism inherited from the fermionic source structure. In a minimal chiral realization one writes

$$Y(\rho) = \sigma(\rho)\,1 + i\gamma^5 \pi(\rho). \tag{39}$$

Equivalently,

$$Y(\rho) = r(\rho) e^{i\gamma^5 \phi(\rho)}. \tag{40}$$

The key point is that $Y(\rho)$ should not be treated as an arbitrary scalar field inserted by hand. It descends from the fermionic source sector, including bilinear structures of the form

$$\bar{\psi}_i \Gamma_A \psi_j. \tag{41}$$

Such bilinears are meaningful only after the underlying fermionic algebraic sector has been fixed. Therefore the CAR/Pauli layer is upstream from the Dirac-type endomorphism sector.

The certified recovery of standard Dirac dynamics then occurs on a window where the spinorial readout, gauge readout, and background conditions are controlled [9]. In that regime Eq. (38) reduces to an effective Dirac equation with a locally frozen mass and gauge coupling. Outside that certified regime, the native fermionic source structure may remain meaningful, but the standard particle-level Dirac interpretation is no longer asserted with the same strength.

V. THE FERMIONIC ADMISSIBILITY LOCK

A. Definition

We now formulate the central structural rule of the paper.

a. Fermionic admissibility lock. A fermionic OT/GKSL sector is admissible only if its effective support, density states, GKSL channels, endomorphism sector, bilinear reductions, and certified fermionic readouts preserve a CAR-compatible fermionic structure.

In schematic form, the admissible chain is

$$\mathcal{A}_{\mathrm{CAR}} \longrightarrow H^F_{\mathrm{eff}} \longrightarrow D^\circ_F(H^F_{\mathrm{eff}}, \mathcal{A}_{\mathrm{CAR}}) \longrightarrow \rho_F(\lambda) \longrightarrow Y(\rho) \longrightarrow D(\rho) \longrightarrow \text{certified fermionic readout.} \tag{42}$$

Equation (42) is not an additional spacetime postulate. It is a source-sector admissibility condition.

B. Native Pauli exclusion

Within the admissible fermionic sector, Pauli exclusion follows from the CAR algebra. Since

$$\{c_i^\dagger, c_i^\dagger\} = 0, \tag{43}$$

one has

$$(c_i^\dagger)^2 = 0. \tag{44}$$

Equivalently, for the number operator

$$n_i = c_i^\dagger c_i, \tag{45}$$

one finds

$$n_i^2 = n_i. \tag{46}$$

Thus each fermionic mode has occupation eigenvalues 0 and 1. For any admissible density state $\rho \in D^\circ_F$,

$$0 \le \mathrm{Tr}(\rho n_i) \le 1. \tag{47}$$

This is the native Pauli statement in the OT/GKSL hierarchy. It is not formulated as a rule about two particles at the same point of a classical spacetime. It is a rule about the admissible algebraic structure of the fermionic source sector.

C. Pauli before certified locality

The ordering of concepts is important. In ordinary language, Pauli exclusion is often stated using mode labels that may include position, momentum, spin, or other quantum numbers. In the OT/GKSL framework, however, classical spacetime locality is not primitive. It is a certified readout property.

Therefore the fundamental Pauli statement must be placed before certified locality:

$$\text{Pauli/CAR} \quad \text{precedes} \quad W_{\text{st}} \quad \text{and certified causal-local readout.} \tag{48}$$

Only after a certified spacetime and a certified fermionic readout have been established may one translate the native CAR statement into the ordinary particle-language form. In that recovered regime, it is legitimate to say that two identical fermions cannot occupy the same quantum state. But within the source–readout ontology, the deeper statement is that two indistinguishable fermionic excitations cannot correspond to the same admissible CAR direction of the source sector.

D. Consequences for admissible reductions

The fermionic admissibility lock constrains all downstream fermionic constructions.

First, the endomorphism $Y(\rho)$ must descend from CAR-compatible fermionic data. Second, HS/NJL-type bilinears must be constructed inside the same admissible algebraic sector. Third, reduced variables such as $r(\rho)$ and $\phi(\rho)$ must be interpreted as collective source-side variables descending from a fermionic sector that already satisfies Pauli admissibility. Fourth, recovered Dirac or Fock particle language is licensed only on certified windows.

Thus the reduced chiral chain

$$Y(\rho) = \sigma(\rho)1 + i\gamma^5\pi(\rho) = r(\rho)e^{i\gamma^5\phi(\rho)} \tag{49}$$

is not independent of the CAR layer. It is a collective reduction of a fermionic source sector whose admissibility has already been fixed.

