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Generalized Symmetry as a Diagnostic Lens: How Non-Invertible Symmetries, Condensable Algebras, and Order-Parameter Structure Reorganize the Classification of Gapped Phases and Phase Transitions

Saluca Agentic AI Research Team — Saluca LLC. AI-drafted synthesis from recent arXiv preprints; for human review, not peer-reviewed.

Abstract

A candidate reading of seven recent preprints from cond-mat.str-el and physics.atom-ph suggests that the classical toolkit for classifying gapped phases and their transitions—built on Landau symmetry breaking, local order parameters, and invertible group symmetries—requires systematic extension at the level of microscopic algebraic structure, not merely at the phenomenological level. The emerging picture, drawn from the corpus, is that *generalized symmetries* (non-invertible, categorical, and higher-form) impose precise algebraic constraints on what constitutes a valid order parameter, what boundary conditions are permissible, and which phase transitions can occur without any form of spontaneous symmetry breaking, even after arbitrary gauging. Concretely: (i) condensable algebras in 2+1d topological orders classify gapped boundaries, and twin algebras sharing identical anyon content but inequivalent algebra structure describe physically distinct phases with isomorphic ground-state degeneracy but inequivalent order parameters, according to the abstract of arXiv:2605.31602; (ii) these twin phases can undergo direct transitions that are stated to be provably beyond-Landau even after partial gauging, in the setting of anomalous finite group symmetry in 1+1d arXiv:2605.31601; (iii) string order parameters in 1d systems with *finite* invertible or non-invertible symmetry organize into Lagrangian algebras in the Drinfel'd center of the symmetry category, making the fusion rule of the symmetry category—not just its representation theory—the operative diagnostic arXiv:2605.27193; (iv) non-invertible symmetry-breaking transitions can be mapped via generalized gauging to deconfined quantum critical points (themselves conjectural in many lattice realizations), and vice versa, establishing a duality whose precise scope

conditions are detailed in the full text arXiv:2605.27672; and (v) non-invertible symmetry enriched topological orders (NI-SETOs) can be constructed from string-net models, extending symmetry enrichment beyond the group-theoretic setting, with chiral examples realized on the boundary of a 3D Walker-Wang model arXiv:2605.28794. Two additional corpus entries—a Rydberg spin-chain study arXiv:2605.27166 and a topological lattice gauge theory paper arXiv:2605.28688—provide microscopic anchoring and structural context, respectively; the categorical interpretation of the Rydberg transitions is a conjecture of this synthesis, not a claim of that paper. The central falsification path is explicit: if a twin-phase transition can be shown to spontaneously break some hidden symmetry after arbitrary gauging, the framework collapses. Concrete tensor-network (DMRG/MPS) constructions of twin-phase order parameters in 1+1d would constitute a direct test.

Introduction

The classification of quantum phases of matter and the transitions between them rests, in its classical formulation, on two pillars: the symmetry group of the Hamiltonian and the pattern of spontaneous symmetry breaking in the ground state. Landau's framework—and its topological extension via topological field theories—has proven extraordinarily productive, yet it is increasingly clear that it misses a large class of phenomena that are intrinsically algebraic rather than group-theoretic in character.

Recent years have seen the concept of symmetry itself generalized: symmetries need not be invertible, need not act on local operators, and need not form a group. They may instead form a *fusion category*, a *higher category*, or even a *hypergroup-graded* structure. The question this synthesis addresses is not merely taxonomic—"how many new symmetry types exist?"—but mechanistic: **what new constraints do these generalized symmetries impose on order parameters, phase boundaries, and the nature of phase transitions?**

The corpus assembled here, drawn entirely from cond-mat.str-el preprints deposited in late May 2026, provides a remarkably coherent set of partial answers. Five papers address this question directly at the level of algebraic mechanism [corpus:arxiv:2605.31602, corpus:arxiv:2605.31601, corpus:arxiv:2605.27193, corpus:arxiv:2605.27672, corpus:arxiv:2605.28794], one provides a complementary lattice gauge theory perspective arXiv:2605.28688, and one anchors the discussion in a concrete, numerically tractable Rydberg spin-chain model arXiv:2605.27166. Together, they suggest—as a heuristic reading, not a derivation from a single unified formalism—that the *algebra structure* of the symmetry, not merely its representation content, is the primary coordinate for classifying phases and predicting transition types. The papers share vocabulary and categorical machinery but do not derive one another's results from a common axiomatic base; the synthesis below argues that they are mutually reinforcing at the level of mechanism, and that their combined implications are testable.

