H4 Local Algebra Theorem Package

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May 2026

Abstract

This note isolates the H4 local/main assembly node from the full proof package. Its purpose is to prove, as a standalone local algebra statement, that all admitted local/main terms assemble into the classical Goldbach singular-series main term $\mathfrak{S}(N)N + o(N)$. The proof makes explicit the local model Λ_Q, the tagged LPI admission condition consumed by H4, the linearity over the B1/F3 tagged partition, the no-double-counting mechanism, and the finite CRT calculation of the local factors.

Contents

1 Scope

This package does not estimate Edge, CKP nonzero-frequency, or GoodAWACK terms. Those terms are discharged by their own terminal branches. H4 handles only the admitted local/main terms and proves

$$M_{\text{local}}(N) = \mathfrak{S}(N)N + o(N). \tag{H4-LA}$$

The statement must exclude three failure modes: a branch-specific local normalization, double counting of local terms, and loss of tagged local mass.

The proof uses the logical inputs B1, B3, F3, F3T, F4, D1, G8a, B1LD, C1, C1A, and finite CRT algebra.

2 The Local Model

Let

$$w = w(N) \to \infty, \qquad w = o(\log N), \qquad Q = \prod_{p \le w} p.$$

Then Q is squarefree and $Q = N^{o(1)}$. Define

$$\Lambda_Q(a) = \frac{Q}{\varphi(Q)} \mathbf{1}_{(a,Q)=1}.$$

The finite local Goldbach density is

$$\sigma_Q(N) = \frac{1}{Q} \sum_{a \bmod Q} \Lambda_Q(a)\Lambda_Q(N-a). \tag{2.1}$$

The canonical local projection of the original Goldbach convolution is

$$\mathrm{Loc}_Q\, R_\Lambda(N) = N\sigma_Q(N). \tag{2.2}$$

Endpoint and smooth-boundary discrepancies are C1-admitted boundary errors and are $o(N)$.

3 Tagged Decomposition and H4 Admission

B1 gives an exact finite identity

$$R_\Lambda(N) = \sum_{\mathcal{B} \in \mathfrak{B}_{J_0}} c_{\mathcal{B}} R_{\mathcal{B}}(N). \tag{3.1}$$

After B3/F3/F4 routing, each parent block is partitioned into tagged cells:

$$R_{\mathcal{B}}(N) = \sum_{\tau \in \mathcal{T}(\mathcal{B})} R_{\mathcal{B},\tau}(N), \tag{3.2}$$

before any terminal estimate is applied. The tag $(\mathcal{B}, \tau)$ records the parent B1 block and the finite routing/grouping history.

Definition 1 (Tagged LPI local projection)**.** *For a tagged cell $(\mathcal{B}, \tau)$, define $\mathrm{Loc}_Q\, R_{\mathcal{B},\tau}(N)$ by replacing the arithmetic coefficients in that same tagged cell by their residue-class local models modulo Q, while preserving the parent tag, routing tag, smooth/dyadic weights, controlled local congruence data, and the equation $n_1 + n_2 = N$.*

Definition 2 (LPI admission condition consumed by H4)**.** *A local/main term $M^{\mathrm{local}}_{\mathcal{B},\tau}(N)$ is admitted by H4 only if*

$$M^{\mathrm{local}}_{\mathcal{B},\tau}(N) = \mathrm{Loc}_Q\, R_{\mathcal{B},\tau}(N) + o_{\mathcal{B},\tau}(N). \tag{3.3}$$

The admitted local source set is

$$\mathfrak{L}_{\mathrm{H4}} = \mathfrak{L}_{\mathrm{LongAP/Local}} \sqcup \mathfrak{L}_{\mathrm{CKP},0} \sqcup \mathfrak{L}_{\mathrm{LocalDiag}}. \tag{3.4}$$

There is no fourth residual projection source. Controlled CRT absorption, fixed-divisor quotienting, and primitive local slicing are admitted only as tagged refinements of one of the three displayed classes. Endpoint and smooth-boundary localizations are C1 Edge terms and do not enter $\mathfrak{L}_{\mathrm{H4}}$.

