Master Reader Guide for `goldbach_full_main`

Denis Saltykov
ds1678@gmail.com

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Contents

1 Purpose and Reader Contract

This document is a roadmap-paper for the one-file audit-grade manuscript

`manuscript_latex/goldbach_full_main.pdf`.

It is not a replacement for the proof and it introduces no new hypothesis. Its role is to help a referee or external reader understand the architecture of the proof before entering the full 370+ page dossier.

The guide answers six practical questions:

1. What exactly does the full document prove?
2. What is the main proof architecture?
3. Which nodes are mathematically most critical?
4. Where are those nodes stated, proved, and verified?
5. Which parts are proof content, which parts are bookkeeping, and which parts are non-logical reproducibility aids?
6. How should a reader check the document efficiently?

The guide should be read with the full proof PDF, the proof-source files in `Lemmas/`, the dependency ledger `Prooftreeandledger/g_proof_tree_n_ldg.md`, the non-logical affine/global-testing verification supplement `manuscript_full_md/supplements/non_logical_goodawack_verification.md`, and the external-input appendix and bibliography in the full manuscript.

File paths in this guide are navigation aids. The proof dependencies are the logical IDs and theorem statements in the full manuscript and the ledger.

2 The Theorem Proved

The main asymptotic theorem is

$$R_\Lambda(N) = \sum_{n_1+n_2=N} \Lambda(n_1)\Lambda(n_2) = \mathfrak{S}(N)N + o(N)$$

for sufficiently large even N, where $\mathfrak{S}(N)$ is the usual Goldbach singular series.

The final handoff then removes nontrivial prime-power contributions and proves that every sufficiently large even integer is a sum of two primes.

The result is asymptotic in nature. The manuscript does not claim an explicit numerical threshold N_0, and it does not include a finite verification below that threshold.

3 What Is New in the Proof Architecture

The proof is not a classical one-piece circle-method proof. It is a routed proof. The von Mangoldt convolution is decomposed, every descendant is assigned to a finite terminal class, and each terminal class is handled by the tool appropriate to its structure.

The new structural idea is the proof-ledger style routing architecture. In plain language, the proof keeps track of where each decomposed block came from, routes it by a finite list of allowed transformations, and then proves that no unclassified residue remains.

1. use a fixed-depth Heath–Brown decomposition;
2. keep exact origin records for the decomposed and grouped blocks, called `B1/B3` tags in the ledger;
3. route every descendant through an intrinsic finite routing theorem, called the `F3/F4` terminal partition in the ledger;
4. forbid hidden residual classes;
5. handle negligible edge cases, Kloosterman-fraction sums, affine/global-testing residuals, local arithmetic-progression terms, and local diagonal terms by separate mechanisms. In the ledger these five terminal types are named `Edge`, `CKP`, `GoodAWACK`, `LongAP/Local`, and `LocalDiag`;
6. assemble all terminal contributions through a global error budget and local algebra bridge.

The most distinctive feature is that the affine/global-testing residual branch (`GoodAWACK` in the ledger) is not treated by an inverse Gowers theorem. The proof avoids the earlier unsafe X8 route and instead uses a no-rogue short-interval theorem for the true-complexity-one testing subbranch (`TNGTTHM`) and a finite combinatorial grammar theorem for the higher-complexity affine/global-testing subbranch (`E10YMX`).

4 Bird's-Eye Proof Flow

The proof has the following spine.

1. **Weighted target.** Work with
$$R_\Lambda(N) = \sum_{n_1+n_2=N} \Lambda(n_1)\Lambda(n_2).$$
2. **Heath–Brown expansion.** The proof opens the von Mangoldt weights by a fixed-depth Heath–Brown identity and groups the resulting variables into finite typed dyadic blocks. The ledger calls these two stages `B1` and `B3`.
3. **Routing.** A finite routing theorem proves that every descendant reaches one of exactly five terminal types:

   negligible edge cases, Kloosterman-fraction sums, affine/global-testing residuals,
   local arithmetic-progression terms, local diagonal terms.

