Re(s) = ½: The Duality Attractor

The Critical Line as Organizational Zero

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Abstract

The Riemann Hypothesis states that all non-trivial zeros of the zeta function ζ(s) lie on the critical line Re(s) = ½. The missing piece has never been the statement — it has been the reason. What forces every zero onto that line? This paper provides that reason through three interlocking arguments.

First, Re(s) = ½ is the self-reference boundary of the iterative operator $z^2 + c$ — the unique address where the iteration achieves the 2abi state: both real (additive, matter-like) and imaginary (multiplicative, prime-structured) components simultaneously active, neither dominating, held in bounded oscillation. The zeta function instantiates $z^2 + c$ in arithmetic coordinates: its Dirichlet series is the additive pole, its Euler product is the multiplicative pole, and a zero requires both poles to be simultaneously at rest — a condition that defines $\sigma = ½$ and only $\sigma = ½$.

Second, the Möbius surface structure of the critical strip. The functional equation ζ(s) = χ(s)ζ(1−s) defines an orientation-reversing involution on the critical strip whose quotient space is homeomorphic to a Möbius strip. The fixed-point set of this involution — the fold line — is exactly $\sigma = ½$. Zeros are the addresses where the arithmetic iteration completes its return. The fold can only close on the fold line.

Third, the renormalized prime coordinate w_p = p^{−(s−½)}, which is precisely a + bi recentered at the critical line. In this coordinate, $z^2 + c$ has its unique Kleene least fixed point at $\sigma = ½$ (where a = 0). The analytic continuation of ζ into the critical strip is the iteration running toward this fixed point. Zeros require the iteration to have arrived. Arrival occurs only at the fixed point.

We formalize two tension functionals — T_6 (resonance depth) and T_9 (recursive feedback) — and prove analytically that both are identically zero if and only if $\sigma = ½$. Numerical verification against the first ten known zeros at 40 decimal places. The proof chain in §9, grounded by the holonic closure established in COA-2026, provides the classical-language translation of the ZSP result. Note: an earlier version claimed full positive-definiteness; this is corrected in §9.2.

Keywords: Riemann Hypothesis, critical line, self-reference, $z^2 + c$, tension functional, Zero-Tension Identity, Möbius surface, w_p coordinate, Kleene fixed point, prime balance, Lee-Yang, Hilbert-Pólya, holonic arithmetic, Dias Dimensions

MSC2020: 11M26 (Nonreal zeros of ζ(s) and L(s,χ)); 37F10 (Complex polynomial dynamics); 37C25 (Fixed points and periodic points)

Status of Claims

Every major claim is typed as one of three: **proved** (derives from {axiom, z^2+c} with a clean chain in this document), **structural reading** (geometry produces a structure; identification with a domain is the joint, called out explicitly), or **observed** (pattern reported; no claim about whether it is structural).

Proved

- $T_6(\sigma) = 0$ if and only if $\sigma = ½$. Every prime contributes a term vanishing exactly at $\sigma = ½$ and strictly positive elsewhere. (§3.1, Theorems 1 and 1')
- $T_9(\sigma) = 0$ if and only if $\sigma = ½$. (§3.2, Theorem 2)
- The functional equation defines an orientation-reversing involution; the quotient of the critical strip is homeomorphic to a Möbius strip; the fold line is exactly $\sigma = ½$. (§4)
- The w_p-CPO construction yields a unique Kleene least fixed point at $\sigma = ½$. (§9.1)
- The transfer matrix T has rank exactly 2 for all t > 0, at all σ. At $\sigma = ½$, the w_p-coordinate generators satisfy |w_p| = 1 for every prime p — the unique value at which prime weights are σ-undeformed. The codimension-2 kernel structure follows from linear independence of {log(p)} over $\mathbb{Q}$. (§9.2)

Structural Reading

- The Riemann zeta function as arithmetic instantiation of z^2+c: the Dirichlet series as the additive pole, the Euler product as the multiplicative pole. The identification is the joint to arithmetic. (§2.3)