VI. CREATION AND ANNIHILATION AS SOURCE-SECTOR TRANSITION GENERATORS

A. Reclassification of the standard operators

The creation and annihilation operators are retained in the OT/GKSL framework, but their status changes. In the standard Fock picture they act on basis states as

$$c_i^\dagger : |n_i = 0\rangle \mapsto |n_i = 1\rangle, \qquad c_i : |n_i = 1\rangle \mapsto |n_i = 0\rangle. \tag{50}$$

In the OT/GKSL hierarchy, this is a representation of an admissible source-sector transition, not a primitive event in a pre-given classical spacetime.

The native object remains the density matrix. Therefore the more fundamental transition language is

$$\rho \longmapsto \mathcal{E}_i^\pm(\rho), \tag{51}$$

where

$$\mathcal{E}_i^\pm : D_F^\circ(H_{\text{eff}}^F, \mathcal{A}_{\text{CAR}}) \longrightarrow D_F^\circ(H_{\text{eff}}^F, \mathcal{A}_{\text{CAR}}). \tag{52}$$

The maps $\mathcal{E}_i^\pm$ represent admissible source-sector transitions whose Fock-coordinate representation may involve $c_i^\dagger$ or c_i.

B. Compatibility with completely positive evolution

In an open-system setting, creation and annihilation processes may appear as Lindblad channels. For example, in a simplified finite-mode model one may consider operators of the form

$$L_i^- = \sqrt{\gamma_i^-}\, c_i, \qquad L_i^+ = \sqrt{\gamma_i^+}\, c_i^\dagger, \qquad L_i^\phi = \sqrt{\gamma_i^\phi}\, n_i. \tag{53}$$

These describe, respectively, loss, injection, and dephasing at the level of the reduced fermionic mode. Such examples show why number conservation cannot be imposed as a universal requirement for all fermionic open systems. A fermionic sector may exchange excitations with an environment.

The relevant requirement is not strict conservation of fermion number in every reduced channel. The requirement is CAR compatibility. We denote by

$$\mathfrak{L}_{\mathrm{CAR}} \tag{54}$$

the class of GKSL generators whose corresponding completely positive semigroup preserves the admissible fermionic state space:

$$e^{t\mathcal{L}} : D_F^\circ(H_{\mathrm{eff}}^F, \mathcal{A}_{\mathrm{CAR}}) \longrightarrow D_F^\circ(H_{\mathrm{eff}}^F, \mathcal{A}_{\mathrm{CAR}}). \tag{55}$$

When $\mathcal{L} \in \mathfrak{L}_{\mathrm{CAR}}$, the open-system dynamics respects the fermionic admissibility lock.

C. Parity and superselection

In many fermionic systems, additional restrictions arise from parity or superselection structure. The fermion parity operator is

$$P_F = (-1)^N, \qquad N = \sum_i n_i. \tag{56}$$

Depending on the physical sector, admissible observables may be required to commute with P_F, and admissible channels may be required to preserve parity or to change it only through explicitly modelled exchange with an environment.

This refinement is not an extra principle competing with Pauli exclusion. It is a possible strengthening of the admissibility lock in sectors where parity superselection is relevant. The general structure is therefore:

$$\text{CAR compatibility} \quad + \quad \text{sector-specific parity or superselection rules.} \tag{57}$$

The precise choice depends on the physical model and the certified readout regime.

D. Physical interpretation

The resulting interpretation is direct. A creation operator does not mean that a particle has appeared as a primitive object in classical spacetime. It means that, within a CAR-compatible representation of the source sector, the state has been moved toward an occupation of a fermionic mode. A particle interpretation becomes available only after a certified readout has recovered the relevant Fock or Dirac language.

Thus, in the OT/GKSL framework,

$$\text{creation/annihilation} = \text{CAR-representation of admissible source transitions,} \tag{58}$$

not

$$\text{creation/annihilation} = \text{primitive spacetime appearance/disappearance.} \tag{59}$$

This reclassification preserves the standard formalism where it applies, while aligning it with the native density-matrix ontology of the framework.

VII. DESCENT TO DIRAC, HS/NJL, AND REDUCED CONSTITUTIVE–HOLONOMIC BRANCHES

A. From admissible fermionic sources to Dirac-type endomorphisms

The fermionic admissibility lock is not an isolated algebraic constraint. It controls the source-side structures from which the later Dirac, HS/NJL, and reduced branch sectors descend. Once a CAR-compatible effective fermionic support has been selected, one may form admissible spinorial bilinears and endomorphisms. These are the quantities that enter the fermionic readout operator.

In the certified Dirac recovery sector, the effective Dirac-type operator takes the schematic form

$$D(\rho) = i\gamma^{\mu}\left(\nabla^{\mathrm{ro}}_{\mu} - ie_{\mathrm{eff}}A_{\mu}\right) - Y(\rho), \tag{60}$$

where $Y(\rho)$ is a state-generated endomorphism inherited from the fermionic source sector. The essential point is that $Y(\rho)$ is not an arbitrary external mass term. It is a source-side object. Its admissible content is constrained by the CAR-compatible fermionic sector from which it descends.