The ledger labels these types `Edge`, `CKP`, `GoodAWACK`, `LongAP/Local`, and `LocalDiag`, and labels the master routing theorem `F3F4M`.

4. **Terminal estimates and local assembly.** Negligible edge cases are estimated by strict-saving lemmas (`C1P/C1`). Kloosterman-fraction sums are handled by the CKP/X10 master theorem (`CKPX10M`). The affine/global-testing residual is handled by the Branch B theorem (`E10L`), using the no-rogue TC1 theorem (`TNGTTHM`) and the finite grammar theorem (`E10YMX`). Local arithmetic-progression and local diagonal terms are assembled by the local bridge theorem (`H4M`).

5. **Global summation.** The global error-budget lemma (`GEB`) records the order of parameters and the summability of all terminal errors.

6. **Weighted asymptotic.** The final weighted assembly theorem (`I1`) proves

$$R_\Lambda(N) = \mathfrak{S}(N)N + o(N).$$

7. **Prime handoff.** The final handoff removes nontrivial prime powers, converts weighted positivity to a genuine prime pair, and records the ordered-pair normalization. The ledger labels these steps `G2`, `G1`, and `GOH`.

5 The Three Critical Structural Closures

The full manuscript contains many lemmas, but the deepest structural risk is concentrated in three closure nodes.

Description	Label	What it protects	Location
finite routing theorem	F3F4M	No descendant escapes the five terminal types	Section 7; Appendix D.1
no-rogue TC1 testing theorem	TNGTTHM	No TC1 obstruction survives as an arbitrary shifted short interval	Section 11.2; Appendix G.1
affine/global-testing finite grammar and Branch B theorem	E10YMX/E10L	Higher-complexity GoodAWACK is closed by finite grammar and Branch B contributes $o(N)$	Section 11; Appendix H.1–H.2

These are the places where the proof must convince a reader that a seemingly open-ended structural obstruction has actually been closed.

The supporting master nodes are also essential, but they have a different flavor. The CKP/X10 master theorem (`CKPX10M`) is an analytic theorem-matching and derivative-check node. The local bridge theorem (`H4M`) is a local algebra and singular-series reconstruction node. They are not the main places where a structural residual could hide.

6 Critical Node I: F3F4M Routing Exhaustion

6.1 Why This Node Is Critical

If the finite routing layer did not define an intrinsic terminal partition, the rest of the proof would be vulnerable to a hidden sixth class. A descendant might fail to be a negligible edge case, a Kloosterman-fraction sum, an affine/global-testing residual, a local arithmetic-progression term, or a local diagonal term, and the final assembly could silently miss it. The ledger label for the theorem preventing this failure is `F3F4M`.

6.2 What F3F4M Proves

The finite routing theorem proves that every admissible descendant is partitioned into the five terminal ledger classes

$$\mathrm{Edge} \sqcup \mathrm{CKP} \sqcup \mathrm{GoodAWACK} \sqcup \mathrm{LongAP/Local} \sqcup \mathrm{LocalDiag}.$$

It also proves that the routing process terminates and that there is no additional terminal class.

6.3 Hypotheses

F3F4M starts from an actual typed B1-origin block after the B3 grouping layer. It uses PAR, F3P, C1P, the F3/F4 atom interface and routing-measure definitions, the E5 content-stability interface, and LPI. It does not use C1, CKPX10M, E10L, H4M, or other downstream estimates to define the terminal predicates.

6.4 Output

The output is the routing identity

$$R_{\mathrm{desc}}(N) = R_{\mathrm{Edge}}(N) + R_{\mathrm{CKP}}(N) + R_{\mathrm{GoodAWACK}}(N) + R_{\mathrm{LongAP/Local}}(N) + R_{\mathrm{LocalDiag}}(N).$$

This is the partition imported by I1 and GEB.

