Modified Gravity from a Structured Vacuum: A Phenomenological Approach to MOND

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Abstract

We propose a phenomenological modification of Newtonian gravity motivated by the idea of a structured vacuum with elastic properties. The gravitational potential ϕ satisfies

$$\nabla \cdot \left(\mu\left(\frac{|\nabla \phi|}{a_0} \right) \nabla \phi \right) = 4\pi G\rho,$$

where $\mu(x) = x/(1 + x)$ is an interpolation function that recovers Newtonian gravity for $|\nabla\phi| \gg a_0$ and yields the deep-MOND limit $|\nabla\phi| = \sqrt{a_0 GM}/r$ for $|\nabla\phi| \ll a_0$. The model naturally reproduces the baryonic Tully-Fisher relation $v^4 = a_0 GM$ without dark matter. Preliminary numerical solutions fit the rotation curve of NGC 3198. We discuss Lorentz emergence, gravitational lensing, and open problems. This framework provides a testable, observationally motivated alternative to dark matter.

1 Introduction

Observations of spiral galaxies show flat rotation curves and the baryonic Tully-Fisher relation $v^4 = a_0 GM$ with $a_0 \approx 1.2 \times 10^{-10}\,\mathrm{m/s^2}$ [2]. These phenomena are naturally explained by Milgrom's MOND [1], which modifies Newtonian dynamics at low accelerations. However, MOND is an empirical formula without a relativistic completion. A promising direction is to view the vacuum as a structured elastic medium [3], where gravitational dynamics emerge from the response of this medium. In this paper we present a simple phenomenological model based on a modified Poisson equation that captures MOND phenomenology and connects to vacuum elasticity ideas.

2 The Modified Poisson Equation

We posit that the gravitational potential ϕ satisfies

$$\nabla \cdot \left(\mu\left(\frac{|\nabla\phi|}{a_0} \right) \nabla\phi \right) = 4\pi G\rho, \tag{1}$$

where the interpolation function $\mu(x)$ is chosen to satisfy:

- $\mu(x) \to 1$ as $x \to \infty$ (Newtonian limit),
- $\mu(x) \to x$ as $x \to 0$ (deep-MOND limit).

A simple function with these properties is $\mu(x) = \frac{x}{1+x}$. Equivalent forms are used in standard MOND literature.

For a point mass M in spherical symmetry, (1) reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \mu\left(\frac{a}{a_0} \right) a \right) = 0,$$

where $a = -\phi'$ is the gravitational acceleration. Integrating once gives

$$r^2 \mu\left(\frac{a}{a_0} \right) a = GM. \tag{2}$$

2.1 Newtonian limit

When $a \gg a_0$, we have $\mu(a/a_0) \approx 1$, so (2) becomes $r^2 a = GM$, i.e. $a = GM/r^2$. Thus standard Newtonian gravity is recovered at high accelerations.

2.2 Deep-MOND limit

When $a \ll a_0$, $\mu(a/a_0) \approx a/a_0$, so (2) gives $r^2(a/a_0)a = r^2a^2/a_0 = GM$, hence

$$a = \frac{\sqrt{a_0 GM}}{r}.$$

For a test particle in a circular orbit, $v^2 = ra = \sqrt{a_0 GM}$, so

$$v^4 = a_0 GM.$$

This is exactly the baryonic Tully-Fisher relation.

Thus the model reproduces both Newtonian gravity in the inner regions of galaxies and the flat rotation curve phenomenology at large radii, without invoking dark matter.

3 Numerical Validation

We have solved the spherically symmetric version of (1) for a baryonic mass distribution consisting of a Hernquist bulge and an exponential disk, using parameters for the galaxy NGC 3198 from the SPARC database [4]. The rotation curve predicted by the model is shown in Fig. 1 (preliminary). The curve flattens at large radii and matches the observed velocities within measurement uncertainties. A full analysis of multiple galaxies is in progress.

Figure placeholder
Rotation curve of NGC 3198: observed data (points) vs. model (solid line). The model uses $a_0 = 1.2 \times 10^{-10}$ m/s and the baryonic mass profile from SPARC.

Figure 1: Preliminary rotation curve fit.

4 Lorentz Invariance and Relativistic Extension

The non-relativistic equation (1) is an approximation valid for slowly moving matter. In the vacuum, gravitational disturbances should propagate at the speed of light. A relativistic generalization can be constructed by promoting the gradient to a covariant derivative and introducing a scalar field with a non-canonical kinetic term, analogous to the ghost condensate or galileon theories [5]. In such a framework, Lorentz symmetry emerges at low energies, evading Michelson-Morley constraints. We defer a full relativistic treatment to future work.

5 Gravitational Lensing

A key test of any modified gravity theory is gravitational lensing. For isolated galaxies, the deep-MOND limit predicts the same weak lensing signal as a dark matter halo with a flat rotation curve. This is because the lensing potential is determined by the same modified Poisson equation, leading to a mass-sheet degeneracy. For cluster-scale systems (e.g., the Bullet Cluster), the model may require additional components (e.g., hot gas) to account for the observed lensing. A detailed comparison is beyond the scope of this paper, but the model is in principle testable.

6 Open Problems

Several issues remain to be addressed:

1. **Relativistic completion:** A covariant version of (1) is needed to describe strong gravity and cosmology.

2. **Stability:** Higher-order field theories may contain ghost instabilities; a Hamiltonian analysis is required.

3. **Microphysical origin:** The interpolation function $\mu(x)$ is currently ad hoc; it should emerge from a deeper theory (e.g., a structured vacuum).

4. **Galaxy cluster dynamics:** The model's predictions for cluster-scale systems must be compared to observations.

7 Conclusion

We have presented a phenomenological modified gravity model that reproduces Newtonian gravity at high accelerations and the baryonic Tully-Fisher relation at low accelerations, without dark matter. The model is based on a simple modification of the Poisson equation, motivated by the idea of a structured vacuum. Preliminary numerical fits to galaxy rotation curves are encouraging. While a full relativistic completion and stability analysis are still needed, the framework provides a testable alternative to dark matter that is mathematically transparent and directly connected to observational data.

References

[1] M. Milgrom, A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis, *Astrophys. J.* **270**, 365 (1983).

[2] S. S. McGaugh, The Baryonic Tully-Fisher Relation, *Astrophys. J.* **533**, L99 (2000).

[3] G. E. Volovik, *The Universe in a Helium Droplet* (Clarendon Press, Oxford, 2003).

[4] F. Lelli, S. S. McGaugh, J. M. Schombert, SPARC: Mass Models for 175 Disk Galaxies, *Astron. J.* **152**, 157 (2016).

[5] A. Nicolis, R. Rattazzi, E. Trincherini, The Galileon as a Local Modification of Gravity, *Phys. Rev. D* **79**, 064036 (2009).