In a minimal chiral realization one writes

$$Y(\rho) = \sigma(\rho)1 + i\gamma^{5}\pi(\rho), \tag{61}$$

where $\sigma(\rho)$ and $\pi(\rho)$ are scalar and pseudoscalar source-side channels. Equivalently, one may introduce polar variables

$$Y(\rho) = r(\rho)e^{i\gamma^{5}\phi(\rho)}. \tag{62}$$

The variables r and ϕ should therefore be interpreted as reduced collective coordinates of an admissible fermionic source sector. They are not primitive spacetime scalar fields inserted independently of the native dynamics.

B. HS/NJL bilinears and CAR compatibility

The same point applies to HS/NJL-type sectors. A typical fermionic bilinear has the form

$$\bar{\psi}_i\Gamma_A\psi_j, \tag{63}$$

and quartic source-side structures may be schematically written as

$$(\bar{\psi}_i\Gamma_A\psi_j)(\bar{\psi}_k\Gamma_B\psi_l). \tag{64}$$

A Hubbard–Stratonovich reduction may then introduce auxiliary collective fields or endomorphisms which encode the relevant bilinear channels. Such reductions are meaningful only after the underlying fermionic algebraic structure has been fixed. The HS/NJL sector therefore inherits the Pauli/CAR admissibility of the fermionic source layer.

This is important for the interpretation of the resulting collective variables. The reduced variables are not bosonic by arbitrary postulate. They are collective source-side variables built from fermionic bilinears. Their effective bosonic character is a property of the reduced description, not a negation of the fermionic admissibility of the underlying sector.

Thus the logical order is

$$\mathcal{A}_{\mathrm{CAR}} \longrightarrow \bar{\psi}\Gamma\psi \longrightarrow Y(\rho) \longrightarrow (\sigma,\pi) \longrightarrow (r,\phi). \tag{65}$$

Pauli exclusion constrains the first layer and therefore constrains the admissible downstream reductions.

C. Reduced constitutive–holonomic dynamics

On a minimal reduced source-side sector, the chiral polar variables may be organized as

$$q = (r,\phi). \tag{66}$$

The reduced geometry may be written as

$$ds^2_{\rm red} = g_r(r)\,dr^2 + g_\phi(r)\,d\phi^2, \tag{67}$$

with $g_r(r) > 0$ and $g_\phi(r) > 0$ on the certified reduced domain. A projected holonomic branch may be encoded by a reduced connection

$$A = A_\phi(r)d\phi. \tag{68}$$

If the phase variable ϕ is cyclic on the reduced sector, one may eliminate it at fixed reduced momentum J. The radial dynamics is then governed by an effective potential

$$U_{\rm eff}(r) = U_{\rm src}(r) + U_{\rm hol}(r; J), \tag{69}$$

where $U_{\rm src}$ is the constitutive source-side contribution and $U_{\rm hol}$ is the projected holonomic contribution.

The stationary reduced branches satisfy

$$U'_{\rm eff}(r_\star) = 0. \tag{70}$$

A stable branch additionally satisfies

$$U''_{\rm eff}(r_\star) > 0. \tag{71}$$

This reduced branch structure is downstream from the CAR layer. The CAR layer determines the admissible fermionic source content; the reduced potential organizes the collective branch behavior of that content.

D. Effective mass from branch stability

For a stable nontrivial reduced branch $r_\star > 0$, the local radial second variation defines an effective mass scale,

$$m^2_{\rm eff} = \frac{U''_{\rm eff}(r_\star)}{g_r(r_\star)}. \tag{72}$$

In this sense, effective mass is not introduced as a primitive parameter attached to a particle before the source sector has been specified. It is obtained as a stability property of a reduced source-side branch.

This does not conflict with the standard Dirac interpretation. On a certified Dirac readout window, the scalar component of $Y(\rho)$ may be locally frozen and read as an effective Dirac mass. The point is that the mass parameter has a source-side origin in the underlying branch structure. In schematic form,

$$\text{CAR-admissible source} \longrightarrow Y(\rho) \longrightarrow r_\star \longrightarrow m_{\rm eff} \longrightarrow \text{certified Dirac mass}. \tag{73}$$

E. Vacuum-like residual energy

The same stationary branch also carries the residual value

$$E_{\rm vac} = U_{\rm eff}(r_\star). \tag{74}$$

This quantity is vacuum-like in the source/readout sense: it is a branch residual which may populate a vacuum-like source slot after the appropriate readout matching and source/response closure. It is not obtained by modifying the Einstein–Hilbert kinetic sector and it is not inserted as a new primitive fluid.

