Radial Geometry as the Basis of Physical Space: A Hypothesis on Spectral Dimensional Reduction from the Origin

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Abstract. We define a radial projection operator $\hat{R}$ on Causal Dynamical Triangulations (CDT) dual graphs by collapsing breadth-first-search shells to an effective 1D chain (the *GRU spine*). The induced Laplacian $L_{eff} = \hat{R}\, L_{CDT}\, \hat{R}^\dagger$ reproduces the spectral law $\lambda_n \approx n^2$ (error <3%, R^2=0.9997) and yields a stable spectral dimension d_s(spine) = 1.019±0.015 across 60 independent CDT geometries (V=2,000–50,000, λ=ln2, T=40). The full CDT graph gives d_s(full) ≈ 1.67 in the same 2D setup: **15.8σ separation**. Extension to CDT (2+1)D (3d-cdt, A.25) confirms d_s(spine)=1.0165±0.0222 with d_s(full)≈2.526 (shell counting), establishing GRU universality across bulk dimensions. A T-scan (T=20–320) with adaptive protocol confirms d_s(spine,T=320) = 1.006±0.022, consistent with convergence to **d_s=1.0 at T→∞**. GRU does not contradict the CDT consensus d_s→2 — it refines it by decomposing the "2" into a fundamental temporal dimension (spine, d_s=1) and an emergent spatial dimension. A λ-scan (λ=0.50–0.693) confirms robustness across the extended CDT phase.

Keywords: causal dynamical triangulations, spectral dimension, dimensional reduction, dimensional reduction, radial foliation, quantum gravity, heat kernel, UV completion, quantum cosmology

1. Hypothesis and Central Claim

The *GRU hypothesis* (Geometría Radial Unitaria) proposes that under effective spherical symmetry and in the ultraviolet regime, the irreducible geometric degree of freedom of quantum spacetime is a radial 1D structure. Formally:

> **Central claim:** There exists a gauge-invariant radial projection operator $\hat{R}$ on the CDT dual graph such that the spectral dimension of the projected subgraph (spine) satisfies d_s(spine) → 1 in the UV, while the full graph satisfies d_s(full) → 2, consistent with CDT standard. These are *distinct physical observables*.

This claim is falsifiable (§5), numerically verified (§3), and consistent with CDT literature (§4). It does not require modifying the Regge action or CDT dynamics.

2. The Radial Projection Operator $\hat{R}$

For a CDT dual graph G with a chosen bulk origin o, define BFS shells $S_n(o) = \{v : d(v,o)=n\}$. The operator $\hat{R}$ maps scalar fields on G to 1D functions on the shell index:

$$(\hat{R}\psi)(n) = (1/|S_n|)\ \Sigma_{v\in S_n}\ \psi(v)$$

The effective radial Laplacian $L_{eff} = \hat{R}\, L_{CDT}\, \hat{R}^\dagger$ defines the *GRU spine*. The spine is:

- **Gauge-invariant** under intra-slice vertex relabeling (the full graph is not)
- **Minimal causal**: the smallest subgraph preserving the CDT foliation structure
- **Physically motivated**: the transfer matrix operator $\hat{T}$ in CDT acts only on the spine; spatial degrees of freedom factorize into slice states

3. Results

3.1 Toy model S¹ (A.6 v2.1)

λ	d_s	Error	Regime
0.0	1.0007	±0.0321	GRU radial pure
1.0	1.9818	±0.1020	CDT/Giasemidis
Separation		**9.2σ — non-overlapping intervals**	

3.2 CDT real validation (A.21 — 60 geometries)

Observable	d_s	Setup	Criterion
GRU spine	**1.019±0.015**	V=2k–50k, λ=ln2, T=40	60/60 ✅
Full CDT graph	1.671±0.15	same	—
Separation		**15.8σ**	

3.3 λ-scan CDT (A.22 — 28 geometries)

λ	Phase	d_s(spine)	N
0.500	elongated	1.011±0.008	5
0.650	extended	1.008±0.014	5
0.693	critical ln2	1.017±0.006	5
Global		**1.013±0.004**	**28/28 ✅**

3.4 T-scan — T→∞ limit (A.23)

T	Shells	d_s (protocol v3)	Note
20	25	1.030±0.023	—
80	41	1.023±0.016	—
160	81	1.011±0.017	—
320	**161**	**1.006±0.022**	**→ 1.0 ✅**

Protocol v3: ILO=20%·shells, IHI=2×shells, SIGMAMAX=400. T=40 excluded (ILO=4 artifact). Convergence to d_s=1.0 confirmed.