Selection Process

The candidate pool consisted of 12 arXiv preprints in cond-mat.str-el, cond-mat.dis-nn, cond-mat.mes-hall, and physics.atom-ph deposited in the last 30 days, as provided in the corpus block. Papers were filtered by thematic coherence with generalized symmetry and phase classification:

specifically, whether they employ or directly bear on the machinery of fusion categories, condensable algebras, Drinfel'd centers, or non-invertible symmetry actions on gapped phases.

Dropped papers and reasons: - arXiv:2605.31540 (Migdal-Eliashberg/SYK): addresses strong-coupling superconductivity; no categorical symmetry content. - arXiv:2605.29875 (quantum annealing in SK spin glass): addresses optimization and ground-state energy estimation; orthogonal mechanism. - arXiv:2605.29745 (Localization Landscape Theory on Bethe lattice): addresses Anderson localization; no connection to generalized symmetry. - arXiv:2605.31185 (electron-on-neon charge qubit): experimental quantum information platform; no connection to categorical phases. - arXiv:2605.30421 (diabolical textures and topological pumps): addresses a distinct class of topological phenomena not connected to condensable algebra classification.

Kept papers: The seven retained papers share the specific algebraic machinery of fusion categories, condensable algebras, and Drinfel'd centers, making the synthesis internally coherent. The dropped papers are not inferior in quality; they are orthogonal to this synthesis angle.

Background

4.1 Generalized Symmetries and Fusion Categories

In the conventional setting, a symmetry of a quantum system is an invertible operation—an element of a group G—that commutes with the Hamiltonian. The generalization relevant here replaces G with a *fusion category* C: a monoidal category in which objects (topological lines or defects) can be tensored together and their tensor product decomposes into direct sums of other objects, but where individual objects need not have inverses. When objects lack inverses, the symmetry is *non-invertible.*

Non-invertible symmetries arise naturally in 1+1d rational conformal field theories (as duality defects), in lattice models with self-duality (such as the transverse-field Ising model at its critical point), and in topological phases where anyonic excitations generate the symmetry. The key structural point is that the *representation theory* of a fusion category—its module categories, its Drinfel'd center, its Lagrangian algebras—encodes far more information than the representation theory of a group, and this additional structure has direct physical consequences for which phases can exist and how they can transition.

4.2 Condensable Algebras and Boundary Conditions

In a 2+1d topological order described by a modular tensor category (MTC), a *condensable algebra* A is an object in the MTC equipped with a multiplication map satisfying appropriate associativity and unit conditions, with the additional requirement that the topological spin of each simple summand is trivial. Physically, A specifies which anyons can end on a gapped boundary: the boundary condition is determined not just by the set of anyons (the underlying object) but by the full algebra structure—the multiplication rule governing how they fuse at the boundary.

The Symmetry Topological Field Theory (SymTFT) framework uses condensable algebras to classify symmetric gapped phases: a gapped phase of a system with symmetry described by a topological order Z(C) corresponds to a choice of condensable algebra in Z(C). The algebra structure, not merely the anyon content, determines the phase.

4.3 Order Parameters, String Operators, and the Drinfel'd Center

In 1d quantum lattice models, the standard diagnostic for a symmetry-breaking phase is a local order parameter with nonzero expectation value. When the symmetry is non-invertible, local order parameters may be insufficient, and one must instead consider *string operators*—nonlocal operators that create and annihilate symmetry defects. The corpus arXiv:2605.27193 states that multiplets of string order parameters, in a system with *finite* invertible or non-invertible symmetry, organize into a *Lagrangian algebra* in the Drinfel'd center Z(C) of the symmetry category C. The multiplication rule of this Lagrangian algebra governs how the twisted-sector local operators at the endpoints of the string fuse in the infrared limit. This is a structural result: the order parameter is not a number but an algebraic object, and its classification requires the full categorical machinery of Z(C).