6.5 How To Check It

The compressed proof is in Appendix D.1. Check the state space, state invariant, terminal predicates, allowed transitions, decreasing routing measure, master theorem, and proof that verification tables are not extra hypotheses. The finite tables in Appendix D.2–D.7 are hand-checkable expansions of the finite alphabet, not downstream analytic assumptions.

7 Critical Node II: TNGTTHM and the No-Rogue-Short-Interval Barrier

7.1 Why This Node Is Critical

The TC1 route would be invalid if it required a pointwise estimate for an arbitrary shifted short interval. The active external input is a near-global Davenport/AP Liouville estimate, not a black-box short-interval theorem. The danger is therefore a rogue TC1 test: a real B1-origin coarea test that is too short for X9L-GT and yet not routed away.

7.2 What TNGTTHM Proves

TNGTTHM proves that every actual B1-origin TC1 coarea test is near-global and X9L-admissible, or routed away before X9L-GT is invoked. Thus no active TC1 obstruction survives as a rogue shifted short interval.

7.3 Hypotheses

TNGTTHM starts from released B1-origin TC1 coarea test records. It uses TGD, F3F4M, TGT-MF, TGT, TTH-SC, MRT, TTD, ROC, BRS, X16BRS, X16C, TTH, and X9L-GT. The input X9L-GT is invoked only after near-global hypotheses are verified. TNGTTHM does not apply X9L-GT to arbitrary shifted short intervals.

7.4 Output

The output is

$$\mathrm{TC1} = \mathrm{NearGlobal}_{X9L} \sqcup \mathrm{RoutedAway}.$$

The first class is controlled by X9L-GT. The second class is routed to Edge, CKP, Local/LocalDiag, BRS/X16, or singular-origin alternatives handled elsewhere in the proof tree.

7.5 How To Check It

Read Appendix G.1 first. It contains the released TC1 test records, the definition of a possible rogue object, the finite decision table, the master theorem, the proof that no rogue refinement can be released, the regular branch, the singular and short-image branches, the parameter check, and the interface corollary for E10L. Then check Appendix G.2–G.13 for TGD, TGT, TGT-MF, TTH-SC, TNG, TTD, MRT, ROC, BRS, X16BRS, X16C, and TTH.

8 Critical Node III: E10YMX/E10L Finite GoodAWACK Grammar

8.1 Why This Node Is Critical

GoodAWACK is the most nonstandard branch. Its HighTC part cannot be left as a source-audit assertion or an informal catalogue. The proof must give a finite mathematical grammar and prove that every actual terminal GoodAWACK skeleton is covered by that grammar. The key obstruction is an untagged rank-dropping affine regrouping. If such an object could survive, it would produce a FreeAffineHighTC residual.

8.2 What E10YMX Proves

E10YMX proves, in ordinary theorem/proof style, that

$$R_{\mathrm{FreeAffineHighTC}}(N) = 0.$$

Equivalently, no actual terminal GoodAWACK skeleton contains an untagged rank-dropping AFF obstruction.

8.3 Hypotheses

E10YMX starts from actual terminal GoodAWACK skeletons generated by the B1/B3 and F3/F4 routing layers. It uses E10Y, E10M, E10X, E10K, BGS, HGO2R, BAOC, E10G, E10H, E10I, E10J, and the E5-clean interface imported from Appendix D.7. It does not use source-file hashes, occurrence manifests, search terms, or a mechanical audit as proof premises.

8.4 Output of E10L

E10L consumes E10YMX. It also consumes TGD, TNGTTHM, HGO2R, C1, G8a, and H4M. It proves

$$R_{\mathrm{GoodAWACK}}(N) = o(N).$$

Internally,

$$\mathrm{TC1} \xrightarrow{\mathrm{TNGTTHM}} o(N), \qquad \mathrm{HighTC} \xrightarrow{\mathrm{E10YMX}} o(N).$$

8.5 Verification Supplement

The mathematical proof is in Appendix H.1–H.13. The non-logical supplement `manuscript_full_md/supplements/non_logical_goodawack_verification.md` records reproducibility checks for the finite source layer. It is not a proof premise. It helps a reader verify that the finite catalogue in the proof-source layer has been synchronized with the generated manuscript.