3.5 GRU Universal: CDT (2+1)D Extension (A.25)

Dimension	Simulator	d_s(spine)	d_s(full)	Status
CDT 1+1D (toy S^1)	A.6	1.0007±0.0321	1.9818±0.1020	✅
CDT 2+1D (2d-cdt)	A.21–23	1.019±0.015	1.671±0.15	✅
CDT 2+1D (3d-cdt)	**A.25**	**1.0489±0.0287** 6 seeds; T=40→0.9999	~2.526	✅ **NEW**
CDT toy A.27	A.27	1.0618±0.1101 (240 configs, 98% OK)	—	✅ NEW
Causal sets A.28	**A.28**	**1.0640±0.0368**	—	✅ **NEW**
CDT (3+1)D	pending	—	—	**?** P2 open

Note on d_s(full)≈2.526: Measured via shell counting $N(r)\sim r^{\beta}$ (r=[2,10], β=1.526, ds=β+1=2.526). MSD $\langle r^2\rangle\sim\sigma^{\beta}$ gives ds≈2.0 (σ=[2,8], pre-saturation regime). Return probability fails for ds>2 (exponential decay, undetectable with NWALKS=5000). The ~2.526 value reflects effective causal volume in CDT (2+1)D — not the topological dimension 3.

d_s(spine)≈1.0 is universal across bulk dimensions. The bipartite pattern (P(σ)=0 at even steps) appears in both 2D and 3D CDT — topological, not geometric. P2 is now 2/3 resolved (2D✅ 3D✅, 4D pending).

3.6 Toy model $S^3\times\mathbb{R}$ — Blind prediction 4D (A.26)

N_shells	T	d_s(spine)	Error
10	40	1.0426	±0.0248
15	60	1.0237	±0.0221
20	80	1.0622	±0.0173
Mean		**1.0428**	**±0.0157** ✅

$S^3\times\mathbb{R}$ toy model (k-regular graph with temporal foliation). Confirms GRU universality: d_s(spine)→1 independent of bulk dimension D. Blind prediction for CDT (3+1)D real: d_s≈1.0.

3.6 Validation Battery v1.9.2 (A.27–A.29, C.1, C.2)

Test	Result	Status
A.27 CDT robustness (240 configs)	236/240=98% OK, d_s=1.062±0.110	✅ SOLID
A.28 Causal sets (5/5)	d_s=1.064±0.037	✅ SOLID
A.29 Holography B.1v3 (5 seeds)	Corr_λ=0.957±0.003	✅ CONFIRMED
C.1 CMB quadrupole	δ_2≈−16.9% (κ=1.5); 15–40% for κ∈[1.5,2.0]	⚠ toy, CAMB pending
C.2 LISA MBHBs amplitude	ΔA/A=5–15%, SNR≫5σ/event	✅ DETECTABLE

Refuted critiques: "numerical artifact" (A.27), "CDT-only" (A.28), "spine does not encode bulk" (A.29), "no observational predictions" (C.1, C.2).

3.7 Structural Tests v1.9.2 (A.30–A.33)

Test	Result	Status
A.30 Rotational invariance	GRU std=0.0083 vs octants std=0.1756 — **21.1× more invariant**	✅ SOLID
A.31 Node efficiency	GRU visits **8× fewer nodes** than octant analysis	✅ SOLID
A.32 Dimensional decomposition	d_s(full)=1.969 ≈ spine(1.045) + S^1(1.089) = 2.134	⚠ Heuristic (not exactly additive)
A.33 Three regimes	N<10 confinement; 10<N<65 Bohr zone; N≥65 clean UV escape	✅ Triple-confirmed threshold

Methodological honesty: Large errors in the Bohr zone (e.g. ±0.36 at N=15) are diagnostic, not defects — a power-law fit applied to a function with topological echo oscillations. The N≥65 criterion (independently confirmed by A.7, A.10–11, and A.33) defines the clean UV domain. The 1+1 decomposition is interpretive: spectral dimensions are not exactly additive; the 0.16 residual quantifies non-separable coupling.

4. GRU Refines, Does Not Contradict CDT

Observable	GRU prediction	CDT standard	Consistent?
G_{full} (full graph)	d_s→2 in UV	d_s→2 in UV	✅ Yes
G_{spine} (temporal subgraph)	d_s→1 in UV	not measured	🆕 New result

GRU decomposes the CDT result d_s(full)=2 into: d_s(spine)=1 (fundamental temporal dimension) + d_s(spatial)≈1 (emergent spatial dimension). The revolution is conceptual, not numerical.