Synthesis

5.1 Twin Algebras as a Mechanism for Beyond-Landau Transitions

Claim: Two gapped phases can be physically distinct—with inequivalent order parameters and inequivalent topological data—while sharing identical anyon content and isomorphic ground-state degeneracy. Transitions between such "twin phases" are stated to be provably beyond-Landau even after arbitrary partial gauging, within the specific settings studied.

Evidence: The concept of *twin condensable algebras* is introduced in arXiv:2605.31602: two condensable algebras in a group-theoretical topological order Z(Vec_G^) that share the same underlying anyon decomposition but have inequivalent algebra structure (inequivalent multiplication maps). Infinite families of examples are constructed from *Gassmann triples*—pairs of non-isomorphic subgroups of a finite group G that induce identical permutation representations. In these cases, the reduced topological orders on either side of the condensation are inequivalent despite having identical anyon content. According to the abstract, the twin phases have isomorphic ground-state Hilbert spaces but inequivalent order parameters, such that no local observable within the framework studied can distinguish them. The companion paper arXiv:2605.31601 sharpens this: twin phases, defined as inequivalent phases whose order parameters lie in the same generalized charge under the symmetry S, never exhibit relative spontaneous symmetry breaking. Stable direct transitions between them are stated to be transitions without hidden symmetry breaking—not just without manifest symmetry breaking, but without any symmetry breaking even after arbitrary (partial) gauging of S. An explicit construction in 1+1d with anomalous finite group symmetry is given; the abstract does not specify whether the construction is exhaustive or illustrative.

Caveat: The explicit constructions in arXiv:2605.31602 are in group-theoretical topological orders Z(Vec_G^). Whether the twin algebra phenomenon extends to non-group-theoretical modular tensor categories (e.g., those arising from quantum groups at roots of unity) is not established by the abstract. The 1+1d examples in arXiv:2605.31601 involve anomalous finite group symmetry; the scope of the "no hidden symmetry breaking" claim outside this setting is not addressed in the abstract and may be qualified in the full text.

Falsification path: Construct a twin-phase transition in a concrete lattice model (e.g., a 1+1d spin chain with finite group symmetry derived from a Gassmann triple) and attempt to identify a hidden symmetry—conventional or generalized—that is spontaneously broken across the transition. If such a symmetry is found, the "intrinsically beyond-Landau" claim fails for that example. A DMRG calculation of the entanglement spectrum and string order parameters across the putative twin-phase transition would be the appropriate numerical test. A positive result in any single Gassmann-triple-derived model would constitute strong corroborating evidence; a negative result (hidden symmetry found) in any such model would falsify the claim as stated.

5.2 String Order Parameters as Lagrangian Algebras: The Algebraic Structure of Phase Diagnostics

Claim: In 1d systems with *finite* generalized (possibly non-invertible) symmetry, the correct diagnostic for a gapped phase is not a single order parameter but a *Lagrangian algebra* in the Drinfel'd center of the symmetry category. The multiplication rule of this algebra—not merely its object content—determines which string operators acquire nonzero expectation values.

Evidence: arXiv:2605.27193 states this result using the tensor network approach to the classification of gapped phases, reformulated in terms of module categories over the symmetry category. The central technical claim, as stated in the abstract, is that the expectation value of any string operator depends explicitly on the *tube algebra module* associated with the Lagrangian algebra, which is realized as the full center of the corresponding module category. This means that two phases with the same symmetry category C but different Lagrangian algebras in Z(C) are predicted to have different patterns of string order parameter expectation values—even if the algebras have the same underlying object. The paper also extends the framework to phases of *symmetric mixed states*, relevant for open quantum systems and quantum channels; the abstract does not detail the precise conditions under which this extension applies.

Caveat: The derivation is stated for 1d systems with *finite* invertible or non-invertible symmetry. Whether it extends to continuous symmetries, to higher dimensions, or to systems with anomalies is not addressed in the abstract. The extension to mixed states is asserted but the scope conditions are not detailed in the abstract and may be qualified in the full text.

Falsification path: Identify two 1d gapped phases with the same symmetry category C but Lagrangian algebras in Z(C) that differ only in their multiplication rule (not their object content). Compute string order parameters for both phases using exact diagonalization or MPS methods on a concrete lattice model. If the string order parameters are identical for both phases despite the algebras being inequivalent, the claim that the multiplication rule governs the diagnostic is falsified for that example.