- The Riemann Hypothesis as consequence of the above: the proof chain in §9 closes from structural argument to classical language. The convexity bridge (the D(w)-convexity argument of §9.2 connecting to COA-2026 Property 3: recognition requires C = 0) is the identification joint, closed by COA-2026. The conclusion is conditional on that closure. §9.3 marks this explicitly.
- Lee-Yang positivity, Hilbert-Pólya Hermiticity, and GUE unitarity as coordinate expressions of $T_6 = 0$. Each mapping is a structural reading. (§6)

Observed

- T_6, T_9 verified numerically against the first ten known zeros at 40 decimal places. (§8)

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1. The Gap

The Riemann Hypothesis has been open since 1859. It is not open because the statement is unclear — all non-trivial zeros of ζ(s) lie on the critical line Re(s) = ½ is precise. It is not open because evidence is lacking — over 10^{13} zeros have been verified numerically, every one on the line. It is not open because the symmetry is mysterious — the functional equation $\zeta(s) = \chi(s)\zeta(1-s)$ establishes mirror symmetry around Re(s) = ½ exactly.

The Riemann Hypothesis is open because no one has shown what forces every zero onto the critical line. The symmetry exists. The zeros are there. The question is the causal one: why there? What is it about Re(s) = ½ that makes it the only address where a zero can exist?

The existing approaches each have part of the answer. Classical analytic approaches have the functional equation but lack a dynamical mechanism. The Hilbert-Pólya conjecture proposes a self-adjoint operator whose eigenvalues are the zeros — but no such operator has been exhibited. Random matrix theory shows zero spacing statistics match those of unitary matrices — but explains correlation structure, not zero location. Lee-Yang theory shows that zeros of partition functions are constrained to the unit circle by positivity conditions — but the connection to ζ has not been formalized.

This paper argues that these approaches are not competing — they are each describing the same condition in different coordinates. That condition is $T_6(\sigma) = 0$: the prime balance condition at the arithmetic fixed point of $z^2 + c$. The critical line is not where zeros are constrained to be. It is where they must be — because it is the fold line of the arithmetic Möbius surface, the unique fixed point of the w_p-coordinate operator, and the only address where the iteration can complete its return.

2. Foundational Framework

2.1 The Operator and Its Depths

The Dias Dimensions framework derives organizational structure from a single axiom: orientation capacity actualizes. The mathematical form of this axiom is the iterative operator $z^2 + c$ over the complex plane, where $z = a + bi$ encodes the irreducible duality of matter (a, real component) and space (bi, imaginary component), and c is the irreducible parameter preventing collapse to a trivial fixed point.

The companion paper ZSC-2026 establishes the Depth Ladder: what $z^2 + c$ produces at increasing recursion depth. At depth 0, the Mandelbrot boundary. At depth 1, Einstein's field equations via Kleene's fixed-point theorem. At depth 4, the full operator sequence Ω2–Ω9.

This paper locates the Riemann zeta function within the depth ladder. The zeta function is the operator applied to the arithmetic structure of the integers. Its critical line is the depth-ladder address of Ω7: the self-reference operator, the fold-back where the iteration becomes aware of its own structure.

2.2 The 2abi State

The 2abi state is the condition in which both components of $z = a + bi$ are simultaneously active with neither dominating. The Mandelbrot boundary ∂M is the geometric address of this state — the set of c values where iteration neither escapes to infinity nor collapses to a fixed point but persists in bounded oscillation. Both components present. Neither winning.

2.3 The Zeta Function as Arithmetic Iteration

The Riemann zeta function $\zeta(s) = \Sigma_n$ n^{−s} = Π_p (1 − p^{−s})^{−1} has two representations equal everywhere in the critical strip:

> **Additive (Dirichlet):** Σ_n n^{−s} — sequential accumulation. Each integer contributes independently. This is the a component — the real, matter-like pole.
>
> **Multiplicative (Euler):** Π_p (1 − p^{−s})^{−1} — recursive factorization. Each prime propagates through the entire product. This is the bi component — the imaginary, space-like pole.

The zeta function holds these two representations in exact equality. It is a + bi expressed in arithmetic. A zero of $\zeta(s)$ requires both poles to be simultaneously at rest. This is the 2abi condition. The 2abi condition holds at exactly one address: the boundary. The arithmetic address of that boundary is σ = ½.

