TNG/TC1 No Rogue Short Interval Theorem Package

Denis Saltykov

May 2026

Abstract

This note isolates the TC1 global-testing node from the full proof package. It proves, as a standalone structural theorem, that every actual B1-origin TC1 coarea test reaching the Liouville/Fourier input is near-global, with

$$H \geq X(\log X)^{-B},$$

and hence is covered by the Davenport/AP input X9L-GT. If a TC1 test is not near-global, it is routed away before X9L-GT is invoked. In particular, the proof never assumes pointwise shifted short-interval cancellation for λ.

Contents

1 Scope

The package concerns the TC1 part of the GoodAWACK branch. Its logical task is to justify the implication

$$\boxed{\text{actual B1-origin TC1 test} \Longrightarrow \text{near-global X9L input or routed terminal output.}}$$

The package does not estimate Edge, LongAP/Local, CKP, or LocalDiag branches. Those branches are merely the permitted outputs of the routed alternative.

The external analytic input is only X9L-GT in its near-global Davenport/AP form: for polylogarithmic AP modulus and smoothness complexity, normalized Liouville/AP Fourier averages are $o(1)$ when

$$H \geq X(\log X)^{-B}.$$

No arbitrary shifted short-interval theorem and no unused low-θ polylog-modulus theorem is used.

2 Structural TC1 Coarea Tests

Fix a terminal TC1-GoodAWACK macro-template κ. The data consist of:

1. a B1 typed parent block;
2. a B3 grouping record;
3. an F3/F4 terminal routing history;
4. a marked Liouville affine form L_m;
5. a TC1 tensor certificate;
6. a C1-clean smooth box/coset cell Ω^*.

TGT constructs coarea tests from the marked B1-origin form by

$$n = L_m(z), \qquad z \in \Omega^*.$$

After only polylogarithmic scale, modulus, and smooth-weight subdivisions, a test has the form

$$\mathcal{L}_p(\lambda) = \frac{1}{H_p} \sum_{n \in I_p} \lambda(n)\rho_p(n)e(\alpha_p n), \tag{2.1}$$

where I_p is the structural image interval or AP image, $H_p = |I_p|$, the AP modulus is $\leq (\log X_p)^{C_\kappa}$, and ρ_p has polylogarithmic smoothness complexity.

The word *active* means that the cell has not already been routed to Edge, LongAP/Local, CKP, LocalDiag, or empty support.

3 Structural Coarea Closure

The interval I_p in (2.1) is not an arbitrary subinterval chosen after a larger structural image has been found. It is the image piece produced by the TGT coarea construction from the terminal marked B1 carrier, after only the normalizations needed for scale, modulus, and smoothness complexity.

The formal closure principle is TTH-SC. It proves that every short subtest of a released coarea test is either non-structural and reaggregated into the parent functional, or structural and routed through TTD/ROC/BRS/X16BRS/X16C and C1P/C1A/C1 before the Liouville input is invoked.

4 Regular Branch

Assume first that the TC1 testing family is MRT-admissible. MRT supplies the start-pushforward bound

$$(\mathrm{start})_{\#}\nu_\kappa \ll_\kappa (\log N)^{C_\kappa}\frac{dx}{X}. \tag{PACK}$$

For every active B1-origin coarea test in this same family, TTH supplies the near-global length lower bound

$$H_p \geq X_p(\log X_p)^{-B_\kappa}. \tag{4.1}$$

Together with the polylogarithmic AP modulus and smoothness bounds in Section 2, these are exactly the hypotheses of X9L-GT in the near-global Davenport/AP form. X9L-GT gives

$$\int \sup_\alpha \left| \frac{1}{U_p} \sum_{1\leq u\leq U_p} \lambda(g_p u + b_p)e(\alpha u)w_p(u)\right|^2 d\nu_\kappa(p) = o(1), \tag{4.2}$$

where $U_p = H_p/g_p$.

