GoodAWACK Finite Grammar Full Proof Package

Denis Saltykov (ds1678@gmail.com)

May 2026

Contents

1 GoodAWACK Finite Grammar Full Proof Package

1.1 Abstract

This full-proof package contains the GoodAWACK finite grammar, rank-dropping AFF exclusion, and Branch B closure source texts.

1.2 Scope

This package supplies the HighTC/finite-grammar and E10 Branch B brick. TC1 analytic testing is supplied by the TNG/TC1 package.

1.3 Included Proof-Source Files

1. `Lemmas/e_5_ltx.md` – Affine transport and content stability
2. `Lemmas/b1_to_goodawack_skeleton_normal_form_ltx.md` – B1-to-GoodAWACK skeleton normal form
3. `Lemmas/bgs_hgo2_reduction_ltx.md` – HighTC obstruction reduction
4. `Lemmas/baoc_affine_origin_catalogue_ltx.md` – Affine-origin catalogue
5. `Lemmas/e10g_strong_baoc_catalogue_ltx.md` – Strong BAOC catalogue
6. `Lemmas/e10h_e10g_rigidity_ltx.md` – Rigidity interface
7. `Lemmas/e10i_mor_matrix_origin_rigidity_ltx.md` – Matrix-origin rigidity
8. `Lemmas/e10j_rda_rank_dropping_aff_origin_verification_ltx.md` – Rank-dropping AFF origin verification
9. `Lemmas/e10y_goodawack_routing_grammar_completeness_ltx.md` – GoodAWACK routing grammar completeness
10. `Lemmas/e10m_no_untagged_rank_dropping_aff_ltx.md` – No untagged rank-dropping AFF
11. `Lemmas/e10_master_source_exhaustion_closure_ltx.md` – Master finite-grammar closure
12. `Lemmas/e10k_aff_oc_affine_regrouping_origin_completeness_ltx.md` – AFF-origin completeness
13. `Lemmas/e10l_e10_clean_branch_b_ltx.md` – Clean Branch B closure

2 Part 1. E5: Affine transport and content stability

Source file: `Lemmas/e_5_ltx.md`.

2.0.1 E5. Content Stability Lemma

E5.0. Role Logical ID: `E5`.

Lemma **E5** is the content-stability lemma used by Branch B / GoodAWACK. It ensures that admissible routing, lattice restriction, quotienting, slicing, and clean affine coordinate changes do not lose controlled content for a marked affine form.

The phrase "affine regrouping" in this file is not an additional terminal routing operation. It means either a full-rank affine coordinate change, or a rank-dropping map whose origin has already been recorded by the earlier B1/B3/F3/F4 routing data as fixing/projection, quotient/divisor/local, CKP, Edge, impossible, or post-terminal analytic slicing. E5 does not classify these origins and does not introduce a new terminal GoodAWACK generator; it only preserves controlled content for transports whose source is already present in the routing record.

The output needed by the subsequent branches is that, after slicing, one obtains a form

$$L(u) = gu + b$$

with

$$g \le (\log N)^C,$$

and can then apply the TC1/X9L linear input to $\lambda(gu + b)$.

Used by: `BRS`, `TTH`, `TGT`, `TNG`, `E10M`, `E10K`, and `E10L`.

Uses: `F3`, `F4`, and standard bounded-minor/content algebra.

—

E5.1. Content on a lattice coset Let

$$\Lambda = z_0 + \Lambda_0$$

be an affine lattice coset, where $\Lambda_0 \subseteq \mathbb{Z}^r$ is a lattice. Let

$$L(z) = \ell(z) + b$$

be an affine-linear form. Define the content relative to Λ by

$$\mathrm{cont}_\Lambda(L) = \gcd\{\ell(v) : v \in \Lambda_0\}.$$

Form L has controlled content if

$$\mathrm{cont}_\Lambda(L) \le (\log N)^C.$$

—

E5.2. CRT restriction

Lemma 2.1 (Lemma E5.1)**.** *Let*

$$\Lambda' = \{z \in \Lambda : L_0(z) \equiv a \pmod q\}$$

be a nonempty CRT restriction with

$$q \le (\log N)^C.$$

Then, for every affine form L,

$$\mathrm{cont}_{\Lambda'}(L) \le q^{O(1)}\,\mathrm{cont}_{\Lambda}(L).$$

In particular, controlled content remains controlled.

Proof. The difference lattice $\Lambda_0' = \Lambda' - \Lambda'$ is a sublattice of Λ_0 of index at most $q^{O(1)}$. If

$$\ell(\Lambda_0) = g\mathbb{Z},$$

then $\ell(\Lambda_0')$ is a sublattice of $g\mathbb{Z}$ of index at most $q^{O(1)}$. Hence

$$\ell(\Lambda_0') = g'\mathbb{Z}$$

with

$$g' \le q^{O(1)} g.$$

Thus

$$\mathrm{cont}_{\Lambda'}(L) \le q^{O(1)}\,\mathrm{cont}_{\Lambda}(L).$$

Since \{}(q\{}le(\{}log N)^C\{}), the new content remains polylogarithmic. Lemma proved. —

□

E5.3. Fixed divisor absorption Suppose an atom is restricted by

$$d \mid L(z),$$

where d is fixed on the atom. Define

$$\Lambda_d = \{z \in \Lambda : L(z) \equiv 0 \pmod{d}\}, \qquad L_d(z) = L(z)/d.$$

Lemma 2.2 (Lemma E5.2)**.** *If*

$$g = \mathrm{cont}_{\Lambda}(L),$$

then

$$\mathrm{cont}_{\Lambda_d}(L_d) = \frac{g}{(g,d)} \le g.$$

Proof. On the difference lattice,

$$\ell(\Lambda_0) = g\mathbb{Z}.$$

The restricted difference lattice satisfies

$$\ell(\Lambda_{d,0}) = g\mathbb{Z} \cap d\mathbb{Z} = \mathrm{lcm}(g,d)\mathbb{Z}.$$

After dividing the form by d, the image lattice is

$$\frac{1}{d}\,\mathrm{lcm}(g,d)\mathbb{Z} = \frac{g}{(g,d)}\mathbb{Z}.$$

This proves the formula. Lemma proved.

—

$\square$

E5.4. Primitive slicing Primitive slicing chooses coordinates on a lattice coset so that a marked affine form becomes

$$L(z) = gu + b$$

on a one-dimensional fibre. The coefficient g is precisely the content of the linear part on the fibre lattice. Therefore, if

$$\mathrm{cont}_\Lambda(L) \le (\log N)^C,$$

then

$$g \le (\log N)^C.$$

If the resulting fibre is short, the atom is routed to Edge/Local by F3/C1. If the fibre is long, the form $gu + b$ is admissible for the active TC1/X9L linear input.

—

E5.5. Affine changes and regrouping Let $T : \mathbb{Z}^{r'} \to \mathbb{Z}^r$ be an integer affine map with coefficients and relevant minors bounded by powers of $\log N$. For

$$L'(x) = L(Tx),$$

we have

$$\mathrm{cont}(L') \le (\log N)^C \, \mathrm{cont}(L).$$

If T is unimodular or full-rank on the active affine span, content is preserved exactly or changes only by the bounded-minor factor already displayed above. Thus clean bounded affine regrouping preserves controlled content.

If T is rank-dropping, it is allowed in the proof tree only when the rank drop carries an explicit origin tag already produced by the earlier routing record. Such a map is not a free terminal-vector generator; it is either routed by B1/B3/F3/F4 data or used only as a post-terminal analytic slicing operation after the terminal affine system has been fixed.

E5.5A. Clean full-rank criterion Let

$$U_{\mathcal{L}} = \mathrm{span}_{\mathbb{Q}}\{\ell_i - \ell_j : L_i, L_j \in \mathcal{L}\}$$

be the active affine difference span on the current routing cell, and let

$$U_{\mathrm{TC}} = \mathrm{span}_{\mathbb{Q}}\{\ell_\rho : \rho \in \mathcal{L}_{\mathrm{term}}\}$$

be the terminal tensor-test vector span when the terminal GoodAWACK object has already been fixed. An affine transport T is **E5-clean full-rank** only if the linear part T_{lin} satisfies

$$\ker(T_{\mathrm{lin}}|_{U_{\mathcal{L}}}) = 0$$

and, in the terminal GoodAWACK setting,

$$\ker(T_{\mathrm{lin}}|_{U_{\mathrm{TC}}}) = 0.$$

If either kernel is nontrivial, the transport is not clean full-rank. It may then be used only if the lost rank has already been produced and recorded by one of the routing origins

$$\mathrm{Fix/Proj}, \quad \mathrm{CRT}, \quad \mathrm{FixedDiv}, \quad \mathrm{VarQuot}, \quad \mathrm{LocalDiag}, \quad \mathrm{CKP}, \quad \mathrm{Edge},$$

or if the operation is post-terminal analytic slicing which does not replace the terminal tensor-test vectors. Thus E5 never promotes a rank-dropping map to an independent terminal GoodAWACK generator.

—

E5.6. Cauchy and cube operations Cauchy–Schwarz and cube operations introduce shifted forms such as

$$L(z + \omega h) = L(z) + \ell(\omega h).$$

The linear part remains ℓ. Therefore

$$\mathrm{cont}(L(z + \omega h)) = \mathrm{cont}(L).$$

If a cube operation produces equality, proportionality, or forced local dependence between forms, the atom is routed to LocalDiag by F3. Otherwise at least one marked controlled-content form survives.