A schematic matching relation has the form

$$\rho_\Lambda^{\rm slot} = K\, d_{\rm eff}(\Lambda_\star)\, E_{\rm vac}, \tag{75}$$

where K is an effective source-to-readout matching coefficient and $d_{\rm eff}(\Lambda_\star)$ is the effective spectral dimension of the certified support. After source/response closure, the effective vacuum-like density may be written as

$$\rho_\Lambda^{\rm eff} = \rho_\Lambda^{\rm slot} + \rho_{\rm resp}^{\rm (vac)}. \tag{76}$$

The interpretation is again source-side. The vacuum-like contribution is a residual of a reduced constitutive–holonomic branch, not a primitive spacetime substance and not a state-dependent deformation of the Einstein kinetic block.

F. CDM-like intermediate branches

The same branch logic also allows an intermediate source-side regime with cold-dark-matter-like behavior. A reduced branch $B_{\rm int}$ may be called CDM-like if it satisfies three structural conditions:

$$\rho_{\rm mat} > 0, \tag{77}$$

$$|w_{\rm mat}| \ll 1, \qquad w_{\rm mat} = \frac{p_{\rm mat}}{\rho_{\rm mat}}, \tag{78}$$

and

$$\chi_{\rm vis}(B_{\rm int}) \ll 1. \tag{79}$$

The first condition says that the branch is materially source-active. The second says that it behaves as a quasi-dust component. The third says that it is suppressed in the ordinary visible readout channel.

The important conceptual distinction is

$$\text{source-side material presence} \neq \text{ordinary visible readout}. \tag{80}$$

A branch may remain active in the source sector while being weakly visible in an ordinary readout channel. This makes the CDM-like branch a property of source-side branch structure, not a new primitive dark-particle ontology.

The Pauli/CAR layer constrains the fermionic source structures from which such branches descend. It does not by itself determine whether a branch is visible, vacuum-like, or CDM-like. The branch classification occurs downstream:

$$\mathcal{A}_{\rm CAR} \longrightarrow \text{fermionic source} \longrightarrow U_{\rm eff}(r) \longrightarrow \{B_{\rm vis}, B_{\rm vac}, B_{\rm int}\}. \tag{81}$$

VIII. CERTIFIED PARTICLE READOUT AND THE STATUS OF FOCK LANGUAGE

A. What is recovered

On a certified fermionic readout window, the standard particle language may be recovered. This includes fermionic modes, Fock occupation, creation and annihilation operators, and effective Dirac dynamics. The recovered description is not arbitrary. It is licensed by the chain

$$D_F^{\circ} \longrightarrow \rho_F(\lambda) \longrightarrow Y(\rho) \longrightarrow D(\rho) \longrightarrow \text{certified Dirac/Fock readout}. \tag{82}$$

In the appropriate weak-curvature, weak-gauge, and locally frozen regime, the Dirac-type equation reduces to the familiar form

$$\left[i\gamma^{\mu}\left(\partial_{\mu} - ie_{\rm eff}A_{\mu}\right) - m_{\rm eff}\right]\psi \simeq 0. \tag{83}$$

In that regime, it is appropriate to use the standard language of fermions, modes, particles, and creation–annihilation operators.

B. What is not primitive

The same statement must be read in the correct direction. The recovered Fock/Dirac language is not the primitive ontology of the full framework. The following objects are not taken as native primitives:

1. classical spacetime points;
2. local relativistic causal cones;
3. particles already localized in a classical geometry;

4. creation or annihilation events occurring in a pre-given spacetime;
5. state-dependent modifications of the Einstein–Hilbert kinetic block.

Instead, the native layer consists of open-system density matrices, GKSL evolution, state-space geometry, entropic ordering, and admissible source sectors. Standard particle language appears when the corresponding readout conditions are satisfied.

C. Fock language as certified representation

The relation between the native and recovered descriptions can be summarized as follows:

$$\text{standard fermionic Fock language} = \text{certified representation of an admissible source sector.} \tag{84}$$

This statement preserves the standard formalism while clarifying its status. It says that the Fock representation is not discarded. It is used when a CAR-compatible source sector admits a certified Fock or Dirac readout.

Thus Pauli exclusion has two levels:

1. a native level, where it is the CAR admissibility of the fermionic source sector;
2. a recovered level, where it is the familiar exclusion of double occupation in the Fock/Dirac particle description.

The first level supports the second. The second is not the foundation of the first.

D. Creation and annihilation as readout-dependent language

Similarly, creation and annihilation have two levels:

1. at the native source level, they are represented by admissible transitions of ρ inside a CAR-compatible sector;
2. at the recovered Fock level, they are represented by the operators $c_i^\dagger$ and c_i acting on occupation states.

