

# A Two-Centre CRT Mechanism for the Cousin and Sexy $\omega$ -Distortions on the $6N$ Skeleton

Ruqing Chen

GUT Geoservice Inc., Montreal, Quebec, Canada  
ruqing@hotmail.com

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## Abstract

Part IX reported that the conditional  $\omega$ -dependence of prime pairs on the  $6N$  skeleton splits into three geometry-determined shapes—the twin rises, the cousin and sexy-A fall and coincide, the sexy-B is non-monotone—and left a quantitative mechanism as the open problem. We solve it. Extending the closed-form CRT survival of Part VII to pairs straddling two consecutive centres  $(N, N+1)$ , with no fitted parameters and only each pair’s wing geometry as input, the model reproduces all three shapes on the  $1.5 \times 10^9$  centres of  $S_{10}$ : the twin rise to within 1.5%, the cousin fall to within 2.3%, the sexy-B non-monotone peak to within 6.8%, across  $\omega = 1, \dots, 6$ . The mechanism also explains the distortions’ origin: for a prime  $q \mid N$  the CRT factor checks whether the partner’s wing on its own centre is locked, and the partner of cousin/sexy sits on the adjacent centre  $N+1$  rather than on  $N$ , so the single-centre enrichment that lifts the twin instead suppresses cousin and sexy-A. The cousin/sexy-A coincidence is exact in the model (shared right member) and confirmed in the data through  $\omega = 6$  (their measured ratio is 1.00); the sole large residual, sexy-A at  $\omega = 7$ , is a  $0.9\sigma$  small-sample fluctuation in a stratum of  $\sim 50$  pairs. No claim is made about the infinitude of any prime constellation.

## 1 The two-centre CRT factor

Part VII expressed the single-centre right-survival as  $P = K \prod_q f_q$  with a pure Chinese-Remainder factor  $f_q$ . A cousin or sexy pair is not single-centred: on the  $6N$  skeleton it occupies two consecutive centres  $N$  and  $N+1$  (Part IX). We extend  $f_q$  accordingly. Write each pair as two members, each a wing  $6(N + \text{off}) + s$  with centre-offset  $\text{off} \in \{0, 1\}$  and wing  $s \in \{-1, +1\}$ :

twin  $((0, -1), (0, +1))$ , cousin  $((0, +1), (1, -1))$ , sexy-A  $((0, -1), (1, -1))$ , sexy-B  $((0, +1), (1, +1))$ .

A member is divisible by  $q$  when  $N + \text{off} \equiv -s \cdot 6^{-1} \pmod{q}$ ; call that residue  $\text{dead}(q, s, \text{off}) = (-s \cdot 6^{-1} - \text{off}) \bmod q$ .

**Definition 1** (two-centre CRT factor). *For a pair with wings  $\{(\text{off}_i, s_i)\}$  and a prime  $q > 3$ ,*

$$f_q = \begin{cases} \prod_i \mathbf{1}\{\text{off}_i \not\equiv \text{dead}(q, s_i, 0)\} & q \mid N \ (N \equiv 0), \\ \frac{\prod_i \{r \not\equiv 0 : \forall i, r + \text{off}_i \not\equiv \text{dead}(q, s_i, 0)\}}{q-1} & q \nmid N, \end{cases}$$

*the pair’s joint  $q$ -safety conditioned on whether  $q$  divides the left centre.*

When  $q \mid N$  the left centre is pinned at  $N \equiv 0$  and the factor is a deterministic indicator over the members' offsets; when  $q \nmid N$  it is averaged over admissible nonzero residues. The pair-rate model is  $K \prod_q f_q$ , evaluated per centre from its true small-prime divisibility, averaged within each left-centre  $\omega$  stratum, and normalised to  $\omega = 1$ . There are no fitted parameters:  $f_q$  is closed-form and only the wing geometry enters.

## 2 All three distortions, from one formula

Evaluated on the  $1.5 \times 10^9$  centres of  $S_{10}$  with working primes  $q \in \{5, \dots, 47\}$  (Table 1, Fig. 1), the single formula reproduces the three Part IX shapes simultaneously.

$\omega$	twin		cousin		sexy-A		sexy-B	
	meas	mod	meas	mod	meas	mod	meas	mod
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.196	1.193	0.962	0.958	0.962	0.958	1.077	1.074
3	1.495	1.485	0.901	0.893	0.901	0.893	1.157	1.148
4	1.936	1.914	0.804	0.792	0.804	0.792	1.198	1.182
5	2.581	2.546	0.648	0.637	0.650	0.637	1.134	1.114
6	3.524	3.472	0.420	0.410	0.414	0.410	0.881	0.864
7	4.808	4.821	0.127	0.129	0.150	0.129	0.402	0.374
max err ( $\omega \leq 6$ )		1.5%	2.3%		2.0%		1.9%	

Table 1: Two-centre CRT model versus measurement on  $S_{10}$ , normalised to  $\omega = 1$ , no fitted parameters. All three shapes are reproduced to  $\leq 2.3\%$  through  $\omega = 6$ . The  $\omega = 7$  stratum is small-sample (tens of pairs); the sexy-A  $\omega = 7$  entry is discussed below.