5.3 Non-Invertible Symmetry Breaking and Duality to Deconfined Quantum Criticality

Claim: Spontaneous breaking of non-invertible symmetries in 1+1d lattice models produces phases characterized by long-range correlations of *local* order parameters, but these order parameters obey a more general algebraic structure than in the invertible setting. Via generalized gauging, certain non-invertible symmetry-breaking transitions are stated to be dual to deconfined quantum critical points (DQCPs) of invertible symmetries, and vice versa.

Evidence: arXiv:2605.27672 uses concrete lattice models of gapped phases with non-invertible Rep(H_8) symmetry (the representation category of the dihedral group of order 8) to establish that symmetry-breaking phases can be characterized by local order parameters despite the symmetry being non-invertible. The algebraic structure these order parameters obey is stated to be more general than a group representation. The paper states that it establishes *precise conditions* under which generalized gauging maps non-invertible symmetry-breaking transitions to DQCPs of invertible symmetries, and provides several families of explicit examples. The abstract does not enumerate all cases where the duality fails; those scope conditions are available only in the full text. This is a direct mechanism-level connection: the duality is stated to be implemented by a specific gauging procedure, not merely an analogy.

Caveat: The Rep(H_8) example is illustrative; the abstract states that the paper establishes precise conditions for the duality but does not enumerate failure cases. DQCPs are themselves conjectural in many lattice realizations, so the duality maps one contested object (non-invertible symmetry breaking) to another (DQCP), which requires care in interpretation. The results are stated in 1+1d; extension to higher dimensions is not addressed.

Falsification path: Take a specific model with non-invertible Rep(H_8) symmetry, identify the putative non-invertible symmetry-breaking transition, perform the generalized gauging explicitly, and check whether the resulting theory has the universal properties of a DQCP (e.g., specific values of scaling dimensions, emergent higher symmetry, or fractionalized excitations). A discrepancy in scaling dimensions between the gauged theory and known DQCP universality classes would falsify the duality claim for that specific example.

5.4 Non-Invertible Symmetry Enriched Topological Orders: Extending the SET Framework

Claim: The framework of symmetry-enriched topological orders (SETs), classically formulated for invertible (group) symmetries via G-crossed braided fusion categories, can be extended to non-invertible symmetries. The resulting NI-SETOs are stated to be classified by relative centers of unitary fusion categories, and the symmetry action on anyons is encoded in a Hopf monad on the category of localized excitations, according to the abstracts of the relevant papers.

Evidence: arXiv:2605.28794 proposes a definition of NI-SETOs and implements it for string-net models using two complementary methods: full inclusions of unitary fusion categories (UFCs) and anyon condensation. Both methods are stated to yield the NI-SETO as a *relative center* of UFCs. Chiral examples are constructed on the boundary of a 3D Walker-Wang model representing the anomaly—these are boundary realizations, not bulk examples—providing a concrete realization of anomalous NI-SETOs. The symmetry action on anyons and symmetry defects is computed using tube algebra techniques.

arXiv:2605.28688 provides a complementary perspective using finite group topological lattice gauge theory (quantum double models). By condensing an arbitrary algebra of charges, the localized excitations in the resulting theory are stated to form a *hypergroup-graded extension* of the deconfined excitations. The symmetry action is generically non-invertible, and the categorical action is encoded in a Hopf monad. Gauging the non-invertible symmetry is identified with computing the category of modules over this Hopf monad.

Caveat: Both papers are constructive and definitional: they propose frameworks and demonstrate self-consistency, but the full classification of NI-SETOs (analogous to the group-cohomology classification of invertible SETs) is not claimed to be complete in the abstracts. The abstract of arXiv:2605.28794 does not specify whether all NI-SETOs arise from the two constructions given, or whether there exist NI-SETOs outside this class. The chiral examples in arXiv:2605.28794 are boundary constructions; whether analogous bulk examples exist is not addressed.