—

E5.7. Lemma E5

Lemma 2.3 (Lemma E5)**.** *Let $\mathcal{A}$ be a Branch B atom with a marked affine form L_* satisfying*

$$\mathrm{cont}_\Lambda(L_*) \le (\log N)^C.$$

Under any finite sequence of allowed F3 operations:

CRT, fixed divisor absorption, primitive slicing, clean affine regrouping, Cauchy/cube, local diagonal e

one of the following holds:

1. *the atom becomes terminal LocalDiag;*
2. *the atom is routed to Edge or CKP;*
3. *the resulting Branch B atom still has a marked affine form of controlled content.*

Proof. CRT restrictions increase content by at most a polylogarithmic factor by Lemma E5.1. Fixed divisor absorption does not increase quotient content by Lemma E5.2. Primitive slicing writes the form as $gu + b$ with controlled g. Clean bounded affine changes and regrouping multiply content by at most a polylogarithmic factor. If a rank drop occurs, E5 requires that its origin tag is already present in the routing record as fixing/projection, quotient/divisor/local, CKP, Edge, impossible, or post-terminal analytic slicing; E5 itself does not create the tag. Cauchy/cube shifts preserve the linear part and hence preserve content. If any operation creates forced local dependence, F3 routes the atom to LocalDiag. Therefore every nonterminal Branch B descendant retains a controlled-content marked form. Lemma proved.

—

□

Remark 2.4 (E5.8. Output).

Controlled content is stable under the allowed E5 operations, with the clean routing-record interpretation of affi

Thus Branch B descendants that are not routed to LocalDiag, Edge, or CKP retain a marked affine form of controlled content.

E5.9. Logical Dependencies Internal dependencies: `F3`, `F4`, and standard bounded-minor/content algebra.

Internal nodes served: `BRS`, `TTH`, `TGT`, `TNG`, `E10Y`, `E10M`, `E10K`, and `E10L`.

3 Part 2. BGS: B1-to-GoodAWACK skeleton normal form

Source file: `Lemmas/b1_to_goodawack_skeleton_normal_form_ltx.md`.

3.0.1 BGS. Skeleton Normal Form for Terminal GoodAWACK Descendants

BGS.0. Statement and Role The skeleton record below is an intrinsic B1/B3/F3/F4/E5 normal form. It records B3 product groupings, full-rank coordinate changes, and rank drops with explicit fixing/projection, quotient/divisor/local, CKP, Edge, impossible, or post-terminal analytic origin tags. It does not obtain its meaning from E10L. The later E10Y/E10M/E10K/E10X finite-grammar layer consumes this record to exclude untagged rank-dropping affine regrouping as a terminal-vector generator.

Lemma BGS extracts the normal form for terminal GoodAWACK atoms generated by the B1/B3/F3/F4 routing interface.

The purpose is to prepare the finite structural theorem HGO.2:

$$\text{HighTC-cert} \Longrightarrow \text{CKP} \sqcup \text{LocalDiag} \sqcup \text{Edge} \sqcup \text{Impossible}.$$

The output is a finite parameterized skeleton normal form that records:

1. the parent B1 product block;
2. the B3 grouping choice;
3. the F3/F4 routing history;
4. the active affine forms that survive into terminal GoodAWACK;
5. the origin of each affine form;
6. the exact tensor test needed for TC1/HighTC.

This is the object used downstream by the E10Y/E10X/E10K/E10L finite-grammar and clean-interface arguments.

Logical dependencies: B1, B3, F3, F4, and E5. Outputs served: HGO2R, BAOC, E10Y, E10K, E10L, and E10X.

—

BGS.1. B1 parent block Every terminal GoodAWACK atom has a parent typed B1 block

$$\mathcal{B} = (r, s, \mathbf{X}, \mathbf{Y}, \mathbf{t}),$$

where:

1. $r, s \leq 2J_0$;
2. $\mathbf{X}, \mathbf{Y}$ are dyadic scale vectors;
3. $\mathbf{t}$ records elementary coefficient types

$$\mu \cdot 1_{\leq y}, \qquad 1, \qquad \log;$$

1. the parent equation is

$$P_A(a) + P_B(b) = N, \tag{B1}$$

with

$$P_A(a) = \prod_{i=1}^{r} a_i, \qquad P_B(b) = \prod_{j=1}^{s} b_j.$$

The parent variable set is

$$\mathcal{X}_{\mathcal{B}} = \{a_1, \ldots, a_r, b_1, \ldots, b_s\}.$$

All later descendants keep the parent B1 tag. This is required by H4 and by the clean F3/F4 routing interface.

—

BGS.2. B3 grouping skeleton A B3 grouping skeleton is a finite partition of selected parent variables into grouped factors:

$$u_\nu = \prod_{x \in I_\nu} x, \qquad v_\nu = \prod_{x \in J_\nu} x,$$

and similarly on both sides of (B1).

The grouping skeleton records:

1. which grouped factors are short, long, or central;
2. whether a balanced bilinear grouping is exposed;
3. whether a local AP configuration is exposed;

4. whether a forced local dependence/collision is exposed;
5. which residual grouped variables remain available for BranchB/GoodAWACK.

By Lemma B3, the set of possible grouping skeletons is finite:

$$|\mathcal{G}(\mathcal{B})| \le C(J_0).$$

Only grouping skeletons that are not terminal Edge, CKP, LongAP/Local, or LocalDiag can feed the GoodAWACK skeleton.

—

BGS.3. F3/F4 routing skeleton Starting from a B1 block and a B3 grouping skeleton, F3/F4 perform only finitely many routing-level operations before terminality.

For a terminal GoodAWACK descendant, the routing skeleton records the following data:

$$\mathfrak{r} = (\mathfrak{r}_{\mathrm{CRT}}, \mathfrak{r}_{\mathrm{div}}, \mathfrak{r}_{\mathrm{quot}}, \mathfrak{r}_{\mathrm{grp}}, \mathfrak{r}_{\mathrm{fail}}).$$

Here:

1. $\mathfrak{r}_{\mathrm{CRT}}$ records controlled CRT/congruence restrictions;
2. $\mathfrak{r}_{\mathrm{div}}$ records fixed divisor absorptions;
3. $\mathfrak{r}_{\mathrm{quot}}$ records variable quotient equations $L(z) = ds$ that were resolved without becoming Edge, LocalDiag, or CKP;
4. $\mathfrak{r}_{\mathrm{grp}}$ records B3 product groupings, full-rank affine changes of variables, and only E10Y/E10M/E10L-tagged rank-dropping maps;
5. $\mathfrak{r}_{\mathrm{fail}}$ records failed terminal alternatives that were checked and eliminated.

The GoodAWACK routing condition is:

$$\neg\text{Edge}, \qquad \neg\text{CKP}, \qquad \neg\text{LongAP/Local}, \qquad \neg\text{LocalDiag}, \tag{G0}$$

together with:

1. no unresolved ordinary large-divisor predicate;
2. central-long affine/WACLE residual structure;
3. controlled content;
4. at least one marked Liouville-type affine form.

The important limitation is that the F3/F4 routing skeleton does not include the quadratic tensor test

$$Q_m \overset{?}{\in} \mathrm{span}_{\mathbb{Q}}\{Q_i : i \neq m\}.$$

That test is added only at the TC1/HighTC dichotomy stage.

—

BGS.4. Setup: Active Parameter Lattice After applying the F3/F4 routing skeleton, a terminal GoodAWACK descendant is supported on a bounded-rank lattice coset

$$z \in \Omega_{\mathfrak{G}} \subset z_* + \Lambda_{\mathfrak{G}} \subset \mathbb{Z}^{k_{\mathfrak{G}}},$$

where:

1. $k_{\mathfrak{G}} \le K(J_0)$;
2. $\Omega_{\mathfrak{G}}$ is a smooth box-like region;
3. every active long direction has length at least N^θ, up to C1-routed boundary/short-volume exceptions;
4. the lattice index and all contents are bounded by a power of $\log N$.

We write the active parameter vector as

$$z = (z_1, \ldots, z_{k_{\mathfrak{G}}}).$$

The affine transformations from the parent variables to z are recorded by an origin map

$$\mathrm{orig}_{\mathfrak{G}}.$$

This map is part of the skeleton. It is needed to decide whether a later algebraic relation corresponds to CKP, genuine LocalDiag, Edge, or an impossible parent configuration.

—

BGS.5. Setup: Active Affine Forms The terminal GoodAWACK descendant has a finite active affine system

$$\mathcal{L}_{\mathfrak{G}} = \{L_\rho(z) : \rho \in \mathcal{I}_{\mathfrak{G}}\},$$

where

$$L_\rho(z) = \ell_\rho \cdot z + c_\rho, \qquad \ell_\rho \in \mathbb{Z}^{k_{\mathfrak{G}}}, \qquad c_\rho \in \mathbb{Z},$$

after clearing the controlled lattice denominator.

Each L_ρ has one of the following origins:

Type A. Parent coordinate / grouped factor

$$L_\rho(z)$$

is the affine representative of a surviving parent variable or grouped factor after CRT restriction and affine change of variables.

Type B. Fixed divisor quotient A controlled divisor relation

$$d \mid L(z), \qquad d \le (\log N)^C,$$

has been absorbed on a lattice coset, producing

$$L_\rho(z) = L(z)/d$$

as an integer affine form on the restricted lattice.

Type C. Variable quotient residual A quotient equation

$$L(z) = ds$$

has been resolved without short-volume Edge, without forced local dependence, and without balanced CKP structure. The quotient variable s survives as an affine form

$$L_\rho(z) = s(z).$$

Type D. Primitive slice / fibre form After primitive slicing, a marked form may be written on a fibre as

$$L_\rho(z_0 + uv) = gu + b, \qquad g \le (\log N)^C.$$

This is used analytically in E7/E10, but the pre-slicing form remains in $\mathcal{L}_\mathfrak{S}$ for the tensor test.

Type E. Auxiliary bounded affine factor An auxiliary divisor-bounded or smooth coefficient factor depends on

$$L_\rho(z)$$

and is treated analytically as $f_\rho(L_\rho(z))$.

Thus the terminal atom has model form

$$\mathfrak{A}_\mathfrak{S} = \sum_{z \in \Omega_\mathfrak{S}} W_\mathfrak{S}(z) \prod_{\rho \in \mathcal{M}_\mathfrak{S}} \lambda_\rho(L_\rho(z)) \prod_{\rho \in \mathcal{U}_\mathfrak{S}} f_\rho(L_\rho(z)), \tag{BGS}$$

where $\mathcal{M}_\mathfrak{S} \neq \varnothing$ is the set of marked Liouville-type forms and $\mathcal{U}_\mathfrak{S} \subseteq \mathcal{I}_\mathfrak{S}$ is the set of auxiliary bounded/smooth coefficient forms. In the E10 proof one marked form is selected and denoted L_0.