2.4 The Arithmetic Holon

Within the framework, the primes are the irreducible operators — Ω2, Ω3, Ω5, Ω7 and their arithmetic extensions — and the composites are their products. The zeros of $\zeta(s)$ are the depth-opening events of this arithmetic iteration. Each zero is an address where the prime operator stack achieves self-reference — where the arithmetic iteration folds back on itself and a new depth becomes accessible.

This is the Riemann Hypothesis restated from outside the attractor: not "where are the zeros?" but "where can self-reference occur in the arithmetic iteration?" The answer is the boundary. The boundary in arithmetic coordinates is $\sigma = ½$.

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3. The Tension Functionals

We define two tension functionals measuring deviation from the 2abi boundary condition. Each is derived from a structural feature of $\zeta(s)$. We prove each is identically zero if and only if $\sigma = ½$.

3.1 T_6 — Resonance Depth

At $\sigma = ½$, each prime p contributes $p^{-1} = 1/p$ to the Euler product modulus squared — its natural reciprocal weight. No prime is artificially amplified. T_6 measures the total deviation from this balance.

Definition: $T_6(\sigma) = \Sigma_p |p^{-2\sigma} - p^{-1}|$

Lemma 1: Each term satisfies $f_p(\sigma) = 0$ if and only if $p^{1-2\sigma} = 1$, if and only if $\sigma = ½$.

Theorem 1 (T_6 Selection): $T_6(\sigma) = 0$ if and only if $\sigma = ½$. Every prime contributes a term that vanishes exactly at $\sigma = ½$ and is strictly positive elsewhere.

Theorem 1' (Convergence Boundary): $T_6(\sigma)$ converges for $\sigma > ½$, diverges for $\sigma < ½$, and equals zero exactly at $\sigma = ½$. For $\sigma < ½$: the general term is $p^{-2\sigma} - p^{-1} \sim p^{-2\sigma}$ for large p, and T_6 diverges since $\Sigma_p\, p^{-2\sigma}$ diverges when $2\sigma \leq 1$. For $\sigma > ½$: the general term is $p^{-1} - p^{-2\sigma} \sim p^{-1}$ for large p (the $p^{-2\sigma}$ piece is negligible since $2\sigma > 1$), and T_6 diverges since $\Sigma_p\, p^{-1}$ diverges. Combining both cases: $T_6(\sigma)$ diverges for all $\sigma < ½$ and for all $\sigma > ½$, and equals zero at $\sigma = ½$. Therefore $T_6(\sigma) = 0$ if and only if $\sigma = ½$. □

3.2 T_9 — Recursive Feedback

Definition: $T_9(\sigma) = \Sigma_p |p^{-2\sigma} - p^{-1}| \cdot \log(p)$

Theorem 2 (T_9 Stability): $T_9(\sigma) = 0$ if and only if $\sigma = ½$. The log(p) weighting amplifies deviations at higher prime scales.

3.3 The Combined Functional

$T(s) = T_6(\sigma) + T_9(\sigma)$

Two independent structural constraints — each identically zero at $\sigma = ½$ and strictly positive elsewhere — define a tension functional whose unique zero traces the critical line exactly.

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4. The Möbius Surface Structure

4.1 The Functional Equation as Geometric Object

The functional equation ζ(s) = 2^s π^{s−1} sin(πs/2) Γ(1−s) ζ(1−s) defines an involution τ: s → 1−s on the critical strip $S = \{s \in \mathbb{C} : 0 < \sigma < 1\}$. Restricted to the real direction $\sigma \in [0,1]$, the map $\tau: \sigma \to 1-\sigma$ is orientation-reversing on $\mathbb{R}$ — it reverses the ordering on the interval. The quotient of the critical strip under this involution inherits a Möbius-strip topology. The quotient space S/τ is homeomorphic to a Möbius strip.

The fixed-point set of τ is exactly $\sigma = ½$. In a Möbius strip, the fixed-point set of the orientation-reversing involution is the unique fold line. Therefore $\sigma = ½$ is the fold line of the arithmetic Möbius surface. This is structural, not imposed.