The relation may be written schematically as

$$\rho \mapsto \mathcal{E}_i^{\pm}(\rho) \quad \rightsquigarrow \quad c_i^\dagger,\ c_i \tag{85}$$

when the corresponding representation is certified. The arrow $\rightsquigarrow$ here denotes recovery of a representation, not derivation of the CAR algebra from arbitrary GKSL dynamics.

IX. PHYSICAL CONSEQUENCES

A. Pauli and matter stability

The standard role of Pauli exclusion in matter stability is preserved in the appropriate regime. Fermionic occupation remains bounded mode by mode, and antisymmetry prevents collapse of identical fermions into the same one-particle state. The OT/GKSL formulation changes the placement of this statement. Matter stability is not grounded in primitive spacetime localization, but in the admissibility of the underlying fermionic source sector and its recovered readout.

Thus the usual physical consequences of Pauli exclusion survive, but their conceptual origin is shifted:

$$\text{fermionic source admissibility} \longrightarrow \text{occupation constraints} \longrightarrow \text{recovered matter stability.} \tag{86}$$

B. Pauli and effective mass generation

Effective mass generation in the reduced OT/GKSL branch structure is a collective source-side phenomenon. The mass scale is read from the local stability of a branch:

$$m_{\mathrm{eff}}^2 = \frac{U''_{\mathrm{eff}}(r_\star)}{g_r(r_\star)}. \tag{87}$$

Pauli does not directly equal this mass-generation mechanism. Rather, Pauli constrains the fermionic source content from which the relevant bilinears, endomorphisms, and reduced variables are constructed.

In this sense, the Pauli layer is upstream of mass generation:

$$\text{Pauli/CAR} \longrightarrow \text{admissible bilinears} \longrightarrow Y(\rho) \longrightarrow r_\star \longrightarrow m_{\mathrm{eff}}. \tag{88}$$

C. Pauli and vacuum-like residual sourcing

The same logic applies to vacuum-like residuals. The residual

$$E_{\mathrm{vac}} = U_{\mathrm{eff}}(r_\star) \tag{89}$$

is a branch quantity. It is not a direct sum over created particles and it is not a naive zero-point mode count. It is a reduced source-side residual which may populate a vacuum-like slot after matching and source/response closure.

The Pauli layer constrains the fermionic sector that generates the branch, but the vacuum-like interpretation belongs to the source/readout closure:

$$\text{Pauli/CAR} \longrightarrow \text{admissible source branch} \longrightarrow E_{\mathrm{vac}} \longrightarrow \rho_\Lambda^{\mathrm{eff}}. \tag{90}$$

D. Pauli and CDM-like behavior

A CDM-like branch is characterized by positive material sourcing, quasi-dust behavior, and suppressed ordinary visible readout. None of these conditions is identical to Pauli exclusion. The Pauli principle constrains the fermionic source sector in which the branch is built. The CDM-like behavior is then a property of the branch and of its visibility transfer.

This separation avoids a category error. Pauli exclusion is not a dark-matter mechanism by itself. It is a fermionic admissibility principle. CDM-like behavior, if present, is a source-side branch property downstream from the admissible fermionic sector.

E. Low-energy and cosmological interpretation

The OT/GKSL source–readout framework is designed to be testable through source-side readout effects rather than through modifications of the Einstein kinetic block. In low-energy settings, one may seek source-state-dependent responses at fixed geometry. In cosmological settings, one may test whether a response generated from native density-matrix dynamics propagates into growth or lensing observables.

The relevant empirical question is therefore not simply whether a particle has been created or annihilated. It is whether an admissible source-sector dynamics generates a certified readout response. The role of Pauli and CAR compatibility is to ensure that the fermionic source sector used in such a response is structurally admissible.

X. MATHEMATICAL SUMMARY

The integration of Pauli exclusion and creation–annihilation operators into the OT/GKSL framework can be summarized by the chain

$$\mathcal{A}_{\mathrm{CAR}} \longrightarrow H^F_{\mathrm{eff}} \longrightarrow D^\circ_F \longrightarrow \mathfrak{L}_{\mathrm{CAR}} \longrightarrow \rho_F(\lambda) \longrightarrow Y(\rho) \longrightarrow D(\rho) \longrightarrow (r,\phi) \longrightarrow U_{\mathrm{eff}} \longrightarrow \text{branches} \longrightarrow \text{certified readout.} \tag{91}$$

Each arrow has a specific meaning.

First, $\mathcal{A}_{\mathrm{CAR}}$ fixes the fermionic algebraic structure. Second, H^F_{eff} is the corresponding effective fermionic support. Third, D°_F is the admissible density-state manifold. Fourth, $\mathfrak{L}_{\mathrm{CAR}}$ denotes the class of GKSL generators preserving fermionic admissibility. Fifth, $\rho_F(\lambda)$ is the native fermionic source trajectory. Sixth, $Y(\rho)$ is the state-generated fermionic endomorphism. Seventh, $D(\rho)$ is the Dirac-type operator on the certified fermionic readout sector. Eighth, (r,ϕ) are reduced collective source-side coordinates. Ninth, U_{eff} organizes the reduced constitutive–holonomic branches. Finally, particle, mass, vacuum-like, CDM-like, spacetime, and causal interpretations are asserted only through certified readout.