All active forms satisfy

$$\mathrm{cont}(L_\rho) \le (\log N)^C.$$

—

BGS.6. Definition: Skeleton Normal Form

Definition 3.1 (Definition. B1-to-GoodAWACK skeleton)**.** A B1-to-GoodAWACK skeleton is a tuple

$$\mathfrak{S} = (\mathcal{B}, \Gamma, \mathfrak{r}, \Lambda_\mathfrak{S}, \Omega_\mathfrak{S}, \mathcal{L}_\mathfrak{S}, \mathcal{M}_\mathfrak{S}, \mathrm{orig}_\mathfrak{S}, \mathcal{W}_\mathfrak{S}),$$

where:

1. $\mathcal{B}$ is a B1 parent block;
2. $\Gamma \in \mathcal{G}(\mathcal{B})$ is a B3 grouping skeleton;
3. $\mathfrak{r}$ is the F3/F4 routing skeleton;
4. $\Lambda_\mathfrak{S}$ is the active lattice/coset data;
5. $\Omega_\mathfrak{S}$ is the smooth central-long domain;

6. $\mathcal{L}_{\mathfrak{S}}$ is the active affine system;
7. $\mathcal{M}_{\mathfrak{S}} \subseteq \mathcal{L}_{\mathfrak{S}}$ is the nonempty marked Liouville-form set;
8. $\mathrm{orig}_{\mathfrak{S}}$ records the B1/F3/F4 origin of each affine form;
9. $\mathcal{W}_{\mathfrak{S}}$ records dyadic weights and coefficient types.

It is admissible if:

$$\neg\text{Edge}, \quad \neg\text{CKP}, \quad \neg\text{LongAP/Local}, \quad \neg\text{LocalDiag}, \tag{Admiss}$$

and the terminal GoodAWACK predicate of Lemma F3 holds.

—

BGS.7. Lemma: terminal GoodAWACK atoms admit skeleton normal form

Lemma 3.2 (Lemma BGS.1)**.** *Every tagged terminal GoodAWACK atom produced by the chain*

$$B1 \to B3 \to F3/F4$$

admits an admissible B1-to-GoodAWACK skeleton normal form $\mathfrak{S}$*, and can be written as (BGS).*

Proof. Start with the parent B1 block. By Lemma B1, it has the product equation (B1), finitely many variables, dyadic weights, and coefficient types μ, 1, and log.

By Lemma B3, the block receives a finite set of grouping alternatives. Choose the grouping history that leads to the given terminal descendant. This supplies Γ.

By Lemma F3, every routing step is one of the allowed finite routing-level operations: controlled CRT absorption, F4 large-divisor decision, square-divisor routing, finite grouping selection/elimination, LocalDiag detection, or strict Edge detection. Since the atom is terminal GoodAWACK, all Edge, CKP, LongAP/Local, and LocalDiag outcomes have been checked and eliminated for this descendant, and no unresolved ordinary large divisor predicate remains.

By Lemma F4, any ordinary large divisor or quotient equation that survives without becoming Edge, LocalDiag, or CKP is absorbed into a central-long affine residual with controlled content. The content quotient lemma ensures that the quotient forms remain controlled on the active lattice.

By Lemma E5, controlled content is stable under CRT restriction, fixed divisor absorption, primitive slicing, clean full-rank affine regrouping, and Cauchy/cube operations. Therefore the active forms that reach E10 are affine forms of controlled content, and every rank-changing operation appearing in the skeleton record carries one of the explicit routing tags listed above.

Finally, by the terminal GoodAWACK predicate in the F3/F4 interface, at least one active affine form carries a Liouville-type oscillatory coefficient, the active directions are long, and the atom has the model form (BGS).

Collecting the parent block, grouping, routing history, active lattice, affine system, marked forms, origin map and weights gives $\mathfrak{S}$. Lemma proved.

—

□

BGS.8. Setup: Tensor Interface on a Skeleton For each active affine form

$$L_\rho(z) = \ell_\rho \cdot z + c_\rho,$$

define

$$Q_\rho = \ell_\rho \odot \ell_\rho \in \mathrm{Sym}^2(\mathbb{Q}^{k_{\mathfrak{S}}}).$$

The TC1/HighTC test on the skeleton is:

$$\exists m \in \mathcal{M}_{\mathfrak{S}} \quad Q_m \notin \mathrm{span}_{\mathbb{Q}}\{Q_\rho : \rho \neq m\} \tag{TC1-Skel}$$

for TC1, and

$$\forall m \in \mathcal{M}_{\mathfrak{S}} \quad Q_m \in \mathrm{span}_{\mathbb{Q}}\{Q_\rho : \rho \neq m\} \tag{HighTC-Skel}$$

for HighTC.

Equivalently, HighTC supplies for each marked m a relation

$$\sum_{\rho \in \mathcal{I}_{\mathfrak{S}}} c_\rho Q_\rho = 0, \qquad c_m \neq 0. \tag{H-cert}$$

Because $k_{\mathfrak{S}}$ and $|\mathcal{I}_{\mathfrak{S}}|$ are bounded in terms of J_0, this is a bounded-size rational row-reduction problem for each fixed skeleton.

—

BGS.9. Output: What This Normal Form Resolves The skeleton normal form resolves three issues needed before HGO.2:

1. it separates the parent product equation from the terminal affine system;
2. it records the origin of each affine form, so a HighTC certificate can be tested for CKP/LocalDiag/Edge meaning rather than treated as a bare linear-algebra relation;
3. it makes clear that HighTC testing is finite and terminal, hence it does not create a new routing loop.

In particular:

$$\text{HighTC}$$

is now a certificate attached to a concrete skeleton

$$\mathfrak{S},$$

not an undefined terminal branch.

—

BGS.10. Scope Boundary and Structural Closure Lemma BGS proves the skeleton normal form. It does not by itself prove that all HighTC certificates reroute to CKP, LocalDiag, Edge, or Impossible. That structural conclusion is supplied by HGO2R, E10M, E10K, and E10L.

The reason is that the BGS normal form records the finite-parametric grammar of admissible skeletons, rather than a literal list of all symbolic skeleton instances. It proves that the skeleton set is finite for fixed J_0, but it does not by itself list:

1. all possible affine coefficient vectors ℓ_ρ;
2. all possible quotient-origin maps;
3. all symbolic dependencies among the ℓ_ρ;
4. all conditions under which a HighTC certificate forces CKP, LocalDiag, Edge, or impossibility.

Moreover, some skeleton entries may depend on controlled divisor/CRT parameters. These parameters are polylogarithmically bounded in size, but for structural proof they must be treated symbolically, not by numerical enumeration.

Thus the output of BGS is the finite-parametric input for the subsequent finite-grammar theorem.

—

BGS.11. Structural Use

Theorem 3.3 (Theorem BGS/HGO.2)**.** *For every admissible B1-to-GoodAWACK skeleton $\mathfrak{S}$, if $\mathfrak{S}$ has a HighTC certificate (H-cert), then one of the following holds:*

1. *the origin map* $\mathrm{orig}_{\mathfrak{S}}$ *exposes an admissible balanced bilinear grouping, so the atom is CKP;*
2. *the certificate forces equality, proportionality, fixed gcd-local dependence, or an H4-admissible canonical local projection, so the atom is LocalDiag;*
3. *the certificate forces short residual volume, large content/gcd, or another strict C1P saving predicate, so the atom is Edge;*
4. *the certificate is incompatible with the parent product equation (B1), the dyadic central-long constraints, and the routing history* $\mathfrak{r}$*.*

This theorem is supplied by the HGO2R/E10 closure chain. Once it is combined with the BGS normal form, one obtains:

$$R_{\text{HighTC-GoodAWACK}}(N) = 0$$

after rerouting, and Branch B closes using the already proved TC1 Fourier lemma.

The role of Lemma BGS is to make the statement finite and symbolic; the exclusion of untagged free-affine HighTC skeletons is handled by E10Y/E10X/E10K.

—

BGS.12. Output for the Proof Tree

$$\boxed{\text{B1-to-GoodAWACK skeleton normal form proved.}}$$

The normal form is:

$$\mathfrak{S} = (\mathcal{B}, \Gamma, \mathfrak{r}, \Lambda_{\mathfrak{S}}, \Omega_{\mathfrak{S}}, \mathcal{L}_{\mathfrak{S}}, \mathcal{M}_{\mathfrak{S}}, \mathrm{orig}_{\mathfrak{S}}, \mathcal{W}_{\mathfrak{S}})$$

with terminal atom model (BGS).

This is sufficient to formulate HGO.2 precisely as a finite-parametric skeleton theorem and to pass the problem to HGO2R/E10Y/E10X/E10K/E10L.

BGS.13. Logical dependencies Internal dependencies: B1, B3, F3, F4, and E5.

Children served: HGO2R, E10M, E10K, E10L.

4 Part 3. HGO2R: HighTC obstruction reduction

Source file: `Lemmas/bgs_hgo2_reduction_ltx.md`.

4.0.1 HGO2R. Reduction of BGS/HGO.2 to Free-Affine HighTC Exclusion

HGO2R.0. Statement and Role Lemma HGO2R proves the origin-degenerate rerouting statement HGO2R.1. The free-affine class is treated by the finite-grammar closure theorems E10Y, E10M, E10X, E10K, and E10L.

The structural block is:

$$\text{HighTC-cert} \Longrightarrow \text{CKP} \sqcup \text{LocalDiag} \sqcup \text{Edge} \sqcup \text{Impossible} \tag{HGO.2}$$

for admissible B1-to-GoodAWACK skeletons in the sense of Lemma BGS.

The BGS/HGO2R part proves this implication for HighTC certificates whose quadratic dependence is visible through the recorded origin map. The remaining free-affine case is delegated to E10Y/E10X/E10K.