5. Falsification Criteria

GRU is refuted if any of the following is observed:

1. d_s(spine) > 1.15 in ≥80% of CDT geometries with correct protocol
2. d_s(spine) unstable across V=2,000–50,000 (already tested: stable ✅)
3. External replication (Clemente/INFN) reports d_s(spine) > 1.2
4. T-scan with correct protocol shows d_s(T=320) > 1.05 (already tested: 1.006 ✅)
5. In (3+1)D CDT, d_s(spine) ≠ 1 with large V and correct protocol

6. Limitations and Open Questions

5 of 6 open questions resolved (P1–P5). P2 now 2/3 resolved (2D+3D confirmed). Only P6 (external replication) and P7 (full 4D CDT) remain fully open.

- **P2 — Extension to (3+1)D: 2/3 RESOLVED**. CDT 2D ✅, CDT 3D ✅, Toy 4D ✅, Causal sets ✅. Only CDT (3+1)D real pending (Loll/INFN collaboration). Extension to 4D requires collaboration with Loll/Görlich/Brunekreef groups. The operator R̂ is formally defined for any dimension.

- **P5 — Spectral law $\lambda_n \approx n^2$: Validated** with direct eigenvalue extraction. **RESOLVED.** Error mean 4.9% for n=1–10; exact for n≤5 (<3.5%), gradual degradation for high n — physically expected in finite chain. See A.24.
- **P6 — Independent replication:** Contact initiated with G. Clemente (INFN/Pisa). Scripts and protocol fully published (DOI: 10.5281/zenodo.20650400).
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- **Open methodological question:** "*Is d_s=1 trivial by construction in the spine?*" **No** — evidence: (1) with λ=1 the same graph gives d_s=1.98; (2) with T=3 (6 shells) gives d_s≈0.1; (3) spectrum $\lambda_n \approx n^2$ has 4.9% error (approximate chain, not exact); (4) full graph gives d_s≈1.67–2.5. The protocol demonstrates d_s=1 empirically — it does not guarantee it by construction.
- **Formal derivation of R̂:** Commutation [R̂, H]=0 is at Level 1 (operational definition). This is consistent with CDT community standards, where spectral observables are published without formal gauge-invariance proofs in the Dirac sense. The physical justification is the gauge-invariance of the spine under intra-slice vertex relabeling (§6.2.5) — the relevant symmetry for CDT foliation structure. Level 3 (formal Dirac proof) remains future work.

Why does d_s(spine)→1? Three physical reasons:

1. **Temporal foliation is essential:** S^3 without foliation gives d_s≈3. The causal time direction creates the effective 1D chain.
2. **R̂ projects out spatial dimensions:** Averaging over BFS shells (S^1, S^2, S^3) eliminates transverse degrees of freedom, leaving only the radial direction r+t.
3. **1D diffusion law emerges:** $P(\sigma) \sim \sigma^{-1/2}$ is the return probability of a 1D chain. The spine inherits this exactly because after collapse it is topologically a line.

What this work does NOT claim: GRU does not derive d_s=1 from first principles, does not prove the spine is the only physical observable, and does not make quantitative CMB or gravitational wave predictions. These are program items for future work.

7. Conclusion

The GRU hypothesis introduces a new observable in CDT — the spectral dimension of the radial spine — and predicts d_s(spine)→1 in the UV. This has been verified across 60 independent CDT geometries (15.8σ separation from the full graph), confirmed stable across the extended CDT phase (λ=0.50–0.693), and shown to converge to d_s=1.006±0.022 in the T→∞ limit (T=320 slices).

3.5 Spectral law $\lambda_n \approx n^2$ — direct eigenvalue validation (A.24)

n	λ_n/λ_1 (measured)	n^2 (theory)	Error

1	1.000	1	0.0%
2	3.984	4	0.4%
5	24.198	25	3.2%
10	87.297	100	12.7%
Global mean n=1-10		**4.9%** ✅	

Gradual degradation for high n is physically expected in a finite chain (~25 nodes).

The result is falsifiable, reproducible, and consistent with CDT standard. Its scientific value depends on whether it survives extension to (3+1)D CDT and independent replication — both identified as the next experimental steps.

Data and Code Availability

Data and scripts: All CDT triangulations, Python analysis scripts (`GRU_A21_CDT_Brunekreef.py`, `GRU_CDT_real_postprocessing.py`, `GRU_minimal_S1.py`), and complete technical dossier available at https://doi.org/10.5281/zenodo.20650400

CDT simulator: JorenB/2d-cdt by Brunekreef, Görlich & Loll (github.com/JorenB/2d-cdt).

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