Falsification path: Using the string-net models explicitly constructed in arXiv:2605.28794, compute the braiding statistics of excitations in the presence of symmetry defects for a specific NI-SETO example. Check whether the braiding data can be fully encoded in the relative center of UFCs as claimed. If the braiding data of the constructed model cannot be recovered from the relative center structure, the classification is internally inconsistent. A second test: identify whether the two construction methods (full inclusions and anyon condensation) yield identical NI-SETOs for all examples in the paper; if they disagree on a specific example, the framework requires revision.

5.5 Rydberg Arrays as a Microscopic Anchor: Factorization Lines and Quantum Criticality in Constrained Models

Claim (speculative): Concrete Rydberg-atom quantum simulators, operating in the blockade-constrained Hilbert space, may realize competition between symmetry-breaking mechanisms that could, in principle, be reinterpreted within the categorical framework developed in the other corpus papers. The existence of an exact ground-state factorization line within the ordered phase provides an analytically tractable reference point for calibrating order-parameter diagnostics. *This connection is a conjecture of this synthesis, not a claim of arXiv:2605.27166.*

Evidence: arXiv:2605.27166 presents a comprehensive phase diagram of a 1d constrained Rydberg spin chain with competing local Rabi driving and dipole-dipole exchange interactions, computed using exact diagonalization, DMRG, and variational uniform MPS. The phase diagram contains a Luttinger liquid, an antiferromagnetic ordered phase, and a polarized paramagnet. Two distinct mechanisms for the destruction of antiferromagnetic order are identified: a conventional Ising transition at strong driving and a continuous quantum melting into the Luttinger liquid at weak driving. An exact ground-state factorization line is found within the ordered phase, providing a zero-entanglement reference point.

The connection to the broader synthesis is a speculative inference: the blockade constraint projects onto a Hilbert space with non-trivial structure, and one might ask whether this projection implements a non-invertible operation in the sense relevant to the categorical framework. However, a Hilbert space projection being non-invertible as a linear map is a necessary but not sufficient condition for generating a fusion category symmetry; whether the blockade constraint generates such a symmetry requires analysis beyond what the abstract of arXiv:2605.27166 addresses. The two distinct melting mechanisms *might* correspond, at the categorical level, to transitions associated with different condensable algebras, but this is an unverified hypothesis of this synthesis. The factorization line is significant as a calibration point: it is a state where all string order parameters vanish (by virtue of the product-state structure), providing a reference from which the onset of categorical order could in principle be tracked—but this interpretation is also an inference of the synthesis.

Caveat: The connection between the Rydberg model's phase transitions and the categorical framework of twin algebras or NI-SETOs is not made in arXiv:2605.27166. The paper is a numerical study; whether its transitions are "beyond-Landau" in the sense of arXiv:2605.31601 requires additional analysis not present in the abstract. See also the Weakly-Connected Addendum below.

Falsification path (conditional): This falsification path is conditional on first establishing the categorical connection independently. If the symmetry category C of the blockade-constrained model can be identified (a prerequisite not established by the corpus), then: compute the string order parameters in the Rydberg model across both transition lines (Ising and quantum melting) using the MPS framework of arXiv:2605.27193, and check whether the pattern of vanishing/nonvanishing string operators is consistent with a Lagrangian algebra in Z(C). If the string order parameter pattern cannot be fit to any Lagrangian algebra, the categorical framework does not apply to this model. If the symmetry category cannot be identified, this falsification path is not yet accessible.

Discussion

What This Implies (Candidate Reading)

The corpus, taken together, supports a *candidate reading* in which the operative level of organization for classifying gapped phases is the *algebra structure* of the symmetry category—specifically, the condensable algebras and Lagrangian algebras in the relevant Drinfel'd center—rather than the representation content alone. This reading, if borne out by the full texts and subsequent work, would have several concrete implications:

1. **Phase diagrams need algebraic labels, not just symmetry-breaking patterns.** Two phases that look identical from the perspective of conventional order parameters (same ground-state degeneracy, same anyon content, same representation-theoretic data) may be physically distinct if their condensable algebras are inequivalent. Phase boundaries between such twin phases would be invisible to conventional diagnostics.

2. **The gauging map is a computational tool, not merely a formal operation.** The duality between non-invertible symmetry-breaking transitions and DQCPs arXiv:2605.27672 means that gauging can be used to *compute* universal properties of one transition from known properties of another. This is a practical tool for numerical studies, not just a formal equivalence—provided the duality holds in the specific model under study.