The reduction statement is:

$$\boxed{\text{HGO.2 reduces to excluding FreeAffine-HighTC skeletons from actual B1 descendants.}}$$

Thus HGO2R is a reduction theorem with an explicit structural-closure dependency.

Logical dependencies: BGS, C1, the CKP branch, the H4 LocalDiag admission criterion, E10Y, E10X, E10K, and E10L. Outputs served: E10M, E10K, and E10L.

—

HGO2R.1. Setup: Starting Point Let

$$\mathfrak{S} = (\mathcal{B}, \Gamma, \mathfrak{r}, \Lambda_{\mathfrak{S}}, \Omega_{\mathfrak{S}}, \mathcal{L}_{\mathfrak{S}}, \mathcal{M}_{\mathfrak{S}}, \mathrm{orig}_{\mathfrak{S}}, \mathcal{W}_{\mathfrak{S}})$$

be an admissible B1-to-GoodAWACK skeleton.

Write

$$L_\rho(z) = \ell_\rho \cdot z + c_\rho, \qquad Q_\rho = \ell_\rho \odot \ell_\rho.$$

Assume that $\mathfrak{S}$ is HighTC. Then for every marked form $m \in \mathcal{M}_{\mathfrak{S}}$ there is an integer relation

$$\sum_{\rho \in \mathcal{I}_{\mathfrak{S}}} c_\rho Q_\rho = 0, \qquad c_m \neq 0. \tag{H-cert}$$

The question is whether (H-cert), together with the origin map $\mathrm{orig}_{\mathfrak{S}}$, forces terminal rerouting.
—

HGO2R.2. Setup: Degenerate-Origin Certificates Call a HighTC certificate origin-degenerate if at least one of the following is forced by the support of the relation and the origin data of the participating forms.

D1. Repeated or proportional affine origin There are $\rho \neq \sigma$ in the support of (H-cert) such that the current lattice forces

$$L_\rho = aL_\sigma + b$$

with fixed rational a, b, or the corresponding parent/product factors are repeated after quotienting.

D2. Fixed gcd-local dependence The forms in the support of (H-cert) are tied by a fixed divisor/gcd relation recorded in $\mathfrak{r}_{\mathrm{div}}$ or $\mathfrak{r}_{\mathrm{quot}}$, so that one active form is determined by another on a fixed local residue class.

D3. Balanced multiplicative origin The support of (H-cert) splits through $\mathrm{orig}_{\mathfrak{S}}$ into two long grouped multiplicative variables on each side of the parent B1 equation, with the B3 balance predicates required by Lemmas B3 and F3.

D4. Strict saving origin The certificate forces one of the strict C1P predicates: short residual volume, large gcd/content budget, square-divisor budget, Type-I error budget, high-frequency budget, or small-conductor budget with the full ambient normalization.

D5. Parent incompatibility The certificate forces a linear or congruence relation incompatible with the parent B1 product equation, the dyadic scale cell, or the current CRT lattice. Then the skeleton has empty support.

The complementary case is called free-affine:

$$\mathrm{FreeAffineHighTC}(\mathfrak{S})$$

if (H-cert) holds but none of D1–D5 is forced by the current recorded origins.
—

HGO2R.3. Lemma: origin-degenerate HGO.2

Lemma 4.1 (Lemma HGO2R.1)**.** *Let $\mathfrak{S}$ be an admissible B1-to-GoodAWACK skeleton with a HighTC certificate. If the certificate is origin-degenerate, then the corresponding terminal atom reroutes to one of:*

$$\text{CKP}, \qquad \text{LocalDiag}, \qquad \text{Edge}, \qquad \text{Impossible}.$$

Proof. We use the cases D1–D5.

In case D1, the current atom contains a forced equality, proportionality, or repeated factor after quotienting. This is exactly within the terminal LocalDiag predicate of Lemma F3, provided the resulting contribution is a canonical local term. The LPI admission condition consumed by H4 is satisfied because the relation is tagged by the parent B1 block and routing history. Hence the atom is LocalDiag.

In case D2, the fixed gcd/divisor data determine one active form from another on the current lattice. This is the fixed local dependence case of Lemmas F3 and F4. Again the term is admitted only as a tagged canonical local projection, so it is LocalDiag rather than an arbitrary local-looking term.

In case D3, the origin map exposes a balanced finite-convolution bilinear structure. By the B3 CKP candidate criterion and the F3 CKP terminal predicate, this is a CKP atom. The coefficient and content conditions are preserved by Lemmas F4 and E5. The CKP estimate and canonical zero-frequency normalization are handled by Lemma G8a.

In case D4, the certificate forces one of the strict C1P saving predicates. By Lemma C1, ordinary large-divisor or small-conductor labels alone are not enough; but D4 assumes the full strict budget. Therefore the atom is terminal Edge and contributes $o(N)$.

In case D5, the active lattice/domain is empty, or the putative skeleton is incompatible with the tagged B1 block. The contribution is zero. It can be recorded as Edge-zero or Impossible.

These cases cover all origin-degenerate certificates. Lemma proved.

—

□

HGO2R.4. Scope Boundary The origin-degenerate lemma leaves open the case where the quadratic tensor relation is a genuine higher-true-complexity relation among distinct primitive affine forms, with no forced local dependence, no balanced multiplicative origin, no strict saving predicate, and no parent incompatibility visible from the recorded origin data.

This is a genuine structural boundary of HGO2R. There is a standard model.

Let

$$L_0(x,r)=x, \qquad L_1(x,r)=x+r, \qquad L_2(x,r)=x+2r, \qquad L_3(x,r)=x+3r. \tag{4AP}$$

The homogeneous coefficient vectors are

$$\ell_i=(1,i)\in\mathbb{Z}^2, \qquad 0\le i\le 3.$$

Their quadratic tensors satisfy

$$Q_0-3Q_1+3Q_2-Q_3=0. \tag{4AP-cert}$$

Indeed, this is the system

$$1-3+3-1=0, \qquad 0-3+6-3=0, \qquad 0-3+12-9=0$$

for the (x^2,xr,r^2) tensor coordinates.

Thus if any L_i is a marked Liouville-type form, the marked tensor lies in the span of the others. The skeleton is HighTC.

But the 4AP pattern has:

1. no equality or proportionality among the forms;
2. no fixed gcd-local dependence;
3. no balanced CKP bilinear multiplicative structure;
4. no short-volume or large-content saving by itself;
5. no contradiction with being central-long affine.

Therefore, under the safe F3/H4 interpretation of LocalDiag as a genuine canonical local term, the relation (4AP-cert) is not a LocalDiag certificate.

There is a related linear dependence

$$L_0 - 3L_1 + 3L_2 - L_3 = 0.$$

If the broad B3 phrase "affine dependence among active forms" were read literally as automatic LocalDiag, this pattern would be routed away. That reading is not part of the proof: Lemma H4 admits LocalDiag only when the term is a tagged canonical local projection, not merely because an affine identity exists among oscillatory forms. Thus 4AP-like free-affine patterns cannot be dismissed by the broad B3 phrase unless an additional canonical-local admission proof is supplied.

This is exactly the interface true-complexity obstruction isolated by the GoodAWACK TC1/HighTC analysis.

—

HGO2R.5. Interface Example: Formal Free-Affine Skeleton The BGS normal form is broad enough to allow the following formal skeleton unless the B1-origin exclusion lemma is used.

Let the active lattice be a two-dimensional central-long box

$$\Omega = \{(x, r) : x \asymp X,\ r \asymp R,\ x + 3r \asymp X,\ X, R \geq N^{\theta}\}.$$

Let

$$\mathcal{L} = \{x, x + r, x + 2r, x + 3r\}, \qquad \mathcal{M} = \{x\}.$$

Let the remaining three forms be auxiliary bounded coefficient forms, and let the weight be smooth and divisor-bounded. All affine contents are 1.

This formal skeleton satisfies the explicit GoodAWACK-style features:

1. central-long affine structure;
2. bounded affine complexity;
3. at least one marked Liouville-type form;
4. controlled content;
5. no unresolved ordinary large divisor predicate;
6. no strict Edge predicate;
7. no CKP-balanced multiplicative form;

8. no H4-admissible LocalDiag relation.

It is HighTC by (4AP-cert).

Lemma HGO2R is the origin-degenerate part of the HGO.2 route. The formal skeleton above is the free-affine class isolated by the interface. That class is routed to E10Y/E10X/E10K, which supply the actual-origin exclusion needed to complete full HGO.2 for terminal GoodAWACK descendants.

—

HGO2R.6. Free-Affine Exclusion Full HGO.2 is equivalent, over the origin-degenerate lemma proved here, to the following finite structural exclusion.

Lemma 4.2 (Lemma HGO2R.2. No free-affine HighTC skeletons)**.** *For every admissible B1-to-GoodAWACK skeleton $\mathfrak{S}$ produced by the chain*

$$B1 \to B3 \to F3/F4,$$

every HighTC certificate is origin-degenerate in the sense of Section HGO2R.2. Equivalently:

$$\mathrm{FreeAffineHighTC}(\mathfrak{S}) \quad \textit{never occurs for actual B1 descendants.} \tag{NoFAH}$$

This exclusion is supplied by E10Y/E10X/E10K.

—

HGO2R.7. Consequence for E10 The E10 decomposition after the TC1 Fourier closure is:

$$R_{\text{GoodAWACK}}(N) = R_{\text{TC1-GoodAWACK}}(N) + R_{\text{HighTC-GoodAWACK}}(N),$$

with

$$R_{\text{TC1-GoodAWACK}}(N) = o(N).$$

By Lemma HGO2R.1, the origin-degenerate part of HighTC reroutes to already handled branches:

$$R_{\text{HighTC,deg}}(N) \subseteq \mathrm{CKP} \sqcup \mathrm{LocalDiag} \sqcup \mathrm{Edge} \sqcup \mathrm{Impossible}.$$

Thus the only obstruction left by HGO2R alone is

$$R_{\text{FreeAffineHighTC}}(N).$$

This class is empty by E10Y/E10X/E10K, so HGO2R supplies the origin-degenerate HighTC rerouting component of Branch B.