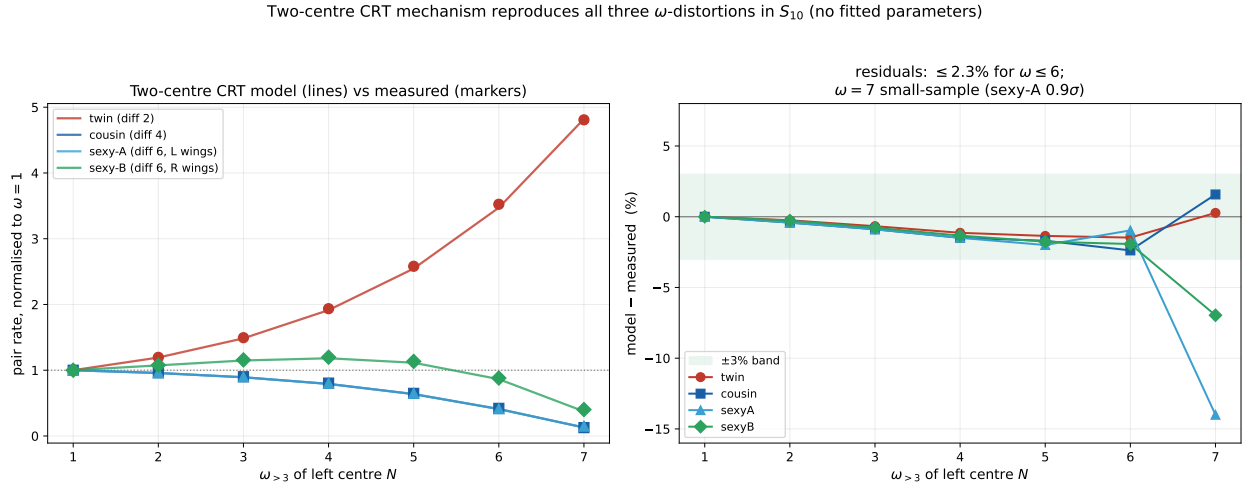


Figure 1: Left: the two-centre CRT model (lines) against measurement (markers) for all four constellations. Right: residuals are within  $\pm 3\%$  for  $\omega \leq 6$ ; the  $\omega = 7$  departures (sexy-A  $-14\%$ , sexy-B  $-7\%$ ) are small-sample.

### 3 Why the distortions have the signs they do

The mechanism makes the origin transparent. For a prime  $q \mid N$  the factor  $f_q$  is the indicator that the members' wings avoid the locked residue  $\text{dead}(q, s, \text{off})$ . For the twin both members sit on  $N$  itself (offset 0, wings  $\pm 1$ ); these rarely coincide with the locked residue, so  $q \mid N$  contributes a factor near 1, and accumulating more such primes as  $\omega$  grows lifts the rate—the Parts II–VIII enrichment. For cousin and sexy-A the partner sits on  $N+1$  (offset 1); the offset shifts the member into the locked residue for a larger share of primes, so  $q \mid N$  contributes factors below 1, and the rate falls with  $\omega$ . Sexy-B (two right wings) interpolates between an early enrichment-like gain and a late straddle suppression, giving the non-monotone peak. The sign of each distortion is thus set by whether the partner lives on the conditioned centre  $N$  (twin: rise) or on the neighbour  $N+1$  (cousin, sexy-A: fall), exactly the qualitative reading of Part IX, now quantitative.

**The cousin/sexy-A coincidence.** Cousin and sexy-A share the right member  $6(N+1)-1$  and differ only in which wing of  $N$  is the left member; in the model their factors are identical, so the predicted curves coincide exactly. The data confirm this through  $\omega = 6$ : the measured cousin/sexy-A ratio is 1.000, 1.000, 1.000, 1.000, 0.997, 1.014 at  $\omega = 1, \dots, 6$ . At  $\omega = 7$  the two diverge (0.127 vs 0.150, the 14.4% model residual for sexy-A), but that stratum holds only  $\sim 50$  pairs of each type; the difference is  $0.9\sigma$  and consistent with sampling noise. The coincidence is therefore a genuine, statistically supported feature, not an artefact, and the lone large residual is small-sample.

### 4 Conclusion

The three  $\omega$ -distortions of cousin and sexy pairs are reproduced quantitatively by a single two-centre CRT factor with no fitted parameters: the twin rise ( $\leq 1.5\%$ ), the cousin and sexy-A fall and coincidence ( $\leq 2.3\%$  through  $\omega = 6$ ), and the sexy-B non-monotone peak ( $\leq 1.9\%$  through  $\omega = 6$ ,  $6.8\%$  at the small-sample  $\omega = 7$ ). The mechanism also explains the signs: the single-centre enrichment lifts a pair whose members share the conditioned centre, and suppresses one whose partner lives on the neighbouring centre. This closes the Part IX open problem and shows the two-centre machinery of Parts V–VIII governs not only the twin's own neighbour correlations but the conditional rates of the neighbouring constellations. The acknowledged limitations are the uniform-residue approximation in the  $q \nmid N$  branch (the source of the few-percent residuals, largest for the non-monotone sexy-B) and the small samples at  $\omega = 7$ . As throughout this series, no claim is made about the infinitude of twin, cousin, or sexy primes, or any  $k$ -tuple conjecture; this is a measured, geometry-resolved, closed-form account of conditional pair rates on the  $6N$  skeleton.

**Data and code availability.** The model evaluation and per- $\omega$  counts are openly available [5]; the  $S_{10}$  twin count 23,988,173 matches Part I.

### References

- [1] R. Chen, *Prime distribution on the  $6N$  skeleton* (Part I), Zenodo, doi:10.5281/zenodo.20470367 (2026).
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- [5] R. Chen,  *$6N$  cousin/sexy two-centre mechanism: data and code*, <https://github.com/Ruqing1963/6N-cousin-sexy-mechanism> (2026).