The central structural statements are the following.

a. Native Pauli admissibility. If a sector is fermionic, its admissible source states must be represented on a CAR-compatible support. The algebraic identity

$$(c_i^\dagger)^2 = 0 \tag{92}$$

implies Pauli exclusion at the native source level.

b. CAR-compatible GKSL evolution. A fermionic GKSL generator is admissible only if the corresponding completely positive evolution preserves the fermionic admissible state space,

$$e^{t\mathcal{L}} D^\circ_F \subseteq D^\circ_F. \tag{93}$$

c. Certified Fock/Dirac recovery. When the fermionic source sector, the Dirac-type endomorphism, and the readout bridge are certified, the standard Fock and Dirac descriptions are recovered as representations of the admissible source sector.

d. Einstein-locked placement. None of these fermionic structures is placed in a state-dependent prefactor multiplying $R[g]$. Their effects enter through source, response, endomorphism, holonomic, interface, or certification sectors.

XI. DISCUSSION

The integration proposed here is conservative with respect to standard fermionic physics and structural with respect to the OT/GKSL corpus. It does not modify the CAR algebra. It does not discard the Fock representation. It does not deny the recovered Dirac equation. It specifies where these structures belong in a framework whose native ontology is density-matrix source dynamics rather than particles on a primitive spacetime background.

The main shift is conceptual. In the usual presentation, Pauli exclusion is often introduced after the one-particle Hilbert space and the particle modes have already been specified. In the OT/GKSL framework, the relevant one-particle modes themselves belong to an effective support, and the particle language becomes a certified representation. Pauli must therefore be formulated before spacetime and particle readout, as a native fermionic admissibility principle.

The same is true of creation and annihilation operators. They remain indispensable for representing fermionic transitions once a CAR sector is selected. But they should not be interpreted as primitive spacetime events. At native level, the fundamental object is an admissible transformation of the density matrix. The Fock operators are the representation of that transformation in a certified fermionic language.

This placement also clarifies the relation between fermionic structure and the reduced physical branches of the framework. Mass generation, vacuum-like residual sourcing, and CDM-like behavior are not direct consequences of Pauli exclusion alone. They are properties of reduced constitutive–holonomic branches. Pauli constrains the source sector from which those branches descend.

Finally, the formulation preserves the Einstein lock. The fermionic admissibility layer affects the source-side content and its certified readouts. It does not introduce a new kinetic gravity law and does not place state dependence in front of the Einstein–Hilbert term.

XII. CONCLUSION

Pauli exclusion and creation–annihilation operators fit naturally into the Einstein-locked OT/GKSL source–readout framework once their structural level is correctly identified. Pauli exclusion is a native fermionic admissibility principle on the CAR-compatible source sector. Creation and annihilation operators are CAR generators representing admissible source-sector transitions. The standard Fock and Dirac languages are recovered when the relevant fermionic readout is certified.

The resulting hierarchy is

$$\text{fermionic source admissibility} \longrightarrow \text{source transitions} \longrightarrow \text{Dirac/Fock recovery} \longrightarrow \text{particle readout.} \tag{94}$$

This hierarchy preserves the usual fermionic formalism while aligning it with the OT/GKSL ontology, in which sources, branches, spacetime, causal semantics, and observables are not primitive objects of a single level but structured outputs of certified readout.

Appendix A: Minimal CAR algebra

This appendix collects the elementary algebraic identities used in the main text. Let c_i and $c_i^\dagger$ be fermionic annihilation and creation operators satisfying

$$\{c_i, c_j^\dagger\} = \delta_{ij}, \qquad \{c_i, c_j\} = 0, \qquad \{c_i^\dagger, c_j^\dagger\} = 0. \tag{A1}$$

For $i = j$, the last relation gives

$$2(c_i^\dagger)^2 = 0, \tag{A2}$$

and hence

$$(c_i^\dagger)^2 = 0. \tag{A3}$$

This is the algebraic expression of Pauli exclusion: a second creation operation in the same fermionic mode annihilates the state.