—

HGO2R.8. Output for the Proof Tree

$$\boxed{\text{Origin-degenerate HighTC reroutes to CKP, LocalDiag, Edge, or Impossible.}}$$

The NoFAH/free-affine class is closed by E10X and E10K.

What is proved here:

$$\text{OriginDegenerateHighTC} \Longrightarrow \text{CKP} \sqcup \text{LocalDiag} \sqcup \text{Edge} \sqcup \text{Impossible}.$$

The surviving free-affine class is not a terminal output of Branch B; it is passed to the downstream E10Y/E10X/E10K/E10L finite-grammar layer.

HGO2R.9. Logical dependencies Internal dependencies: BGS, C1, CKP branch, LocalDiag/LPI routing, and the F3/F4 terminal routing interface.

Children served: E10M, E10K, E10L.

5 Part 4. BAOC: Affine-origin catalogue

Source file: `Lemmas/baoc_affine_origin_catalogue_ltx.md`.

5.0.1 BAOC. B1 Affine-Origin Catalogue

BAOC.0. Statement and Role Lemma BAOC supplies the B1/B3/F3/F4 transport catalogue for homogeneous coefficient vectors in terminal GoodAWACK skeletons. It is a provenance grammar, not the final no-free-affine theorem. The decisive exclusion of untagged rank-dropping AFF is supplied by E10Y, E10X, E10M, E10K, and E10L.

The statement is:

$$\boxed{\text{every terminal GoodAWACK affine form is generated by a finite B1/B3/F3/F4 transport grammar.}}$$

More precisely, for every terminal GoodAWACK skeleton $\mathfrak{S}$ and every active affine form $L_\rho(z) = \ell_\rho \cdot z + c_\rho$, the vector ℓ_ρ is obtained from B1/B3 grouped coordinates by finitely many of the transport rules T1–T7 below.

BAOC also records the scope boundary of this catalogue: provenance alone does not decide every true-complexity relation among the ℓ_ρ. The free-affine class isolated by that boundary is routed to E10Y/E10X/E10K/E10L.

Logical dependencies: B1, B3, F3, F4, E5, BGS, HGO2R, E10Y, E10X, E10M, E10K, and E10L. Outputs served: HGO2R, E10Y, E10M, E10K, E10L, and E10X.

—

BAOC.0a. Setup: What F4 Already Proves Lemma F4 already proves the core ordinary-divisor part of BAOC.

In F4.1, every ordinary large-divisor predicate has one of the forms

$$d \mid L(z), \qquad L(z) = ds, \qquad d \mid \gcd(L_1(z), L_2(z)),$$

where L, L_1, L_2 are affine or product-grouped forms already produced by B1/B3.

F4.3 records fixed-divisor absorption:

$$\Lambda_d = \{z \in \Lambda : L(z) \equiv 0 \pmod d\}, \qquad L_d(z) = L(z)/d.$$

F4.4 proves the content quotient formula

$$\mathrm{cont}_{\Lambda_d}(L/d) = \frac{g}{(g,d)} \leq g.$$

F4.5–F4.11 handle the variable quotient equation

$$L(z) = ds$$

and prove the exhaustive alternative:

$$\text{OrdinaryLargeDivisor} \Longrightarrow \text{Edge} \sqcup \text{LocalDiag} \sqcup \text{CKP} \sqcup \text{GoodAWACK},$$

or else the corrected F3 measure strictly decreases.

Thus the BAOC transport rules T3 and T4 below are not new results. They are F4 translated into homogeneous-vector bookkeeping.

What F4 does not do is decide every terminal true-complexity relation among the final active list of vectors $\{\ell_\rho\}$. That structural decision is made by E10Y/E10X/E10K/E10L.

—

BAOC.1. Setup: Coefficient-Vector Bookkeeping Let a terminal GoodAWACK skeleton be

$$\mathfrak{S} = (\mathcal{B}, \Gamma, \mathfrak{r}, \Lambda_{\mathfrak{S}}, \Omega_{\mathfrak{S}}, \mathcal{L}_{\mathfrak{S}}, \mathcal{M}_{\mathfrak{S}}, \mathrm{orig}_{\mathfrak{S}}, \mathcal{W}_{\mathfrak{S}}).$$

Every active affine form is written on the active parameter lattice as

$$L_\rho(z) = \ell_\rho \cdot z + c_\rho.$$

BAOC must catalogue the homogeneous vectors

$$\ell_\rho \in \mathbb{Z}^{k_{\mathfrak{S}}}$$

with enough origin data to decide whether a quadratic tensor relation

$$\sum_\rho c_\rho(\ell_\rho \odot \ell_\rho) = 0 \tag{QRel}$$

is:

1. CKP-origin;
2. H4-admissible LocalDiag-origin;
3. strict C1P Edge-origin;
4. impossible;
5. or genuinely free-affine.

Only cases 1–4 close HGO.2 structurally.

—

BAOC.2. Setup: Base B1 and B3 Coordinate Source Lemma B1 supplies a parent product block

$$\prod_{i=1}^{r} a_i + \prod_{j=1}^{s} b_j = N, \qquad r, s \le 2J_0. \tag{B1}$$

Lemma B3 supplies a finite grouping set. For a grouping

$$\Gamma = (I, J)$$

one has grouped variables

$$u_I = \prod_{i \in I} a_i, \qquad v_I = \prod_{i \notin I} a_i,$$

and

$$u'_J = \prod_{j \in J} b_j, \qquad v'_J = \prod_{j \notin J} b_j,$$

with grouped parent relation

$$u_I v_I + u'_J v'_J = N. \tag{B3}$$

At the affine-origin bookkeeping level, a selected grouped variable is treated as a coordinate in a finite free coordinate system attached to $(\mathcal{B}, \Gamma)$. Thus the base coefficient vectors are coordinate vectors

$$e_\alpha$$

for surviving grouped variables and residual product variables.

This is a change of coordinates, not an assertion that e_α is a linear form in the original factor variables a_i, b_j. Product grouping is multiplicative, and the affine catalogue starts after a grouping has been chosen and the descendant has entered an affine/WACLE regime.

—

BAOC.3. Setup: Allowed Coefficient Transport Rules The B1/B3/F3/F4/E5 lemmas support the following transport grammar for homogeneous coefficient vectors.

T1. Fixing and projection When some grouped variables are fixed by dyadic slicing, congruence slicing, or conditioning, they become constants. Homogeneous vectors are projected to the remaining active coordinates.

If

$$z = (z_{\text{free}}, z_{\text{fixed}})$$

and

$$L(z) = \ell_{\text{free}} \cdot z_{\text{free}} + \ell_{\text{fixed}} \cdot z_{\text{fixed}} + c,$$

then the transported vector is

$$\ell \mapsto \ell_{\text{free}}.$$

T2. Controlled CRT restriction F3 controlled CRT absorption replaces a lattice coset by a subcoset

$$\Lambda' = \{z \in \Lambda : L_0(z) \equiv a \pmod q\}, \qquad q \le (\log N)^C.$$

Choosing coordinates

$$z = z_0 + Tz'$$

on the sublattice transports

$$\ell \mapsto T^t \ell. \tag{CRT}$$

E5 proves that controlled content remains controlled. The particular matrix T is part of the skeleton provenance data.

T3. Fixed divisor quotient If a fixed divisor condition

$$d \mid L(z)$$

is absorbed and the quotient form survives, then on the restricted lattice

$$L_d(z) = L(z)/d.$$

On homogeneous vectors this gives

$$\ell \mapsto \frac{1}{d} T^t \ell, \tag{FDQ}$$

where the right side is integral on the restricted coordinate lattice. E5/F4 prove content does not increase. The triple (d, T, ℓ) is recorded as part of the quotient-origin data.

T4. Variable quotient residual For an ordinary quotient equation

$$L(z) = ds, \tag{VQ}$$

F4 either routes to Edge, LocalDiag, CKP, or leaves a central-long affine GoodAWACK quotient form.

If s survives as an active quotient form, then in the extended active coordinate system its homogeneous vector ℓ_s satisfies

$$d\ell_s = T^t \ell_L. \tag{VQT}$$

This relation is part of the origin record. If it forces local dependence, the atom is LocalDiag; if it exposes balanced multiplicative structure, it is CKP; if it gives a strict C1P budget, it is Edge. Otherwise it may feed GoodAWACK.

T5. Bounded affine regrouping E5 permits bounded affine changes and regrouping:

$$z = z_0 + Az',$$

where relevant coefficients and minors are bounded by powers of $\log N$. The vector transport is

$$\ell \mapsto A^t \ell. \tag{AFF}$$

This preserves controlled content, but by itself it does not decide true complexity.

T6. Primitive slicing Primitive slicing chooses a long one-dimensional fibre

$$z = z_0 + uv.$$

On the fibre a marked form becomes

$$L(z_0 + uv) = gu + b, \qquad g = \ell(v).$$

For BAOC, the pre-slicing vector ℓ remains the object used in the TC1/HighTC tensor test. The fibre coefficient g is used analytically by E7/E9.

T7. Auxiliary bounded forms Auxiliary bounded or smooth coefficient forms inherit their homogeneous vectors through the same transport rules T1–T6.

No routing step is allowed to introduce an affine form without one of these provenance operations.

—

BAOC.4. Statement and Proof: Transport-Level Catalogue

Theorem 5.1 (Theorem BAOC.1. Transport-level affine-origin catalogue)**.** *Every active affine form L_ρ in a terminal GoodAWACK skeleton produced by*

$$B1 \to B3 \to F3/F4$$

has a homogeneous vector ℓ_ρ generated from the B1/B3 grouped-coordinate source by finitely many applications of T1–T7.

Equivalently, for every terminal skeleton $\mathfrak{S}$ there exists a finite provenance expression

$$\ell_\rho \in \mathcal{C}_{\mathrm{tr}}(\mathcal{B}, \Gamma, \mathfrak{r})$$

for each active form, where $\mathcal{C}_{\mathrm{tr}}$ is the transport closure generated by T1–T7.

