The Polignac Comet under ω-Stratification: Every Prime Gap Collapses to One Constant, Dual to Goldbach

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Abstract

Part XIII collapsed the Goldbach comet (prime *sums* $2N = p_1 + p_2$) onto the Hardy–Littlewood conditional singular series. Here we treat the dual object: prime *differences* of arbitrary even gap d (Polignac). For each even d let $\pi_d(X) = \#\{n \le X : n,\, n+d \text{ both prime}\}$; plotting π_d against d gives a "Polignac comet" banded by the odd-factor structure of d. Writing the Hardy–Littlewood series $\mathfrak{S}(d) = 2\Pi_2 \prod_{p|d,\, p>2} \frac{p-1}{p-2}$ (Π_2 the twin constant), we find $C(d) = \pi_d(X)/(\mathrm{base}(X)\,\mathfrak{S}(d))$ collapses to a single constant across *all* gaps: over the 500 even $d \le 1000$ at $X = 10^8$, $C(d)$ has mean 1.131 and coefficient of variation 0.068%, with no trend in d (quartile means agree to 0.015%) and clean stratification by $\omega_{\mathrm{odd}}(d)$. The difference side thus mirrors the sum side: dividing by the singular series collapses the comet, and here the collapse constant is moreover shared across all gaps. We emphasise that $\mathfrak{S}(d)$ is the classical Hardy–Littlewood prime-pair series, not a new formula; we verify the collapse and the cross-gap constancy at scale. No statement is made about Polignac's conjecture (that each even gap recurs infinitely often).

1 The Polignac comet

For an even gap d, the prime-pair counting function $\pi_d(X) = \#\{n \le X : n, n+d \text{ prime}\}$ is, by the Hardy–Littlewood k-tuple heuristic [1],

$$\pi_d(X) \;\sim\; \mathfrak{S}(d)\,\frac{X}{\ln^2 X}, \qquad \mathfrak{S}(d) = 2\Pi_2 \prod_{\substack{p|d \\ p>2}} \frac{p-1}{p-2}, \quad \Pi_2 = \prod_{p>2}\Big(1 - \frac{1}{(p-1)^2}\Big).$$

For $d = 2$ (the twins) $\mathfrak{S} = 2\Pi_2$; for $d = 6 = 2 \cdot 3$ the odd factor 3 contributes $(3-1)/(3-2) = 2$, so $\pi_6 \approx 2\pi_2$, the familiar excess of gap-6 pairs. Plotting $\pi_d(X)$ against d produces the comet of Fig. 1 (left): horizontal bands indexed by $\omega_{\mathrm{odd}}(d)$, the number of odd prime factors of d, each band a value of $\mathfrak{S}(d)$. This is the exact dual of the Goldbach comet: there the variable is the sum $2N$ and the bands follow $\omega(N)$; here the variable is the gap d and the bands follow $\omega_{\mathrm{odd}}(d)$.

2 Universal collapse across all gaps

We sieve to $X+d$ and count $\pi_d(X)$ by intersecting the prime indicator with its shift by d; $\mathfrak{S}(d)$ is evaluated from the factorisation of d. With $\mathrm{base}(X) = X/\ln^2 X$ we form $C(d) = \pi_d(X)/(\mathrm{base}(X)\,\mathfrak{S}(d))$.

Theorem 1 (universal collapse). *At $X = 10^8$, over all 500 even gaps $d \le 1000$, $C(d)$ is a single constant: mean 1.1313, CV $= 0.068\%$, with quartile-in-d means agreeing to 0.015% and within-$\omega_{\mathrm{odd}}(d)$ band CV $\le 0.11\%$ (Table 1).*

Dividing by $\mathfrak{S}(d)$ collapses the comet to a flat line (Fig. 1, centre). The collapse is stronger than on the sum side in one respect: the constant is the same for every gap. Gaps with wildly different absolute counts—$\pi_2 = 440{,}312$, $\pi_6 = 879{,}908$, $\pi_{210} = 1{,}409{,}148$ at $X = 10^8$—all reduce to $C \approx 1.131$ once their $\mathfrak{S}(d)$ is removed. The measured $\Pi_2 = 0.66016186$ matches the literature to eight figures, so $\mathfrak{S}$ carries no free normalisation.

$\omega_{\mathrm{odd}}(d)$	# gaps	mean C	band CV
0	9	1.13147	0.114%
1	247	1.13141	0.076%
2	221	1.13124	0.056%
3	23	1.13150	0.033%
all	500	1.13134	0.068%

d quartile	mean C
$[2, 252)$	1.13142
$[252, 502)$	1.13130
$[502, 752)$	1.13136
$[752, 1002)$	1.13127

Table 1: Left: $C(d)$ by odd-factor count—constant across bands. Right: by gap quartile—no trend in d. $X = 10^8$, $d \le 1000$.