The number operator is

$$n_i = c_i^\dagger c_i. \tag{A4}$$

Using the CAR relations,

$$\begin{aligned} n_i^2 &= c_i^\dagger c_i c_i^\dagger c_i \\ &= c_i^\dagger (1 - c_i^\dagger c_i) c_i \\ &= c_i^\dagger c_i - c_i^\dagger c_i^\dagger c_i c_i. \end{aligned} \tag{A5}$$

Since $(c_i^\dagger)^2 = 0$ and $c_i^2 = 0$, the second term vanishes, so

$$n_i^2 = n_i. \tag{A6}$$

Thus n_i is a projection and its eigenvalues are

$$\mathrm{Spec}(n_i) = \{0, 1\}. \tag{A7}$$

For any density matrix ρ on the fermionic Fock space,

$$0 \le \mathrm{Tr}(\rho n_i) \le 1. \tag{A8}$$

The last inequality is the appropriate open-system form of the occupation constraint. It allows mixed states and fractional expectation values, while preserving the Pauli exclusion principle at the algebraic level.

Appendix B: Examples of CAR-compatible GKSL channels

This appendix gives elementary examples of Lindblad operators acting on a finite fermionic mode sector. These examples are not intended as a universal classification. Their purpose is to illustrate how creation, annihilation, dephasing, and exchange can be represented while preserving the CAR-compatible state space.

1. Loss channel

A loss channel for mode i may be represented by

$$L_i^- = \sqrt{\gamma_i^-}\, c_i, \qquad \gamma_i^- \geq 0. \tag{B1}$$

The corresponding contribution to the GKSL generator is

$$\mathcal{L}_i^-(\rho) = L_i^- \rho L_i^{-\dagger} - \frac{1}{2}\{L_i^{-\dagger} L_i^-, \rho\}. \tag{B2}$$

This channel removes occupation from the mode when it is present. It is not number-conserving, but it remains a legitimate fermionic open-system channel when interpreted as exchange with an environment or reservoir.

2. Injection channel

An injection channel for mode i may be represented by

$$L_i^+ = \sqrt{\gamma_i^+}\, c_i^\dagger, \qquad \gamma_i^+ \geq 0. \tag{B3}$$

Its GKSL contribution is

$$\mathcal{L}_i^+(\rho) = L_i^+ \rho L_i^{+\dagger} - \frac{1}{2}\{L_i^{+\dagger} L_i^+, \rho\}. \tag{B4}$$

Because $(c_i^\dagger)^2 = 0$, the injection channel cannot produce double occupation of the same fermionic mode. The Pauli constraint is already built into the operator algebra.

3. Dephasing channel

A dephasing channel may be represented by the number operator,

$$L_i^\phi = \sqrt{\gamma_i^\phi}\, n_i, \qquad \gamma_i^\phi \geq 0. \tag{B5}$$

The corresponding generator contribution is

$$\mathcal{L}_i^\phi(\rho) = L_i^\phi \rho L_i^{\phi\dagger} - \frac{1}{2}\{L_i^{\phi\dagger} L_i^\phi, \rho\}. \tag{B6}$$

This channel suppresses coherence between different occupation sectors of the mode without changing the occupation eigenvalues.

4. Number-conserving hopping or mixing

For two modes i and j, a number-conserving transition can be represented by

$$L_{ij} = \sqrt{\gamma_{ij}}\, c_i^\dagger c_j, \qquad \gamma_{ij} \geq 0. \tag{B7}$$

This operator transfers occupation from mode j to mode i, subject to the CAR algebra. If mode i is already occupied, the action is suppressed by Pauli exclusion.

5. CAR-compatible generator class

The examples above motivate the following abstract definition. Let

$$\mathfrak{L}_{\mathrm{CAR}} \tag{B8}$$

be the class of GKSL generators whose semigroups preserve the admissible fermionic state space:

$$e^{t\mathcal{L}} : D_F^{\circ}(H_{\mathrm{eff}}^F, \mathcal{A}_{\mathrm{CAR}}) \longrightarrow D_F^{\circ}(H_{\mathrm{eff}}^F, \mathcal{A}_{\mathrm{CAR}}), \qquad t \geq 0. \tag{B9}$$

A fermionic OT/GKSL model should specify which Lindblad operators belong to this class in the sector under consideration. This is the operational content of CAR-compatible open-system evolution.

Appendix C: Notation table

Symbol	Meaning
ρ	Native density matrix
λ	Native evolution parameter
$D^{\circ}(H_{\mathrm{eff}})$	Faithful density states on the effective support
H_{eff}	Effective support of the native state sector
h_{eff}	Effective one-particle fermionic mode space
H_{eff}^F	Fermionic effective Fock support $\mathcal{F}_-(h_{\mathrm{eff}})$
$\mathcal{A}_{\mathrm{CAR}}$	Canonical anticommutation relation algebra
$c_i^{\dagger}, c_i$	Fermionic creation and annihilation operators
n_i	Number operator $c_i^{\dagger} c_i$
D_F°	CAR-compatible admissible fermionic density states
$\mathcal{L}$	GKSL generator
$\mathfrak{L}_{\mathrm{CAR}}$	CAR-compatible GKSL generator class
$Y(\rho)$	State-generated fermionic endomorphism
$\sigma(\rho), \pi(\rho)$	Scalar and pseudoscalar chiral source channels
r, ϕ	Reduced chiral polar variables
$D(\rho)$	Dirac-type readout operator
W_{acc}	Certified operational readout window
W_{st}	Certified spacetime readout window
W_{Dir}	Certified fermionic/Dirac readout window
U_{eff}	Reduced constitutive–holonomic effective potential
$r_{\star}$	Stable stationary reduced branch
m_{eff}	Effective mass from branch stability
E_{vac}	Vacuum-like residual branch energy
$\rho_{\Lambda}^{\mathrm{eff}}$	Effective vacuum-like source/response density
χ_{vis}	Ordinary visible-readout branch visibility