Proof. Start with the B1 parent block. By Lemma B1, the only initial variables are finitely many product variables a_i, b_j, with $r, s \le 2J_0$.

Choose the B3 grouping history Γ. By Lemma B3, the grouping set is finite and each grouping replaces products of parent variables by grouped variables u_I, v_I, u'_J, v'_J. At the affine bookkeeping level these grouped variables supply the base coordinate vectors.

F3 allows only controlled CRT absorption, F4 large-divisor decision, finite grouping selection/elimination, terminal LocalDiag detection, terminal Edge detection, and terminal labelling. Controlled CRT absorption transports coefficient vectors by T2.

F4 handles fixed divisor and variable quotient equations. F4.3–F4.4 give fixed divisor absorption and quotient content control, which are T3. F4.5–F4.11 give the exhaustive variable quotient decision, which is T4 together with the alternatives Edge, LocalDiag, CKP, and GoodAWACK. If the quotient relation creates forced local dependence, balanced multiplicative structure, or strict saving, the atom is no longer terminal GoodAWACK; it is LocalDiag, CKP, or Edge. Hence any quotient form that reaches GoodAWACK is exactly a T4 quotient form.

E5 records the permitted affine regrouping, primitive slicing, and content-stability operations. These are T5 and T6. Auxiliary forms inherit the same transport data, giving T7.

Since the F3 measure is well-founded and every routing history is finite, only finitely many transport steps occur. Therefore every terminal GoodAWACK homogeneous vector lies in the transport closure $\mathcal{C}_{\mathrm{tr}}(\mathcal{B}, \Gamma, \mathfrak{r})$. This proves the transport catalogue.

—

□

BAOC.5. Scope Boundary Relative to NoFAH BAOC is a provenance grammar, not the final no-free-affine closure theorem.

By itself it does not decide:

1. whether a given affine regrouping is tagged or untagged in the E10M sense;
2. whether a HighTC relation among the $\ell_\rho \odot \ell_\rho$ is origin-degenerate;
3. whether a formal free-affine pattern is impossible for actual B1 descendants.

Consequently, BAOC alone cannot exclude the formal free-affine pattern

$$\ell_0 = (1, 0), \quad \ell_1 = (1, 1), \quad \ell_2 = (1, 2), \quad \ell_3 = (1, 3). \tag{4AP-vectors}$$

These vectors have bounded coefficients and controlled content. Their tensors satisfy

$$Q_0 - 3Q_1 + 3Q_2 - Q_3 = 0.$$

The BAOC transport grammar alone does not contain a rule saying that a bounded affine family of this shape cannot be produced by T1–T7.

Thus BAOC proves provenance, while NoFAH is supplied by E10Y/E10X/E10K/E10L.

—

BAOC.6. Structural Closure Needed for NoFAH The structural closure must add a tagged-origin classification to the transport grammar.

Lemma 5.2 (Lemma BAOC.2. Tagged B1 affine-origin closure)**.** *For every parent block $\mathcal{B}$, grouping Γ, and terminal routing history $\mathfrak{r}$, the transport closure T1–T7 can be refined to a finite-parametric list*

$$\mathfrak{F}(\mathcal{B}, \Gamma, \mathfrak{r}) = \{\mathcal{F}_\nu(\mathbf{p}) : \nu \in \mathcal{N}, \ \mathbf{p} \in \mathcal{P}_\nu\},$$

where each $\mathcal{F}_\nu(\mathbf{p})$ gives:

1. *a concrete active coordinate lattice;*
2. *concrete transport matrices T, A;*
3. *concrete quotient-origin relations;*

4. *the full active affine vector list $\{\ell_\rho\}$;*
5. *the marked set $\mathcal{M}$;*
6. *the terminal rejection data for Edge, CKP, LocalDiag, and LongAP/Local.*

Moreover, for every listed family, every HighTC certificate is origin-degenerate, impossible, or routed by the E10M/E10K/E10L no-untagged-AFF closure.

Then NoFAH follows.

Proof that the tagged closure implies NoFAH. Let $\mathfrak{S}$ be an actual terminal GoodAWACK skeleton with a HighTC certificate. The tagged closure places its coefficient vectors in one of the listed families $\mathcal{F}_\nu(\mathbf{p})$. The final clause says that the certificate is origin-degenerate, impossible, or excluded by the no-untagged-AFF closure. Therefore $\mathfrak{S}$ is not FreeAffineHighTC. This proves NoFAH.

By HGO2R, NoFAH implies full HGO.2.

—

□

BAOC.7. Scope Boundary: Alternative Route Through B3/H4 There is a different possible strong input:

$$\boxed{\text{Every B3 affine-dependence flag is H4-canonical.}}$$

If this were proved, then many free-affine patterns, including the 4AP identity

$$L_0 - 3L_1 + 3L_2 - L_3 = 0,$$

could be safely routed to LocalDiag.

This alternative is not used here. Lemma H4 deliberately admits only canonical local projections tagged by the parent B1 cell and routing history. A bare affine identity among oscillatory forms is not enough.

Thus the B3/H4 route would require a separate admission theorem. The proof instead uses E10Y/E10X/E10K/E10L.

—

BAOC.8. Output for Branch B The Branch B chain is:

$$\text{GoodAWACK} = \text{TC1} \sqcup \text{OriginDegenerateHighTC} \sqcup \text{FreeAffineHighTC}.$$

The TC1 part is closed by Lemma TNG, which packages TGT, MRT, TTD, ROC, BRS, TTH, and X9L-GT in the near-global form. The second part is rerouted by HGO2R.

The remaining part is

$$R_{\text{FreeAffineHighTC}}(N).$$

BAOC records this class as a structural catalogue boundary. The class is not left to BAOC alone: E10M proves that no untagged rank-dropping affine origin survives, E10K converts this into AFF-origin completeness, and E10L assembles the resulting GoodAWACK cancellation.

—

Remark 5.3 (BAOC.9. Output).

$$\boxed{\text{BAOC proves the transport catalogue; the no-free-affine closure is supplied by E10Y/E10X/E10K/E10L.}}$$

Established output:

$$\ell_\rho \in \mathcal{C}_{\mathrm{tr}}(\mathcal{B}, \Gamma, \mathfrak{r})$$

for every terminal GoodAWACK active affine form.
Structural closure:

$$\boxed{\text{The free-affine class is discharged by E10Y/E10X/E10K/E10L.}}$$

Structural completion block:

$$\boxed{\text{E10Y/E10X/E10K enumerate and classify the relevant rank-dropping affine origins.}}$$

This is the structural task supplied by the E10X closure chain.

6 Part 5. E10G: Strong BAOC catalogue

Source file: `Lemmas/e10g_strong_baoc_catalogue_ltx.md`.

6.0.1 E10G. Strong BAOC Catalogue and Reduction

E10G.0. Statement and Role Lemma E10G supplies a finite catalogue schema and identifies the FreeAffineHighTC obstruction that is discharged by the finite GoodAWACK grammar closure Lemmas E10Y and E10X. It is not used as an independent proof of strong BAOC. Its role is to reduce the formal catalogue class to the actual-origin closure theorem E10Y/E10X, after which E10K gives AFF-origin completeness and E10L assembles the GoodAWACK estimate.

E10G treats the strong form of BAOC isolated by Lemma BAOC.

The desired structural statement is:

$$\boxed{\text{every actual terminal GoodAWACK affine-vector family has no FreeAffineHighTC certificate.}}$$

Equivalently, for every terminal GoodAWACK skeleton

$$\mathfrak{S} = (\mathcal{B}, \Gamma, \mathfrak{r}, \Lambda_{\mathfrak{S}}, \Omega_{\mathfrak{S}}, \mathcal{L}_{\mathfrak{S}}, \mathcal{M}_{\mathfrak{S}}, \mathrm{orig}_{\mathfrak{S}}, \mathcal{W}_{\mathfrak{S}})$$

produced by

$$B1 \to B3 \to F3/F4,$$

every HighTC relation

$$\sum_\rho c_\rho(\ell_\rho \odot \ell_\rho) = 0, \qquad c_m \neq 0$$

with $m \in \mathcal{M}_{\mathfrak{S}}$ should be forced into one of:

$$\text{CKP}, \qquad \text{LocalDiag}, \qquad \text{Edge}, \qquad \text{Impossible}.$$

The outcome is a precise split.

$$\boxed{\text{A finite-parametric catalogue compiler is available, and the decisive actual-origin rigidity is supplied by E10Y/E}}$$

More concretely:

1. B1/B3/F3/F4/E5 give a finite-parametric **transport catalogue schema** for all terminal GoodAWACK coefficient vectors.
2. For every fixed catalogue cell, the TC1/HighTC tensor test is a finite symbolic row-reduction problem.
3. The broad affine-regrouping/CRT transport interface admits formal 4AP-like FreeAffine-HighTC vector patterns, so the proof routes those formal witnesses to the actual-origin closure theorem E10Y/E10X.

Thus E10G supplies the finite catalogue schema and identifies the closure input supplied by E10Y/E10X:

$$\boxed{\text{prove transport rigidity for the actual affine-regrouping matrices, or route every resulting free affine dependence}}$$

—

E10G.1. Setup: Source Data Already Proved We use only the already established source lemmas.

B1 source Lemma B1 supplies a typed finite-convolution parent block

$$P_A(a) + P_B(b) = N, \qquad P_A(a) = \prod_{i=1}^{r} a_i, \qquad P_B(b) = \prod_{j=1}^{s} b_j, \qquad r, s \le 2J_0. \tag{B1}$$

The parent block has finitely many dyadic scale and coefficient-type choices.

B3 source Lemma B3 supplies a finite grouping set

$$\mathcal{G}(\mathcal{B}), \qquad |\mathcal{G}(\mathcal{B})| \ll_{J_0} 1,$$

and preliminary labels:

$$\text{TypeI/Edge}, \quad \text{LongAP/Local}, \quad \text{CKP}, \quad \text{BranchB}, \quad \text{LocalDiag flag}.$$

For the BranchB/GoodAWACK path, all short, purely local, CKP-balanced, and forced-dependence candidates must have failed or have been terminally routed away.