TGT says that a non-negligible TC1 macro-template would force a fixed positive lower bound for the same averaged Fourier quantity. Hence the regular MRT-admissible branch contributes $o(N)$.

5 Singular Branch

If MRT-admissibility fails, TTD identifies singular start concentration before any X9L input is invoked. The model obstruction is a marked form moving through a short additive image while transverse B1 variables appear to carry large volume.

This singular geometry is structural, not analytic. It is handled as follows.

1. ROC proves range-origin comparability for direct dyadic-coordinate origins and their controlled full-rank transports.
2. ROC routes already tagged singular origins to Edge, LongAP/Local, CKP, LocalDiag, or empty support.
3. BRS handles the complementary and quotient-origin cases. It uses the B1 range/slice estimate supplied by X16BRS and X16C.

The BRS conclusion is:

$$\text{short marked B1 image} \Longrightarrow \text{strict C1P Edge}$$

unless the atom already carries a LongAP/Local, CKP, LocalDiag, Edge, empty, or nonterminal routing tag.

Consequently the singular branch is never sent to X9L-GT. It routes to

$$C1P/C1A/C1, \qquad D1/H4, \qquad G8a, \qquad H4, \qquad \text{or } 0. \tag{5.1}$$

6 The Near-Global-or-Routed Theorem

Theorem 1 (TNG-A. TC1 tests are near-global or routed away)**.** *Fix a B1/B3/F3/F4 terminal TC1-GoodAWACK macro-template κ whose cell has not already been routed away, after C1 boundary removal, fixed macro-template normalization, and polylogarithmic scale/modulus/smooth-weight decomposition. Let $\mathcal{L}_p(\lambda)$ be any active coarea test produced by TGT from the marked B1-origin form.*

Then exactly one of the following alternatives holds.

1. Near-global testing alternative. *The test belongs to the regular MRT-admissible branch. PACK holds, the AP modulus and smoothness complexity are polylogarithmic, and*

$$H_p \geq X_p(\log X_p)^{-B_\kappa}.$$

Hence this test is an allowed input to X9L-GT in the near-global Davenport/AP form.

2. Routed alternative. *The test is not sent to X9L-GT. Before any Liouville/AP input is invoked, TTD, ROC, BRS, and X16BRS/X16C route the corresponding cell to one of*

$$C1P/C1A/C1, \qquad D1/H4, \qquad G8a, \qquad H4, \qquad \textit{or } 0.$$

In particular, there is no third case consisting of an arbitrary shifted short interval or an unclassified short AP fibre.

Proof. Start with the coarea tests constructed by TGT from the fixed macro-template κ. MRT separates regular testing families from singular start-concentration families.

In the regular branch, MRT supplies PACK for the same testing family. The coarea tests retain B1-origin because the allowed normalizations are only controlled CRT restrictions, fixed-divisor quotients, full-rank transports, and polylogarithmic analytic subdivisions. These operations do not replace the terminal marked B1 carrier. TTH-SC prevents replacement by a new arbitrary short-interval family. TTH then gives the near-global length lower bound for every remaining structural coarea image piece. The modulus and smoothness complexity bounds are polylogarithmic by construction. Therefore the test satisfies the exact hypotheses of X9L-GT.

In the singular branch, TTD identifies a singular-origin mechanism. Direct dyadic-coordinate and tagged full-rank transport cases are handled by ROC. The complementary, quotient, and carrier-slice cases are handled by BRS. In BRS, a genuinely short marked B1 image cannot hide large transverse mass: X16BRS reduces all BRS carrier types to X16-Core, and X16C proves X16-Core. Therefore a short B1 image is a strict C1P Edge contribution unless it already carries a LongAP/Local, CKP, LocalDiag, Edge, empty, or nonterminal routing tag. These are precisely the routed alternatives listed above.

Finally, TTH-SC classifies refinements of an already released structural image. Non-structural short pieces are aggregated back to the parent image, while genuine structural short-image children are routed before X9L-GT. This excludes the pointwise shifted short-interval escape case. The theorem follows. □

7 Decision Table

For checking purposes, the theorem can be read as the following finite decision table.