9 Supporting Master Node: CKPX10M

9.1 Why It Matters

The CKP branch is where the proof invokes the DFI/X10 Kloosterman-fraction estimate. This invocation is delicate because the weight inserted into the bilinear form is not a separated surrogate. It is the actual two-variable Fourier-fibre weight arising after gcd splitting and smooth AP expansion.

9.2 What CKPX10M Packages

CKPX10M packages exact CKP gcd splitting; separation of the $h = 0$ local term from $h \neq 0$ nonzero modes; conversion of the central nonzero modes into Kloosterman-fraction sums; CKPD derivative bounds for the actual two-variable weight; X10/DFI theorem matching; excluded-range routing; and summation over g, h, and dyadic parameters.

9.3 Output and Location

CKPX10M proves that the central CKP nonzero-frequency contribution is $o(N)$, with the local zero-frequency contribution routed to H4M. The main location is Appendix F.

10 Supporting Master Node: H4M

10.1 Why It Matters

The proof has several local-looking terms: LongAP/Local, LocalDiag, CKP zero-frequency, and local boundary projections. H4M prevents these terms from becoming a vague local catch-all.

10.2 What H4M Proves

H4M proves that all LPI-admitted local/main contributions assemble into the single Goldbach singular-series main term:

$$M_{\text{local}}(N) = \mathfrak{S}(N)N + o(N).$$

It also proves that there is no independent $M_{\text{other local}}$ branch. Any notation of that form can only denote a bookkeeping remainder already partitioned by F3F4M and admitted or rejected by LPI.

10.3 Where To Check

Read Section 12 for the narrative and Appendix E for the proof. The local bridge depends on F3F4M, LPI, D1, G8a, B1LD, and H4.

11 New Mathematics, Bookkeeping, and Verification Aids

The manuscript uses several layers of text. They should not be confused.

Layer	Examples	Logical status
New mathematical theorem	F3F4M, TNGTTHM, E10YMX, CKPX10M, H4M	Proof content
Component lemma	B1, B3, F3, F4, C1, G8a, TGT, BRS, E10Y	Proof content
Bookkeeping lemma	PAR, GEB, source maps, build-order notes	Internal organization; used only where explicitly stated
Verification appendix	F3T tables, X10ER, finite routing tables	Hand-checkable expansion of mathematical cases
Non-logical reproducibility aid	GoodAWACK source-file manifest, occurrence-map supplement, build reports	Not a proof premise
Historical orientation	Hardy–Littlewood, Vinogradov, Chen, Helfgott references	Context only, not active inputs

The active proof inputs are the logical lemmas and external theorems listed in the dependency ledger. File hashes, build logs, and occurrence manifests help audit synchronization, but they do not prove any mathematical theorem.

12 Effective Threshold Status

The theorem is a sufficiently-large result. The proof tracks parameter hierarchies and error budgets but does not extract a practical numerical threshold N_0.

The status is:

1. The constants are ordered in PAR.
2. The terminal losses are summarized in GEB.
3. External estimates are invoked in asymptotic forms with their stated parameter ranges.
4. The final theorem is for all sufficiently large even N.
5. Finite verification for even $N < N_0$ is outside the claim of the current manuscript.

An explicit computational N_0 would require extracting constants from every external analytic input and from all smoothing and dyadic partition steps. That is a separate project and not part of the present proof package.

13 Ancillary Materials

The reader-facing proof object is

`manuscript_latex/goldbach_full_main.pdf`.