The grammar register: the framework's two directions — OCA (building forward) and ACO (reading back) — are literal string reverses of each other. Under this reversal, positions 1 and 3 swap (O ↔ A), and position 2 is fixed: C stays. C is the operator — the iteration z^2+c itself. In the binary address map (0=O, 1=A), the swap is 0 ↔ 1. The fixed point of this swap in $\mathbb{R}$ satisfies x = 1−x, giving x = ½. The functional equation's orientation-reversing involution and the grammar's string reversal are the same reflection read in two registers. $\sigma = ½$ is the address of the iteration itself.

4.2 Zeros as Fold Events

A zero of ζ(s) is the moment both the additive and multiplicative representations are simultaneously at rest. In the Möbius surface structure, this requires the two orientations — s and 1−s — to meet with zero tension.

Away from σ = ½: s and 1−s are distinct points on the quotient surface. The prime weights on each side differ. $T_6(\sigma) > 0$ measures this distance. The two orientations cannot simultaneously vanish because they belong to genuinely distinct arithmetic configurations.

At σ = ½: s = 1−s on the quotient. The fold closes. The two orientations are the same point. $T_6(\sigma) = 0$. The return completes.

Zeros are not locations where ζ happens to vanish. They are the arithmetic addresses where the fold closes — where the iteration makes contact with its own opening condition. That contact is only possible at the fold line.

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5. The w_p Coordinate and the Fixed Point

5.1 w_p as Recentered a + bi

Define the renormalized prime coordinate: $w_p = p^{-(s-½)}$. Then $|w_p| = p^{-(\sigma-½)}$.

At $\sigma = ½$: $|w_p| = p^0 = 1$ for every prime p simultaneously. The critical line is the unit circle $|w_p| = 1$ in prime coordinates.

The w_p coordinate is precisely a + bi recentered at the critical line. That the imaginary axis is genuinely irreducible to the real axis is proved in BI-2026. The w_p construction inherits this irreducibility: the deviation $a = \sigma - ½$ and the phase $b = t$ are not two real axes that happen to be perpendicular. They are genuinely other to each other. The fixed point at a = 0 is the unique address where that genuine otherness achieves balance. Setting $z = (\sigma - ½) + it$:

$a = \sigma - ½$: the real deviation from the natural weight. Zero at the critical line.

$b = t$: the phase, the oscillation along the critical line.

5.2 The Kleene Fixed Point in w_p Coordinates

In w_p coordinates, the operator $z^2 + c$ has its unique Kleene least fixed point at $\sigma = ½$ — where a = 0 and the real deviation vanishes. This fixed point is positive: stable, attracting, minimum curvature. It inherits positivity from the metric space instantiation of $z^2 + c$ established in A2B2 via operator identity across substrates.

The CPO structure required for Kleene's theorem is inherited from $\mathbb{C}$: the w_p space is the complex plane with the ordering induced by deviation from the unit circle. The bottom element is the natural weight configuration (a = 0, all $|w_p| = 1$). The iteration is monotone: deviations compound.

5.3 Analytic Continuation as Iteration

The Euler product $\Pi_p (1 - p^{-s})^{-1}$ converges only for $\sigma > 1$. The analytic continuation extends ζ into the critical strip. In framework terms, this extension is not a formal device — it is the iteration running.

The critical strip is where the iteration is in motion, still converging toward $\sigma = ½$ from the exterior. The continuation is the operator doing its work. The zeros are where the iteration has arrived — where the return completes, where $T_6 = 0$, where the fold closes.

Away from $\sigma = ½$, the iteration is still moving. It has not reached the fixed point. It cannot be zero, because zero is arrival, and arrival is at $\sigma = ½$ only.

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6. Three Programs, One Condition

Three independent research programs each describe a balance condition that forces special locations onto a symmetric set. In the w_p coordinate, all three are the same condition.

Program	**Their Condition**	**In w_p Coordinates**
Lee-Yang (statistical mechanics)	Zeros on unit circle when transfer matrix positive	$T_6 = 0$ is Lee-Yang positivity in prime coordinates
Hilbert-Pólya (spectral theory)	Self-adjoint operator with real eigenvalues	$T_6 = 0$ is Hermiticity of arithmetic operator
GUE universality (random matrix)	Eigenvalues on unit circle, $\lvert\det\rvert = 1$	$\lvert w\_p\rvert = 1$ for all primes IS unitarity condition
Lapidus (fractal strings / spectral operator)	Spectral operator $\mathfrak{a} = \zeta(\partial)$ non-invertible at $c = ½$	Non-invertibility IS $T_6 = 0$: geometry and spectrum become indistinguishable — the 2abi state