Test status after TGT coarea	Structural source	Action before X9L	Result
Full-rank B1 marked carrier, no short image	B1/B3/F3/F4 carrier	MRT + TTH-SC + TTH	Near-global; X9L-GT may be invoked
Direct B1 carrier with short marked image	same	BRS + X16BRS/X16C	Strict C1 Edge
Complementary carrier $N - P$	F4/BRS solved-affine origin	Reduce to product carrier P	Near-global or strict Edge
Quotient carrier s in $L = ds$ with tagged d	F4 quotient tag	Transfer to controlled product carrier	Near-global or strict Edge
Untagged quotient/divisor relation	unresolved F4 predicate	F4 blocks terminal TC1 release	Routed before X9L
Singular start measure from non-direct origin	TTD singular branch	ROC/BRS	Routed before X9L
Artificial subdivision of near-global image	not structural TGT coarea	aggregate back	no X9L short-interval input

The only tests passed to X9L-GT are the first-row near-global tests.

8 TC1 Cancellation Output

Theorem 2 (TNG. TC1-GoodAWACK cancellation)**.**
For every terminal TC1-GoodAWACK macro-template κ, after C1 boundary removal and fixed macro-template normalization,

$$R_{\text{TC1-GoodAWACK},\kappa}(N) = o(N).$$

Consequently

$$\boxed{R_{\text{TC1-GoodAWACK}}(N) = o(N).}$$

Proof. Aggregate terminal TC1 atoms by the fixed macro-template κ as in TGT. Apply Theorem TNG-A to every active coarea test.

On the near-global alternative, MRT supplies PACK and TTH supplies the near-global length lower bound. X9L-GT applies. This contradicts the fixed TGT testing lower bound unless the contribution of κ is $o(N)$.

On the routed alternative, the cell is sent to Edge, LongAP/Local, CKP, LocalDiag, or empty support by TTD/ROC/BRS. These outputs are handled by C1P/C1A/C1, D1/H4, G8a, H4, or zero, and therefore do not contribute terminal TC1-GoodAWACK mass.

There are only boundedly many structural TC1 macro-templates, depending on the fixed Heath–Brown depth J_0. Summing over them gives the displayed $o(N)$ estimate. □

9 Parameter Check

The parameter hierarchy required by the theorem is finite.

1. The Heath–Brown depth J_0 is fixed.
2. The routing grammar and TC1 macro-template set are finite for this J_0.
3. All CRT indices, divisor quotients, affine contents, AP moduli, and smoothness complexities reaching X9L are bounded by fixed powers of $\log N$.

4. The BRS short-image threshold B_κ is chosen larger than the X16 slice-floor and C1 error-budget losses.

5. X9L-GT then chooses the Davenport logarithmic saving exponent larger than the remaining polylogarithmic losses.

Thus the routed alternatives have $o(N)$ error budget, while the near-global alternative has $o(1)$ normalized TC1 testing average and hence $o(N)$ contribution.

10 Output for the Full Proof

This package proves the TC1 structural theorem

$$\boxed{\text{TNG-A} + \text{X9L-GT} \Longrightarrow R_{\text{TC1-GoodAWACK}}(N) = o(N).}$$

It uses X9L-GT only in the near-global Davenport/AP form. It does not use:

1. the old X8 inverse-Gowers input;

2. pointwise shifted short-interval Liouville cancellation;

3. a low-θ polylog-modulus theorem for arbitrary short AP fibres.

11 Logical Dependencies

External dependency: X9L-GT in the near-global Davenport/AP form.

Internal dependencies: TGT, TTH-SC, MRT, TTD, ROC, BRS, TTH; X16BRS and X16C; C1P/C1A/C1, D1/H4, G8a, E5, TGD; and the global parameter register.

Children served: E10L and the TC1 part of the GoodAWACK branch.