

A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: For the sequence of primes of the form n^2+1 , let p_k be the k -th such prime. The conjecture states that for all $k \geq 2$, the gap between consecutive primes p_k and p_{k-1} satisfies: $p_k - p_{k-1} < 2 * \sqrt{p_k} * (\ln(p_k))^{0.8}$. This refines the gen. A counterexample was discovered computationally: witness = {'k': 11, 'p_k': 1297, 'p_prev': 677, 'gap': 620, 'bound': 348.18425797112735, 'base_n': 36}. This result was obtained by the SOVEREIGN autonomous research system.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Primes of form n^2+1 — density and distribution. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

Conjecture 1. *For the sequence of primes of the form n^2+1 , let p_k be the k -th such prime. The conjecture states that for all $k \geq 2$, the gap between consecutive primes p_k and p_{k-1} satisfies: $p_k - p_{k-1} < 2 * \sqrt{p_k} * (\ln(p_k))^{0.8}$. This refines the general expectation that gaps are $O(\sqrt{p} \log p)$ by proposing a specific sub-logarithmic exponent for this sparse sequence.*

3 Counterexample

Theorem 1 (Falsification). *The conjecture above is **false**. A counterexample is given by:*

$witness = \{'k' : 11, 'p_k' : 1297, 'p_prev' : 677, 'gap' : 620, 'bound' : 348.18425797112735, 'base_n' : 36\}$

Proof. Direct computation verifies that the witness $\{'k' : 11, 'p_k' : 1297, 'p_prev' : 677, 'gap' : 620, 'bound' : 348.18425797112735, 'base_n' : 36\}$ satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module. \square

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.