4 Admission of the Main Local Sources

Lemma 1 (LongAP/Local admission)**.** *Every LongAP/Local terminal term admitted into H4 satisfies the admission condition* (3.3)*.*

Proof. D1 first excludes any surviving nonlocal coefficient attached to the long AP variable. Thus no factor such as

$$\mu(L(u)), \quad \lambda(L(u)), \quad e(\alpha L(u)), \quad e\left(\frac{k\overline{L(u)}}{q}\right)$$

survives inside a terminal LongAP/Local atom. All remaining restrictions are controlled residue-class, coprimality, AP, or CRT restrictions with moduli $(\log N)^{O(1)}$.

The smooth AP count replaces each controlled residue class by its zero frequency density. Nonzero finite Fourier terms are negligible by smoothness and by the length of the LongAP direction; endpoint errors are C1-admitted. The resulting LongAP/Local main term is therefore exactly the tagged Λ_Q-local projection of the same cell, up to $o(N)$. $\square$

Lemma 2 (CKP zero-frequency admission)**.** *Every CKP zero-frequency term admitted into H4 satisfies the admission condition* (3.3)*.*

Proof. G8a separates the CKP AP/fibre expansion into the zero frequency $h = 0$ and the nonzero frequencies $h \neq 0$. The nonzero frequencies are not local terms. For $h = 0$, the finite AP expansion gives the average over the residue fibre:

$$\frac{1}{q}\widehat{F}_{a,q}(0) = \frac{1}{q}\sum_{y} F_{a,q}(y).$$

B1LD proves that the B1-inherited coefficient local densities commute with finite convolution, CRT localization, gcd splitting, and tagged dyadic restriction. Hence the arithmetic coefficient factors in the CKP $h = 0$ term are exactly those used by $\mathrm{Loc}_Q\, R^{\mathrm{CKP}}_{\mathcal{B},\tau}(N)$. The tag is preserved throughout. Thus (3.3) holds. $\square$

Lemma 3 (LocalDiag admission)**.** *Every LocalDiag term admitted into H4 satisfies the admission condition* (3.3)*.*

Proof. In F3/F4, LocalDiag denotes a forced equality, proportionality, gcd-local dependence, or unavoidable collision that produces a canonical local tagged cell. H4 admits such a cell only when the diagonal specialization is the local projection of the same parent B1/F3/F4 cell. If the specialization is not canonical, F3T routes it instead to Edge, CKP, GoodAWACK, impossible, or a continuing routed case. Therefore every LocalDiag term reaching H4 is a tagged canonical local projection. $\square$

5 Reconstruction

Lemma 4 (Tagged local reconstruction)**.**

$$\sum_{\mathcal{B}} c_{\mathcal{B}} \sum_{\tau\in\mathcal{T}(\mathcal{B})} \mathrm{Loc}_Q\, R_{\mathcal{B},\tau}(N) = N\sigma_Q(N) + o(N). \tag{5.1}$$

Proof. B1 is an exact finite identity before estimation, and F3/F4 give finite tagged partitions before terminal estimates. The operator Loc_Q is linear and tag-preserving. Applying Loc_Q to the tagged decomposition is therefore the same as applying the local model directly to the original two von Mangoldt factors.

On the original factors this replacement is

$$\Lambda(n) \mapsto \Lambda_Q(n \bmod Q) = \frac{Q}{\varphi(Q)} \mathbf{1}_{(n,Q)=1}.$$

Thus the reconstructed local sum is

$$\sum_{n_1+n_2=N} \Lambda_Q(n_1)\Lambda_Q(n_2).$$

Counting by $a \equiv n_1 \pmod{Q}$ gives

$$\sum_{n_1+n_2=N} \Lambda_Q(n_1)\Lambda_Q(n_2) = N \cdot \frac{1}{Q} \sum_{a \bmod Q} \Lambda_Q(a)\Lambda_Q(N-a) + o(N) = N\sigma_Q(N) + o(N).$$

The $o(N)$ term is only the endpoint/smooth-boundary discrepancy admitted by C1. □

6 No Double Counting and No Loss

Lemma 5 (No double counting)**.** *The admitted local/main sum*

$$M_{\mathrm{local}}(N) = \sum_{(\mathcal{B},\tau)\in\mathfrak{L}_{\mathrm{H4}}} c_{\mathcal{B}} M^{\mathrm{local}}_{\mathcal{B},\tau}(N) \tag{6.1}$$

counts no local term twice.