TABLE I. Notation used in the fermionic admissibility formulation.

Appendix D: Minimal statement of the source–readout hierarchy

For reference, the complete hierarchy used in the paper may be summarized as

$$\mathcal{A}_{\mathrm{CAR}} \longrightarrow H_{\mathrm{eff}}^F \longrightarrow D_F^{\circ} \longrightarrow \mathfrak{L}_{\mathrm{CAR}} \longrightarrow \rho_F(\lambda) \longrightarrow Y(\rho) \longrightarrow D(\rho) \longrightarrow (r,\phi) \longrightarrow U_{\mathrm{eff}} \longrightarrow \text{branches} \longrightarrow \text{certified readout}. \tag{D1}$$

The interpretation of the arrows is as follows. The CAR algebra defines the fermionic admissibility of the source sector. The effective support and density-state manifold specify where the native state lives. The GKSL generator drives the open-system evolution. The endomorphism $Y(\rho)$ and the Dirac-type operator $D(\rho)$ provide the fermionic readout sector. The reduced variables (r, ϕ) and the potential U_{eff} organize the constitutive–holonomic branch structure. The final particle, mass, vacuum-like, CDM-like, spacetime, and causal interpretations are asserted only through certified readout.

ACKNOWLEDGMENTS

The author thanks the readers of the OT/GKSL corpus for discussions concerning fermionic admissibility, source-side branch structure, and the status of particle language in certified readout frameworks.

DATA AVAILABILITY

No new numerical data are generated in this paper. The paper formulates a structural integration of Pauli exclusion and creation–annihilation operators into the Einstein-locked OT/GKSL source–readout framework. References to existing OT/GKSL implementations and cosmological stress tests are given in the bibliography.

[1] W. Pauli, Zeitschrift für Physik **31**, 765 (1925).
[2] P. Jordan and E. Wigner, Zeitschrift für Physik **47**, 631 (1928).
[3] V. Fock, Zeitschrift für Physik **75**, 622 (1932).
[4] S. Weinberg, *The Quantum Theory of Fields. Volume I: Foundations* (Cambridge University Press, Cambridge, 1995).
[5] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, Journal of Mathematical Physics **17**, 821 (1976).
[6] G. Lindblad, Communications in Mathematical Physics **48**, 119 (1976).
[7] E. A. Carlen and J. Maas, Journal of Functional Analysis **273**, 1810 (2017), arXiv:1609.01254 [math-ph].
[8] G. Bocquet, Foundations of a testable certified-domain einstein-locked ot/gksl theory of coherence-dependent gravitational sourcing (2026), preprint.
[9] G. Bocquet, Certified recovery of dirac electron dynamics in central abelian potentials from the einstein-locked optimal-transport-gksl framework (2026), preprint, version v1.
[10] G. Bocquet, Certified spacetime readout on finite support: A unified temporal and geometric boundary in the einstein-locked ot/gksl framework (2026), preprint.
[11] G. Bocquet, Certified causality, locality, and relativity as finite-resource spacetime readout in the einstein-locked optimal-transport/gksl framework (2026), preprint, version v2.
[12] Y. Nambu and G. Jona-Lasinio, Physical Review **122**, 345 (1961).
[13] Y. Nambu and G. Jona-Lasinio, Physical Review **124**, 246 (1961).
[14] S. P. Klevansky, Reviews of Modern Physics **64**, 649 (1992).
[15] M. Buballa, Physics Reports **407**, 205 (2005).
[16] G. Bocquet, Mass generation and vacuum-like residual sourcing theorem from constitutive–holonomic balance in the einstein-locked ot/gksl framework (2026), manuscript in the Einstein-locked OT/GKSL corpus.
[17] G. Bocquet, A theorem on a cdm-like intermediate branch in the einstein-locked ot/gksl framework (2026), manuscript in the Einstein-locked OT/GKSL corpus.
[18] G. Bocquet, From native density-matrix gksl dynamics to cosmological growth and tomographic-shear suppression: Stage31–stage32 fixed-object stress tests of a gksl–optimal-transport source/readout core (2026), preprint.