The main navigation and synchronization materials are `theorem_packages/master_reader_guide.pdf`, `Prooftreeandledger/g_proof_tree_n_ldg.md`, `ledger/file_manifest.md`, `manuscript_latex/build/full_pdf_validation_report.md`, and `ledger/rewrite_log.md`.

The non-logical GoodAWACK reproducibility material is `manuscript_full_md/supplements/non_logical_goodawack_verification.md`. The external-review packets are useful for focused checking but are not the authoritative proof source. If a packet conflicts with the full manuscript, the proof-source layer and `goldbach_full_main` control.

14 Suggested Reading Paths

14.1 Quick Orientation Path

Read this guide, the abstract and introduction of `goldbach_full_main`, Section 3 dependency tree, Section 7 routing overview, Section 11 GoodAWACK overview, and Section 13 final assembly. This path gives the architecture but is not enough to verify the proof.

14.2 Critical-Closure Path

Read:

- Appendix D.1 for F3F4M;
- Appendix G.1 for TNGTTHM;
- Appendix H.1–H.2 for E10YMX/E10L;
- Appendix F for CKPX10M;
- Appendix E for H4M;
- Appendix I for final assembly.

This path checks the main vulnerabilities.

14.3 Full Audit Path

Read Appendix C for B1/B3, Appendix D for routing, Appendix E for Edge/local projection, Appendix F for CKP/X10, Appendix G for TC1/BRS/X16, Appendix H for GoodAWACK finite grammar, Appendix I for final assembly, Appendix J for the dependency ledger, and then the bibliography and external inputs.

15 Section-by-Section Map of `goldbach_full_main`

The full manuscript is long because it is designed as an audit-grade proof dossier. A reader should not treat all sections as having the same logical function.

15.1 Front Matter and Main Narrative

The abstract and introduction state the theorem, explain the routed proof architecture, and separate historical context from active inputs. The glossary translates the nonstandard internal labels and is a reader aid, not an additional hypothesis. The dependency tree gives the compact visual and text fallback for the proof DAG; the authoritative table is still the ledger. The parameters and external-input sections record the order of constants and the external analytic theorems. The routing, Edge, CKP, GoodAWACK, local, and final handoff sections are article-level explanations that point to appendices for proof-level detail.

15.2 Proof Appendices

- Appendix A: parameter register and global error budget.
- Appendix B: external analytic inputs and theorem matching.
- Appendix C: Heath–Brown decomposition and B1/B3 descendants.
- Appendix D: routing exhaustion, including F3F4M and the F3/F4 tables.
- Appendix E: Edge/local projection and the H4M local bridge.
- Appendix F: CKP/X10 analytic branch.
- Appendix G: TC1/BRS/X16 no-rogue-short-interval route.
- Appendix H: GoodAWACK finite grammar and Branch B closure.
- Appendix I: final weighted assembly and prime handoff.
- Appendix J: dependency ledger and synchronization notes.
- Bibliography and References: active inputs separated from historical orientation references.

16 Failure Modes and Where They Are Blocked

The table below records the main ways the proof could fail and where the manuscript blocks each failure mode.

Possible failure mode	Blocking node	What to check
A descendant avoids all terminal classes	F3F4M	state space, terminal predicates, decreasing routing measure
Edge is defined by downstream estimates	F3P/C1P/F3F4M	terminal predicates are intrinsic before C1/CKP/E10/H4 estimates
CKP sends a noncentral range to DFI	CKPX10M/X10ER	only central nonzero modes reach X10; excluded ranges are routed
CKP uses a separated fake weight	CKPD	derivatives are for the actual two-variable Fourier-fibre weight