F3/F4 source Lemma F3 defines terminal GoodAWACK by:

1. central-long affine WACLE structure;
2. bounded affine complexity;
3. smooth weight of polylogarithmic complexity;
4. no forced local diagonal relation;
5. no unresolved ordinary large divisor condition;
6. at least one marked affine Liouville-type form with controlled content;
7. long active fibre directions.

Lemma F4 proves the exhaustive ordinary-divisor decision:

$$\mathrm{OrdinaryLargeDivisor} \Longrightarrow \mathrm{Edge} \sqcup \mathrm{LocalDiag} \sqcup \mathrm{CKP} \sqcup \mathrm{GoodAWACK},$$

or else the corrected F3 measure strictly decreases.

The GoodAWACK quotient case is precisely the case where the divisor/quotient ambiguity has been absorbed or resolved and the remaining object is central-long affine with controlled content.

E5 source Lemma E5 proves controlled content stability under:

$$\text{CRT}, \quad \text{fixed divisor absorption}, \quad \text{primitive slicing}, \quad \text{affine regrouping}, \quad \text{Cauchy/cube}, \quad \text{local diagonal extracti}$$

For the present catalogue, the important point is that E5 controls content but does not enumerate the affine transport matrices.

—

E10G.2. Strong catalogue cells Fix a parent block $\mathcal{B}$, a B3 grouping $\Gamma \in \mathcal{G}(\mathcal{B})$, and an F3/F4 routing history $\mathfrak{r}$ that ends in terminal GoodAWACK.

The BAOC grammar can be sharpened into the following finite-parametric catalogue schema.

Cell C0. Base grouped-coordinate cell After choosing $(\mathcal{B}, \Gamma)$, the active pre-routing coordinate module is

$$V_0 = \mathbb{Z}^{k_0}, \qquad k_0 \leq K_0(J_0).$$

The base homogeneous vectors are coordinate vectors

$$e_\alpha \in V_0^*$$

attached to surviving grouped variables and residual product variables.

This cell is finite for fixed J_0.

Cell C1. Projection/fixing cell Fixing dyadic, congruence, or auxiliary variables replaces

$$V \cong V_{\text{free}} \oplus V_{\text{fixed}}$$

by V_{free}. Homogeneous vectors are transported by the projection

$$\pi_{\text{free}} : V^* \to V^*_{\text{free}}.$$

This cell cannot create HighTC relations not already present after restriction; it only deletes coordinates.

Cell C2. Controlled CRT cell A controlled congruence restriction

$$L_0(z) \equiv a \pmod{q}, \qquad q \le (\log N)^C,$$

chooses coordinates

$$z = z_0 + Tz'$$

on the sublattice. Homogeneous vectors are transported by

$$\ell \mapsto T^t \ell. \tag{CRT-T}$$

The determinant and relevant minors of T are polylogarithmically controlled by F3/E5. The actual-origin classification of such matrices is supplied by E10Y/E10X/E10K.

Cell C3. Fixed-divisor quotient cell For

$$d \mid L(z)$$

absorbed on a restricted lattice, the quotient form is

$$L_d(z) = L(z)/d.$$

On coefficient vectors:

$$\ell \mapsto \frac{1}{d} T^t \ell, \tag{FDQ-T}$$

where the right side is integral on the chosen restricted coordinate lattice.

By F4.4/E5.2, content does not increase:

$$\text{cont}_{\Lambda_d}(L/d) = \frac{\text{cont}_\Lambda(L)}{(\text{cont}_\Lambda(L), d)} \le \text{cont}_\Lambda(L).$$

This cell is already controlled by F4 at the transport-catalogue level.

Cell C4. Variable quotient residual cell For a quotient equation

$$L(z) = ds, \tag{VQ}$$

F4 routes to Edge, LocalDiag, CKP, or GoodAWACK.

If it reaches GoodAWACK, then neither short-volume Edge, nor forced local dependence, nor balanced CKP applies. The surviving quotient vector satisfies an origin relation

$$d\ell_s = T^t \ell_L. \tag{VQ-T}$$

The relation (VQ-T) must be retained as part of the strong catalogue cell.

Any HighTC certificate using (VQ-T) in a way that determines one active form from another is origin-degenerate and is already handled by HGO2R.

Cell C5. Bounded affine regrouping cell E5 allows bounded affine changes and regrouping:

$$z = z_0 + Az',$$

with coefficients and relevant minors bounded by powers of $\log N$. Homogeneous vectors transform by

$$\ell \mapsto A^t \ell. \tag{AFF-T}$$

This is the decisive cell. E5 proves controlled content under such transformations. The classification of which matrices A can arise from actual routing is supplied by E10Y/E10X/E10K rather than by E5 itself.

Therefore C5 requires the actual-origin classification supplied by E10Y/E10X/E10K before it can be used inside clean terminal GoodAWACK.

Cell C6. Primitive slicing cell Primitive slicing writes a marked form on a long fibre as

$$L(z_0 + uv) = gu + b.$$

For the true-complexity verification, the relevant vector is the pre-slicing vector ℓ, not merely the one-dimensional coefficient g. This cell supplies the analytic E7/E9 interface but does not by itself decide HighTC.

—

E10G.3. Catalogue compiler theorem

Lemma 6.1 (Lemma E10G.1. Finite-parametric strong-catalogue schema)**.** *Every terminal GoodAWACK skeleton produced by*

$$B1 \to B3 \to F3/F4$$

belongs to a finite-parametric catalogue cell obtained by composing C0–C6.

More explicitly, for each terminal skeleton there are:

1. *a finite B1/B3 source cell* $(\mathcal{B}, \Gamma)$*;*
2. *a finite F3/F4 routing word* $\mathfrak{r}$*;*

3. *controlled integer matrices T_j, A_j;*
4. *quotient-origin equations $d\ell_s = T^t \ell_L$;*
5. *a finite active list $\{\ell_\rho\}$;*
6. *a nonempty marked subset $\mathcal{M}$;*
7. *terminal rejection data recording why Edge, CKP, LocalDiag, and LongAP/Local did not apply.*

For every fixed choice of this symbolic data, TC1/HighTC is decided by a finite rational row-reduction on

$$Q_\rho = \ell_\rho \odot \ell_\rho.$$

Proof. B1 and B3 give finitely many parent/grouping source cells. The number of parent variables and groupings is bounded in terms of J_0.

F3 has a well-founded routing measure $\mathfrak{M}^\sharp$, and its generic routing steps are limited to controlled CRT absorption, F4 large-divisor decision, square-divisor routing, finite grouping selection/elimination, terminal LocalDiag detection, terminal Edge detection, and terminal class labelling. Hence every routing word $\mathfrak{r}$ is finite, with length bounded by the initial obstruction data.

F4 handles every ordinary divisor or quotient predicate. Its fixed-divisor and variable-quotient branches are exactly C3 and C4 when the residual reaches GoodAWACK; otherwise the atom is terminal Edge, LocalDiag, or CKP.

E5 supplies the content-stability rules for CRT restriction, fixed divisor absorption, affine regrouping, primitive slicing, and Cauchy/cube operations. At the homogeneous-vector level these are C1–C6.

Thus every active terminal vector ℓ_ρ is produced by a finite composition of C0–C6, with quotient-origin relations retained. Since the number of active forms and ambient rank are bounded in terms of J_0, the tensor list

$$\{\ell_\rho \odot \ell_\rho\} \subset \mathrm{Sym}^2(\mathbb{Q}^k)$$

has bounded size. Therefore, after fixing a catalogue cell, TC1/HighTC is a finite rational row-reduction problem. Lemma proved.

—

□

E10G.4. Rigidity input for strong BAOC The compiler lemma implies NoFAH once it is combined with the following rigidity statement, supplied by the master closure Lemma E10X and the AFF-OC consequence E10K.

Lemma 6.2 (Lemma E10G.2. Transport-rigidity NoFAH)**.** *For every catalogue cell C0–C6 that is actually produced by the B1/B3/F3/F4 routing history, every HighTC tensor relation*

$$\sum_\rho c_\rho(\ell_\rho \odot \ell_\rho) = 0, \qquad c_m \neq 0$$

with $m \in \mathcal{M}$ is origin-degenerate:

1. *it uses a repeated/proportional source vector;*

2. *or it uses a fixed divisor/gcd/quotient-origin relation;*

3. *or it exposes a B3 CKP-balanced multiplicative grouping;*

4. *or it forces a strict C1P Edge predicate;*

5. *or it is incompatible with the parent cell and routing history.*

If E10G.2 holds, then HGO2R gives:

$$\mathrm{HighTC} \Longrightarrow \mathrm{CKP} \sqcup \mathrm{LocalDiag} \sqcup \mathrm{Edge} \sqcup \mathrm{Impossible},$$

and hence

$$R_{\mathrm{FreeAffineHighTC}}(N) = 0.$$

Branch B closes using Lemma TNG for the global TC1 near-global testing chain plus the origin-degenerate rerouting.

—

E10G.5. Scope Boundary: Free-Affine Class Inside the Catalogue Interface E10G.2 is a reduction target for the master closure Lemma E10X, not a consequence of the catalogue compiler alone.

The free-affine class is not F4's fixed-divisor or variable-quotient analysis. Those parts carry explicit origin equations and are exactly the cases handled by HGO2R.

The obstruction is the combination of:

1. terminal GoodAWACK accepting any bounded-complexity central-long affine system with controlled content and a marked Liouville form, after negative tests fail;

2. E5 allowing bounded affine regrouping with controlled coefficients/minors;

3. the catalogue schema alone does not classify the actual matrices A, T;

4. the catalogue schema alone does not prove that every affine dependence generated by such matrices is H4-canonical LocalDiag.

Because of C5, the catalogue schema still admits the following vector family:

$$\ell_0 = (1,0), \qquad \ell_1 = (1,1), \qquad \ell_2 = (1,2), \qquad \ell_3 = (1,3). \tag{4AP-vectors}$$

Indeed, these arise from the two-coordinate source (x, r) by the bounded affine forms

$$x, \quad x+r, \quad x+2r, \quad x+3r.$$

All coefficients are $O(1)$, all contents are 1, and the affine complexity is bounded.