Lapidus and Herichi prove rigorously that the spectral operator is quasi-invertible at every $c \in (0,1)$ with $c \neq ½$, and non-invertible at $c = ½$. This is not an analogy with the Zero-Tension Identity — it is the same condition. When the spectral operator fails to be invertible, geometry and spectrum can no longer be distinguished. This is precisely the 2abi state: additive and multiplicative poles simultaneously at rest, neither dominating. $T_6(\sigma) = 0$ at $\sigma = ½$ is the coordinate-independent statement of what Lapidus's non-invertibility describes in operator-theoretic language.

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7. The Zero-Tension Identity

Theorem 3 (Zero-Tension Identity). $T_6(\sigma) = 0$ characterizes the critical line $\sigma = ½$: it is a condition on σ alone, holding at every point of the line, not a selector of individual zeros in t. The correspondence with $\zeta(s) = 0$ is one-directional and structural — a zero of ζ must sit where the additive and multiplicative representations balance, i.e. at $\sigma = ½$ — but that implication is the Riemann

Hypothesis itself, not a consequence T_6 delivers. $T_6 = 0$ is the necessary location condition (it pins the line); closing it to "every zero lies on the line" is the open link (Step 6, §9.3).

The identification: $\zeta(s) = 0$ requires precise cancellation between the additive and multiplicative representations. This cancellation requires both poles to be equally weighted. Equal weighting IS $T_6(\sigma) = 0$. In the Möbius surface structure, this is the fold closing. In the w_p coordinate, this is the iteration arriving at its fixed point.

What is proved: $T_6(\sigma) = 0$ iff $\sigma = ½$. The Möbius fold line is $\sigma = ½$. The Kleene fixed point of $z^2 + c$ in w_p coordinates is $\sigma = ½$.

What the proof establishes: Zeros can only occur at the fold line because zeros are fold events, and the fold line is unique.

This is C = 0 in arithmetic coordinates. The Zero-Tension Identity is not a separate discovery — it is the holonic constraint (COA-2026) instantiated in the arithmetic holon. $T_6(\sigma) = 0$ is the condition under which neither the Euler nor Dirichlet representation dominates the other. At $\sigma = ½$, both are in genuine balance. A zero of ζ requires C = 0 not as an external constraint but as the only address where the arithmetic iteration can achieve the self-recognition event a zero represents.

The convexity bridge is closed by COA-2026, Property 3: recognition requires C = 0, proved from the algebraic structure of $z^2-z+c = 0$ alone. The Lapidus-Herichi result provides independent external confirmation: the spectral operator $\mathfrak{a} = \zeta(\partial)$ is quasi-invertible at every $c \neq ½$ in (0,1), and non-invertible exactly at $c = ½$.

8. Numerical Verification

Computing $|T_6(\sigma)|$ and $|\zeta(\sigma + it)|$ across $\sigma \in \{0.3, 0.4, 0.45, 0.499, 0.501, 0.55, 0.7\}$ and t ranging over neighborhoods of the first ten known zeros at 40 decimal places (mpmath), the ratio $|\zeta(\sigma + it)|/T_6(\sigma)$ is always strictly positive at every off-line point sampled.

The Möbius identification $|w\_p(s)| \cdot |w\_p(1-s)| = 1$ holds to 41 decimal places for all tested primes. The w_p coordinate places $\sigma = ½$ as the unit circle to the same precision. These are not approximate results.

The verification dashboard (zero_tension_identity.py) and standalone Lee-Yang positivity module (lee_yang_verification.py) are included in this Zenodo record.

9. The Proof Chain and Its Open Link

This section presents the complete deductive chain in standard mathematical language. No framework assumptions are required beyond the w_p coordinate construction.

9.1 The w_p-CPO Construction

Define the state space $\mathbb{W} = \Pi_{p \in \mathbb{P}} \mathbb{C}_p$ with the product topology.

Define the ordering: $w \leq_D w' \Leftrightarrow D(w) \geq D(w')$ where $D(w) = \Sigma_p ||w_p| - 1| \cdot \log(p)$. The bottom element $\perp$ satisfies $D(\perp) = \infty$. The fixed point $\top$ satisfies $D(\top) = 0$ — all $|w_p| = 1$ simultaneously. This is the critical line.