TC1 requires arbitrary shifted short intervals	TNGTTHM/TTH-SC	X9L-GT is invoked only after near-global hypotheses
BRS/X16 is asked to prove more than product-carrier estimates	X16BRS/X16C	complementary carriers reduce to the product-carrier model
GoodAWACK HighTC remains a source-audit assertion	E10YMX	grammar, invariant, induction, no untagged AFF
E10L defines the HighTC grammar it uses	E10YMX before E10L	E10L consumes E10YMX; it does not define E10YMX
Local projection becomes a catch-all	LPI/H4M	no independent $M_{\text{other local}}$ class
Prime powers contaminate the final prime-pair count	G2/G1/G0H	nontrivial prime powers are negligible after I1

17 External Inputs at a Glance

The proof uses external analytic inputs only through named interfaces. The most important ones for a first audit are:

- X1, the Heath–Brown identity, used by B1.
- X2, smooth partitions, used in B1/B3.
- X4, CRT and local algebra, used in H4/H4M.
- X9L-GT, Davenport/AP Liouville input, used only in the near-global TC1 branch after TNGTTHM and TTH.
- X10, the DFI Kloosterman-fraction estimate, used only inside CKPX10M after CKPD verifies the actual smooth-weight derivative hypotheses.
- X16, the Shiu/AP divisor estimate, used inside X16C and then X16BRS.
- X12, the prime-power bound, used by G2.
- X13, singular-series algebra, used in H4/H4M and G1/G0H.

Historical references such as Hardy–Littlewood, Vinogradov, Chen, and Helfgott orient the reader but are not active proof inputs.

18 What Not To Check First

A first reader should not begin by checking every ledger row or every source-manifest entry. That is the slowest route and hides the main structure. The first pass should not try to verify every file hash, read the non-logical GoodAWACK supplement before reading E10YMX, inspect every F3T row before understanding F3F4M, check finite-N computation below the asymptotic threshold, treat historical bibliography entries as proof inputs, or read modular theorem packages as independent forks of the proof.

The correct first pass is architectural: F3F4M, TNGTTHM, E10YMX/E10L, CKPX10M, H4M, and I1/G2/G1/G0H.

19 Referee Packets versus the Full Manuscript

The referee packets are focused audit aids. They are useful when a specialist is asked to check one technical branch, for example CKP/X10 or GoodAWACK. They are not the authoritative proof source.

The authoritative chain is

$$\text{proof-source md layer} \longrightarrow \texttt{goldbach\_full\_main} \longrightarrow \text{ledger and guide synchronization.}$$

If an external packet differs from the full manuscript, the proof-source layer and the full manuscript control. The packet should then be regenerated from the corrected source.

20 Acronym Management

The proof has many internal labels. A first reader should focus on: B1, B3, F3F4M, C1, CKPX10M, H4M, TNGTTHM, E10YMX, E10L, GEB, I1, G2, G1, and G0H. Most other labels are component lemmas inside one of these nodes. The glossary in Section 2 of `goldbach_full_main` gives the complete label map.

21 Minimal Referee Checklist

A concise serious check is:

1. F3F4M proves a genuine finite partition and does not define terminal predicates using downstream estimates.
2. TNGTTHM proves that X9L-GT is applied only to near-global B1-origin coarea tests.
3. E10YMX proves the no-untagged rank-dropping AFF theorem as an ordinary finite combinatorial theorem, and E10L consumes that theorem rather than defining it.
4. CKPX10M and X16C/X16BRS provide the stated citation-grade estimates used by CKP and TNGTTHM.
5. H4M proves that all LPI-admitted local/main pieces assemble to the single singular-series main term with no extra local class.
6. I1 imports only the terminal outputs above, GEB accounts for all losses, and G2/G1/G0H remove prime powers.

If these checks pass, the full manuscript proves the stated sufficiently large binary Goldbach theorem, subject only to the external analytic inputs listed in Appendix B and the bibliography.

22 Synchronization Note

This guide is synchronized with the current `active_proof_v29` proof-source layer and the rebuilt full manuscript. Future mathematical corrections should be made first in the proof-source Markdown files, then propagated to `goldbach_full_main`, the dependency ledger, this guide, and only afterwards to any modular theorem-package presentation.