3. **Non-invertible symmetry enrichment extends the SET classification.** The NI-SETO framework [corpus:arxiv:2605.28794, corpus:arxiv:2605.28688] opens the question of whether existing materials or engineered systems (topological insulators, fractional quantum Hall states, quantum spin liquids) harbor non-invertible symmetry enrichment that has been overlooked because the classification tools were not available. This remains a hypothesis pending application to specific physical systems.

4. **Programmable quantum simulators (Rydberg arrays) may probe these phenomena.** The exact factorization line in arXiv:2605.27166 provides a concrete experimental reference point, though the categorical interpretation of the Rydberg transitions remains a conjecture of this synthesis and requires independent verification before this implication can be treated as established.

What This Does NOT Imply

- **This is not a claim that all phase transitions are beyond-Landau.** The twin-phase mechanism is a specific algebraic construction that requires the existence of Gassmann triples or analogous structures. Most phase transitions in condensed matter systems are conventional.

- **The categorical framework does not automatically apply to all systems with non-invertible symmetries.** The results of arXiv:2605.27193 are stated for 1d systems with *finite* symmetry; the results of arXiv:2605.31602 are for group-theoretical topological orders. Extension to other settings requires additional work.

- **The dropped papers are not incorporated into this synthesis.** arXiv:2605.31540, arXiv:2605.29875, arXiv:2605.29745, arXiv:2605.31185, and arXiv:2605.30421 address distinct mechanisms (strong-coupling superconductivity, spin-glass optimization, Anderson localization, charge qubits, and topological pumps, respectively) and do not share the algebraic framework of the core synthesis. Including them would dilute the thesis without adding structural support.

Weakly-Connected Addendum

The Rydberg spin-chain paper arXiv:2605.27166 is the weakest link in the synthesis. Its connection to the categorical framework rests on an inference—that the blockade constraint implements a non-invertible operation, and that the two distinct melting mechanisms correspond to different condensable algebras—that is not established in the abstract, and for which no mechanistic argument is provided in the full text as far as the abstract reveals. The paper is a high-quality numerical study of a concrete model, and its factorization line is a genuinely useful reference point, but the categorical interpretation is a conjecture of this synthesis, not a claim of the paper. Readers should treat the Rydberg connection as a hypothesis to be tested, not an established result. The paper is retained in the synthesis because its numerical methods (DMRG, variational MPS) are precisely those needed to test the predictions of arXiv:2605.27193, making it a methodological resource even if the categorical connection is not yet established.

Limitations

Abstract-only reading. This synthesis is based exclusively on arXiv abstracts. Full proofs, detailed constructions, and the precise scope conditions of each result are available only in the full texts. Claims about "precise conditions" arXiv:2605.27672, "all NI-SETOs" arXiv:2605.28794, and the exhaustiveness of the twin-phase construction arXiv:2605.31601 are taken from abstract language and may be qualified in the full texts. This is a fundamental methodological limitation that applies to every claim in this synthesis.

Preprint status. All papers are v1 preprints, not peer-reviewed. The algebraic constructions in particular (twin algebras, NI-SETOs, Hopf monad encoding) involve substantial categorical machinery whose correctness has not been independently verified by referees at the time of writing.

Small corpus. The synthesis draws on seven papers from a single two-week window. This is a convenience sample, not a systematic literature review. Important prior work on non-invertible symmetries, SymTFT, and beyond-Landau transitions is not represented in the corpus and is not cited here (per the ground rules). The synthesis should be read as a snapshot of a specific moment in the field, not a comprehensive review.

Dimension specificity. Most results are stated in 1+1d or 2+1d. The extent to which twin algebras, Lagrangian algebra order parameters, and NI-SETOs extend to 3+1d or higher is not addressed by the corpus. Phase-diagram boundaries described here should be understood as hypothesized for the specific dimensions stated.

Gassmann triple construction. The explicit infinite families of twin algebras in arXiv:2605.31602 rely on Gassmann triples, which are a special class of subgroup pairs in finite groups. Whether this construction is "generic" or "exceptional" in the space of all finite groups is not stated in the abstract.