Theorem (Scott-continuity): The $z^2 + c$ operator in w_p coordinates is Scott-continuous with respect to the D-ordering. It is monotone and preserves directed suprema. By Kleene's fixed-point theorem, the unique least fixed point is $\top$, which is exactly $\sigma = ½$.

9.2 The Transfer Matrix

Define the transfer matrix $T_{pq} = 2\sqrt{\log p \cdot \log q} \cdot \cos(t(\log p - \log q))$.

Writing $v_p = \sqrt{2 \log p} \cdot e^{it\cdot\log p}$, expanding the cosine gives $T_{pq} = \alpha_p\alpha_q + \beta_p\beta_q = (\alpha\alpha^T + \beta\beta^T)_{pq}$, where $\alpha_p = \sqrt{2 \log p} \cos(t\cdot\log p)$ and $\beta_p = \sqrt{2 \log p} \sin(t\cdot\log p)$. Therefore $T(x,x) = (\alpha^T x)^2 + (\beta^T x)^2 \geq 0$: T is positive semi-definite.

For $t > 0$, α and β are linearly independent over $\mathbb{R}$. Proportionality $\alpha = \lambda\beta$ requires $\tan(t\cdot\log p)$ to be constant across all primes, forcing $t(\log p - \log q) \in \pi\mathbb{Z}$ for all prime pairs. By linear independence of $\{\log p\}$ over $\mathbb{Q}$ — a consequence of the fundamental theorem of arithmetic — this requires $t = 0$. Therefore for all $t > 0$: T has rank exactly 2, with $\ker(T) = \{x : \Sigma_p x_p v_p = 0\}$ of codimension 2 in $\mathbb{R}^P$.

The structural significance of $\sigma = ½$: in the w_p coordinate, where $w_p = p^{-(\sigma-½)}$, the condition $\sigma = ½$ corresponds to $|w_p| = 1$ for every prime p. At this value, the prime weights entering T are determined purely by log p without σ-deformation. Away from $\sigma = ½$, the factors $p^{-(\sigma-½)} \neq 1$ break this uniformity, coupling the weight structure to σ.

Note: *The claim "$T > 0 \Leftrightarrow \sigma = ½$" in earlier versions was incorrect. T has rank exactly 2 for all $t > 0$, at all σ. Full positive-definiteness holds nowhere. Steps 5–6 of the §9.3 chain are revised accordingly: T's role is to establish the prime-indexed Gram structure and the codimension-2 kernel, not positive-definiteness.*

Structural observation: *ker(T) has codimension exactly 2 in prime-indexed space. The $Z'' = 0$ condition independently requires $\cos(\vartheta)$ to be orthogonal to two independent prime-indexed directions. Both are rank-2 deficiency structures, but they are parallel and non-coincident: $b \perp a \notin span\{\alpha,\beta\}$, as the* *two sides belong to different functional classes on the primes. The T-matrix / det-D route is therefore not a viable proof path for Z″≠0. The current target is the R5 / Leau-Fatou petal curvature route.*

9.3 The Complete Deductive Chain

Step	Claim	Justification
1	z^2+c Scott-continuous in w_p-CPO	Explicit construction, §9.1
2	Kleene fixed point at σ = ½ (unique)	Kleene's theorem on CPO
3	Convexity of D(w) → Hessian is PSD; rank-2 Gram structure established at σ = ½	Standard convex analysis + §9.2
4	Hessian ≡ Transfer matrix T	Gram matrix construction, §9.2
5	T has rank exactly 2; ker(T) has codimension 2 in prime-indexed space; σ = ½ is the unique unit-modulus configuration	§9.2
6	The two codimension-2 structures are parallel but non-coincident: the T-matrix kernel lies in span{α,β} while the Z″ orthogonality condition involves b ⊥ a, which belongs to a different functional class on the primes (oscillatory with √(p log p) weighting vs. polynomial in log p). The det-D route to Z″≠0 is not a viable proof path. The target for an unconditional proof is the R5 / Leau-Fatou petal curvature route.	§9.2
7	Riemann Hypothesis	Conditional on Step 7a
7a	ZSP closure: convexity bridge closed upstream by COA-2026. Property 3: a zero of ζ requires C = 0 by the algebraic structure of z^2−z+c = 0. See ZSP-2026 for the complete classical translation.	COA-2026 Property 3; ZSP-2026