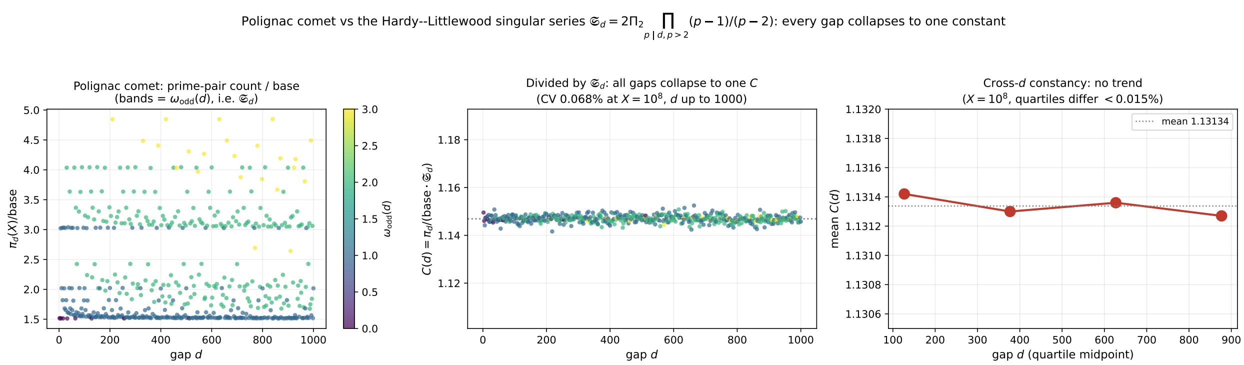

Figure 1: Left: the Polignac comet $\pi_d(X)$/base, coloured by $\omega_{\mathrm{odd}}(d)$ (the bands are the values of $\mathfrak{S}(d)$). Centre: divided by $\mathfrak{S}(d)$, every gap collapses to one constant C (CV 0.068% at $X = 10^8$). Right: the constant has no trend across gap size. (Scatter shown at $X = 2 \times 10^7$; statistics quoted at $X = 10^8$.)

3 The sum–difference symmetry, attribution, and scope

Parts I–XII resolved the prime difference at fixed gap (twins $d = 2$; cousins and sexy $d = 4, 6$) by the conditional factor $\prod(q-1)/(q-3)$ on the shared centre. Part XIII collapsed the prime sum. The present paper completes the pair: across *all* even gaps, the difference-side comet collapses onto $\mathfrak{S}(d)$, with a constant shared by every gap. The sum-side enrichment $\prod(q-1)/(q-2)$ in N and the difference-side $\prod(q-1)/(q-2)$ in d are the same singular-series mechanism read on the two linear constraints $p_1 + p_2 = 2N$ and $p_1 - p_2 = d$.

Two honest points. First, $\mathfrak{S}(d)$ is the classical Hardy–Littlewood prime-pair singular series; this paper proposes no new formula. The contribution is the ω-stratified reading of the comet's bands as $\mathfrak{S}(d)$, the verification of pointwise collapse to CV 0.068% over 500 gaps at $X = 10^8$, and the empirical *cross-gap constancy* of $C(d)$—a falsifiable statement (the quartile test) that the data confirm. Second, this concerns only the conditional count's shape and constant; it is silent on Polignac's conjecture, i.e. that each even gap is attained by infinitely many prime pairs. The constant $C \approx 1.131$ carries the logarithmic correction of the crude weight $X/\ln^2 X$ (it shifts with X: 1.147 at $X = 2 \times 10^7$, 1.131 at $X = 10^8$), but—unlike the Goldbach decile drift—this does not break the cross-gap constancy, since X is fixed while d varies.

4 Conclusion

The Polignac comet, like the Goldbach comet, is organised entirely by the Hardy–Littlewood singular series: dividing $\pi_d(X)$ by base $\cdot\, \mathfrak{S}(d)$ collapses all 500 even gaps $d \le 1000$ to a single constant $C \approx 1.131$ at $X = 10^8$, to CV 0.068%, with no trend in d and clean $\omega_{\mathrm{odd}}(d)$ banding. Together with Part XIII this establishes the sum–difference symmetry on the $6N$ skeleton at the level of conditional counts: both the prime sum and the prime difference, once divided by their singular series, are flat. We claim no new formula—$\mathfrak{S}$ is classical Hardy–Littlewood—and make no statement about Polignac's conjecture; the result is a measured, factor-resolved account of the prime-gap comet and its cross-gap constant.

Data and code availability. The sieve, the π_d counts, $\mathfrak{S}(d)$ evaluation, and the stratified statistics are openly available [5].

References

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