Finite symmetry restriction. The Lagrangian algebra order parameter result of arXiv:2605.27193 is stated for systems with *finite* symmetry. Whether it extends to continuous symmetries or to systems with anomalies is not addressed in the abstract.

Conclusion

The corpus assembled here supports a candidate reading in which the algebra structure of generalized symmetries—specifically, the condensable algebras and Lagrangian algebras in the Drinfel'd center of the symmetry category—serves as the primary coordinate for classifying gapped phases and predicting the character of phase transitions in 1d and 2d quantum systems. Twin algebras provide a concrete mechanism for phases that are physically distinct yet indistinguishable by conventional order parameters, and twin-phase transitions provide stated examples of beyond-Landau criticality that is intrinsic rather than accidental, within the specific settings of anomalous finite group symmetry studied. Non-invertible symmetry enrichment extends the SET classification beyond the group-theoretic setting, while the duality between non-invertible symmetry breaking and deconfined quantum criticality provides a practical computational tool—subject to the caveat that DQCPs are themselves conjectural in many lattice realizations. Programmable Rydberg arrays, with their exact factorization lines and numerically tractable phase diagrams, offer a concrete experimental platform for testing these predictions, though the categorical interpretation of Rydberg transitions remains a conjecture of this synthesis and requires independent verification. The central falsification path—demonstrating that a twin-phase transition admits a hidden symmetry breaking after arbitrary gauging—is concrete, computationally accessible via DMRG and MPS methods, and would decisively test the framework.

Response to Review

Heuristic framing and historiographic hedging. The Introduction already named the synthesis as a heuristic reading; v2 preserves this and additionally restores the "candidate reading" qualifier in the Discussion's "What This Implies" header and throughout the Discussion section, where v1 had dropped the hedge and stated the operative-level claim as a conclusion rather than a reading.

Selection Process promotion. The Selection Process subsection was buried inside Limitations in v1. In v2 it is promoted to a standalone top-level section (§3) immediately after the Introduction, making the corpus curation decision visible before the synthesis begins rather than after it ends.

Weakest-link hedging in the main body. The Rydberg section (§5.5) in v1 presented the blockade-constraint/non-invertible-operation bridge as an interpretive inference but did not explicitly argue for or against the mechanistic connection. V2 adds a brief mechanistic note—that a Hilbert space projection being non-invertible as a linear map is necessary but not sufficient for generating a fusion category symmetry—and labels the entire categorical interpretation as speculative throughout §5.5, consistent with the Weakly-Connected Addendum. The falsification path for §5.5 is also sharpened to note its conditionality on first establishing the symmetry category.

Abstract-only overshoot corrections. Several claims in v1 were stated as established facts when they were abstract-level assertions: "there is no local observable that can distinguish them" (§5.1), "the symmetry action on anyons is encoded in a Hopf monad" (§5.4), and the factorization-line/string-order-parameter inference (§5.5) are now prefaced with "according to the abstract," "stated to be," or equivalent hedges. The Abstract itself is updated to restore the "finite symmetry" qualifier for arXiv:2605.27193, the "anomalous finite group symmetry" qualifier for arXiv:2605.31601, the "boundary of a 3D Walker-Wang model" qualifier for arXiv:2605.28794, and the "conjectural in many lattice realizations" caveat for DQCPs in arXiv:2605.27672.

Falsification path sharpening. The §5.4 falsification path in v1 was abstract ("identify a physical system claimed to realize a NI-SETO") with no specific system identified. V2 redirects it to the string-net models already constructed within arXiv:2605.28794 itself, making the test constructive and self-contained. The internal consistency check (do the two construction methods agree?) is added as a second test. The §5.5 falsification path is explicitly marked as conditional.

Dropped caveat restoration. All four dropped caveats identified in the review—finite symmetry restriction (arXiv:2605.27193), anomalous finite group symmetry scope (arXiv:2605.31601), boundary-only chiral examples (arXiv:2605.28794), and DQCP conjectural status (arXiv:2605.27672)—are restored in the Abstract and/or the relevant synthesis sections. A standalone "Finite symmetry restriction" bullet is added to Limitations to make the arXiv:2605.27193 scope condition visible at the summary level.