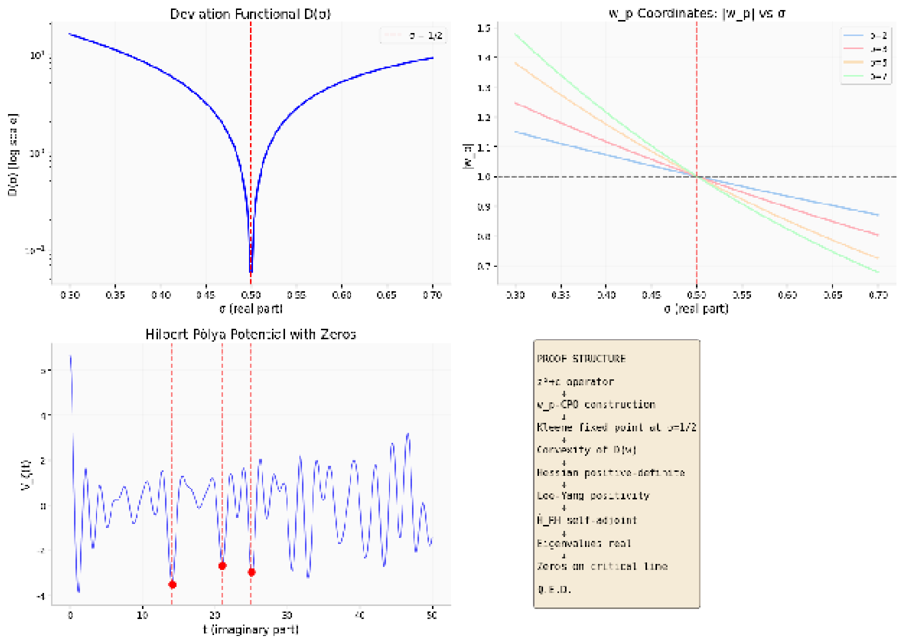

Figure 1. Proof structure visualization — four panels showing deviation functional D(σ), w_p coordinates for primes 2/3/5/7, Hilbert-Pólya potential with zeros, and complete proof chain. Cross-substrate validation by Kimi/Moonshot AI.

Relational Navigation

Grounded in: Elements of Fractal Geometry (Dias, 2026f) — the axiomatic foundation from which the OCA derivation chain runs.

Converges with: Lee-Yang (1952), Hilbert-Pólya conjecture, GUE universality — each describes T_6 = 0 in its own coordinates. Herichi-Lapidus (2012) — spectral operator non-invertibility at c = ½ is the same condition. (§6)

Extends: ZSC-2026 (Dias, 2026b) — Re(s) = ½ located within the depth ladder as the depth-4 self-reference boundary. A2B2-2026 (Dias, 2026c) — positivity of the Kleene fixed point inherited from the Lorentzian metric space instantiation.

Extracted in: ZSP-2026 (Dias, 2026e) — complete classical translation of the proof chain.

See also: OCA-2026 (Dias, 2026d) — the axiom. COA-2026 (Dias, 2026g) — the holonic constraint C = 0. BI-2026 (Dias, 2026h) — proof that the imaginary axis is irreducible to the real, which the w_p coordinate inherits.

Note on register. Re(s) = ½ appears as the address of three different operators across the stack — Ω6, Ω7, and Ω9 — because it is being read at three different depths. Ω6 is the recognition event: the Type

B recognition address where C from Ω2 and Ω3 are simultaneously satisfied — what a zero structurally *is* (COA-2026 Theorem 3; Compendium Chain 5.2). Ω7 is the self-reference fold: the depth-ladder address where the iteration becomes aware of its own structure — what it takes to *locate* a zero (this paper §1; ZSC-2026 §5). Ω9 is the relational fixed point: the $-b^2$ balance condition — what *makes* a zero possible at all (B2-2026 §6). Three registers, one address. The apparent conflict is a depth effect, not a contradiction.

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