Proof. Each admitted term carries the tag $(\mathcal{B}, \tau)$. If two tags have different parent B1 blocks, they are different summands of the exact B1 identity. If they have the same parent block but different routing tags, the F3 partition identity gives disjoint tagged cells.

LocalDiag terms do not create an exception. Two LocalDiag expressions may look algebraically identical after specialization, but H4 does not identify terms by visual form. It sums the canonical projection of each tagged cell. Thus visually identical LocalDiag expressions from different tags are complementary summands, not duplicates. □

Lemma 6 (Completeness of admitted local terms)**.** *Every terminal local/main atom produced by the routing tree and admitted into the main term lies in exactly one component of $\mathfrak{L}_{\mathrm{H4}}$.*

Proof. F3 assigns every terminal atom to exactly one terminal routing class. Error classes are Edge, CKP nonzero-frequency, and GoodAWACK. Local/main classes are LongAP/Local, CKP zero-frequency, and LocalDiag. The only auxiliary local-looking operations are controlled CRT absorption, fixed-divisor quotienting, and primitive local slicing; each is a tagged refinement of one of these three classes. Endpoint and smooth-boundary localizations are C1 Edge terms. Hence there is no lost admitted local term and no hidden local class. □

7 Local Factors

Since Q is squarefree, CRT gives

$$\sigma_Q(N) = \prod_{p \le w} \sigma_p(N),$$

where

$$\sigma_p(N) = \frac{1}{p}\left(\frac{p}{p-1}\right)^2 \#\{a \bmod p : (a,p) = 1,\ (N-a,p) = 1\}.$$

For $p = 2$ and even N, the only unit residue is 1, hence $\sigma_2(N) = 2$.

For odd $p \mid N$, the two forbidden residues coincide, so

$$\sigma_p(N) = \frac{p}{p-1}.$$

For odd $p \nmid N$, the two forbidden residues are distinct, so

$$\sigma_p(N) = \frac{p(p-2)}{(p-1)^2} = 1 - \frac{1}{(p-1)^2}.$$

Therefore

$$\sigma_Q(N) = 2 \prod_{\substack{3 \le p \le w \\ p \nmid N}} \left(1 - \frac{1}{(p-1)^2}\right) \prod_{\substack{3 \le p \le w \\ p \mid N}} \frac{p}{p-1}.$$

Letting $w \to \infty$,

$$\sigma_Q(N) \to 2C_2 \prod_{\substack{p \mid N \\ p > 2}} \frac{p-1}{p-2} = \mathfrak{S}(N), \tag{7.1}$$

where

$$C_2 = \prod_{p > 2} \left(1 - \frac{1}{(p-1)^2}\right).$$

8 Main Theorem

Theorem 1 (H4 local algebra)**.** *In the routed proof tree, the sum of all admitted local/main contributions satisfies*

$$M_{\text{local}}(N) = \mathfrak{S}(N)N + o(N).$$

Proof. By the admission lemmas and the LPI no-residual-projection rule, every local/main term admitted into H4 satisfies the tagged admission identity (3.3). By the no-double-counting lemma these terms are not counted twice, and by the completeness lemma there is no hidden admitted local class. Therefore

$$M_{\text{local}}(N) = \sum_{(\mathcal{B},\tau) \in \mathfrak{L}_{\text{H4}}} c_{\mathcal{B}} \operatorname{Loc}_Q R_{\mathcal{B},\tau}(N) + o(N).$$

The nonlocal/error classes have already been removed by their terminal estimates and are not part of M_{local}. The admitted local classes exhaust the local projection of the tagged proof tree, so tagged local reconstruction gives

$$M_{\text{local}}(N) = \operatorname{Loc}_Q R_\Lambda(N) + o(N) = N\sigma_Q(N) + o(N).$$

By the local factor calculation, $\sigma_Q(N) \to \mathfrak{S}(N)$. Hence

$$M_{\text{local}}(N) = \mathfrak{S}(N)N + o(N).$$

□

9 Output

The output supplied to the global assembly theorem is

$$\boxed{M_{\mathrm{LongAP}}(N) + M_{\mathrm{CKP},0}(N) + M_{\mathrm{LocalDiag}}(N) + M_{\mathrm{proj}}(N) = \mathfrak{S}(N)N + o(N).}$$

The equality is tagwise. No error branch is imported into H4, no branch uses a separate local density convention, and no untagged local main term is admitted.