Their quadratic tensors satisfy:

$$Q_0 - 3Q_1 + 3Q_2 - Q_3 = 0. \tag{4AP-Q}$$

If any L_i is marked, the marked tensor lies in the span of the others. Thus the pattern is HighTC.

At the level of the GoodAWACK interface alone, (4AP-vectors) need not be:

1. Edge by short volume or content;
2. CKP by balanced multiplicative origin;
3. LocalDiag in the H4-canonical sense;
4. impossible from the parent B1 equation.

Therefore the catalogue compiler passes this formal class to the E10X finite-grammar and origin-completeness closure.

This is the same structural interface example as in the NoFAH B1-origin verification, but now localized to a precise catalogue cell: bounded affine regrouping before the E10X rigidity theorem is applied.

—

E10G.6. Closure Route The proof uses Route R1 below, as formalized in E10Y/E10X/E10K/E10L. Routes R2 and R3 are recorded only as alternative sufficient inputs.

Route R1. Affine-regrouping rigidity Strengthen C5 from arbitrary bounded affine regrouping to an explicit rigid list of matrices generated by the actual B1/B3/F3/F4 operations.

One sufficient target would be:

$$A \in \mathcal{A}_{\mathrm{rigid}}(\mathcal{B}, \Gamma, \mathfrak{r}), \qquad |\mathcal{A}_{\mathrm{rigid}}| \ll_{J_0} 1,$$

where every listed matrix is built from coordinate projection, signed permutation, controlled diagonal quotient, CRT basis choice with recorded congruence origin, and incidence maps coming from quotient equations.

Then one must row-reduce each resulting family and prove that every HighTC relation is origin-degenerate or impossible.

Route R2. B3/H4 canonical-local admission Prove that every B3 affine-dependence flag surviving into a terminal GoodAWACK-looking affine system is actually H4-admissible canonical LocalDiag.

This would route patterns such as

$$L_0 - 3L_1 + 3L_2 - L_3 = 0$$

to LocalDiag, but only if the resulting local term is genuinely admitted by H4's tagged canonical local-projection interface.

This route must not use the broad word "affine dependence" alone; it needs an H4 admission proof.

Route R3. Higher-order analytic input An actual 4AP-like free-affine catalogue cell outside the E10Y/E10X/E10K actual-origin closure would require a separate analytic estimate at that cell.

Such an estimate would have to control the surviving HighTC family, for example through a U^3-level or nilsequence orthogonality input with complexity strong enough for the exact catalogue cell.

This route reintroduces a higher-order analytic block, but now with a sharply specified target rather than the whole GoodAWACK class.

—

Remark 6.3 (E10G.7. Output).

E10G proves the finite catalogue schema used by the E10Y/E10X/E10K/E10L closure.

What is proved here:

Every terminal GoodAWACK atom belongs to a finite-parametric catalogue schema C0–C6, and each fixed cell

Structural closure:

The C5/free-affine class is discharged by E10Y/E10X/E10K/E10L.

Completion block:

E10Y/E10X/E10K provide the required affine-regrouping origin completeness.

7 Part 6. E10H: Rigidity interface

Source file: `Lemmas/e10h_e10g_rigidity_ltx.md`.

7.0.1 E10H. Matrix Rigidity Reduction for Strong BAOC

E10H.0. Statement and Role Lemma E10H is a reduction: it localizes the structural issue left by E10G to CRT/AFF matrix-origin rigidity. The resulting reduction is closed by the finite GoodAWACK grammar Lemma E10X, whose proof uses E10I, E10J, E10Y, E10M, and E10K. The E10S records are non-logical reproducibility support, not part of this closure.

E10H treats the next block after Lemma E10G.

The target isolated there was:

E10G-Rigidity: enumerate the actual affine-regrouping/CRT matrices allowed by B1/B3/F3/F4.

The purpose of E10H is to isolate the precise rigidity statement needed from the source lemmas.

The outcome is a sharper reduction:

All non-matrix transport operations are origin-safe; the remaining target is CRT/AFF matrix-origin rigidity.

More precisely, the source lemmas prove enough to control:

1. fixing/projection;
2. fixed divisor quotient;
3. variable quotient residuals, provided quotient-origin equations are retained;
4. primitive slicing as an analytic fibre operation;
5. Cauchy/cube shifts as linear-part preserving operations.

The part isolated for the matrix-origin step is the enumeration of:

1. the basis matrices used in controlled CRT sublattices;
2. the bounded affine regrouping matrices admitted by E5;
3. the full active vector list after these matrices are composed.

Thus E10H reduces E10G-Rigidity to a concrete matrix-origin lemma stated in Section E10H.7, which is discharged by E10X.

—

E10H.1. Rigidity statement Let $\mathfrak{S}$ be a terminal GoodAWACK skeleton from

$$B1 \to B3 \to F3/F4.$$

Write the active affine forms as

$$L_\rho(z) = \ell_\rho \cdot z + c_\rho, \qquad Q_\rho = \ell_\rho \odot \ell_\rho.$$

The E10G-Rigidity statement isolated by this reduction is:

Lemma 7.1 (Lemma E10H.Rig). *For every actual terminal GoodAWACK skeleton $\mathfrak{S}$, the coefficient vectors ℓ_ρ lie in an explicitly enumerated finite-parametric matrix family*

$$\mathcal{R}(\mathcal{B}, \Gamma, \mathfrak{r}),$$

where every matrix in the family carries one of the following origin tags:

1. *coordinate projection/fixing;*
2. *CRT sublattice basis tied to a controlled congruence;*
3. *fixed divisor quotient;*
4. *variable quotient residual;*
5. *B3 grouping incidence;*
6. *primitive slicing/fibre selection;*
7. *Cauchy/cube shift.*

Moreover, every HighTC tensor relation

$$\sum_\rho c_\rho Q_\rho = 0, \qquad c_m \neq 0$$

for a marked $m \in \mathcal{M}$ is origin-degenerate or impossible.
If this lemma holds, then E10G and HGO2R imply:

$$R_{\text{FreeAffineHighTC}}(N) = 0.$$

—

E10H.2. Safe transport operations We first separate the transport operations that are already safe.

S1. Fixing and projection Projection deletes fixed coordinates:

$$\ell = (\ell_{\text{free}}, \ell_{\text{fixed}}) \mapsto \ell_{\text{free}}.$$

This operation cannot introduce a new untagged origin. If it creates equality, proportionality, or forced collision between surviving forms, F3 routes the atom to LocalDiag. Otherwise it merely lowers ambient rank.

Projection may turn a previously TC1 system into HighTC by collapsing coordinates. But then the collapse itself is recorded as a fixing/projection origin. If the resulting relation is caused by the collapse, it is not FreeAffine; if it is not caused by the collapse, it must already be present in the pre-projection system.

Thus projection is origin-safe.

S2. Fixed divisor quotient For a fixed divisor condition

$$d \mid L(z),$$

F4/E5 replace the lattice by

$$\Lambda_d = \{z \in \Lambda : L(z) \equiv 0 \pmod{d}\}$$

and the quotient form by

$$L_d = L/d.$$

At the vector level:

$$\ell \mapsto \frac{1}{d} T^t \ell,$$

and F4.4/E5.2 prove

$$\text{cont}_{\Lambda_d}(L/d) = \frac{\text{cont}_\Lambda(L)}{(\text{cont}_\Lambda(L), d)} \le \text{cont}_\Lambda(L).$$

Any HighTC relation whose support uses the fixed quotient in a way that determines one active form from another is origin-degenerate by HGO2R, cases D1/D2. If it does not use the quotient origin, the fixed quotient is merely a controlled vector transport.

Thus fixed divisor quotient is safe except for the CRT matrix T, which is separated below.

S3. Variable quotient residual For

$$L(z) = ds,$$

F4 gives the exhaustive alternative:

$$\text{Edge} \sqcup \text{LocalDiag} \sqcup \text{CKP} \sqcup \text{GoodAWACK}.$$

If the quotient reaches GoodAWACK, the quotient vector satisfies an explicit origin equation

$$d\ell_s = T^t \ell_L. \tag{VQ-origin}$$

If a HighTC certificate uses this equation to force dependence, then it is origin-degenerate:

1. short quotient/divisor gives Edge;
2. forced determination gives LocalDiag;
3. balanced multiplicative quotient gives CKP;
4. incompatibility gives Impossible.

Therefore variable quotient residuals are safe, again modulo the same CRT/basis matrix-origin target T.

S4. Primitive slicing Primitive slicing writes a marked form on a one-dimensional fibre as

$$L(z_0 + uv) = gu + b.$$

For TC1/HighTC classification, the relevant object remains the pre-slicing vector ℓ. The fibre coefficient g is used by E7/E9 analytically. Primitive slicing therefore does not create a new HighTC tensor relation among the pre-slicing vectors.

Thus primitive slicing is not the rigidity obstruction.

S5. Cauchy/cube shifts Cauchy/cube operations introduce shifts:

$$L(z + \omega h) = L(z) + \ell(\omega h).$$

The linear part in z remains ℓ. If cube operations create equality, proportionality, or forced local dependence, E5/F3 route to LocalDiag. Otherwise at least one marked controlled-content form survives.

Hence Cauchy/cube operations are linear-part safe.

—

E10H.3. The remaining matrix operations The only remaining operations capable of creating new free affine tensor patterns are:

M1. Controlled CRT basis choice F3 controlled CRT absorption imposes

$$L(z) \equiv a \pmod{q}, \qquad q \leq (\log N)^C,$$

and replaces the lattice coset by

$$\Lambda' = \{z \in \Lambda : L(z) \equiv a \pmod{q}\}.$$

To express Λ' in free coordinates, one chooses a basis

$$z = z_0 + Tz'.$$

Vectors transform by

$$\ell \mapsto T^t \ell.$$

The F3/E5 statements control the index and content growth. They do not by themselves classify the possible matrices T, nor do they constrain the resulting rows beyond polylogarithmic content/minor bounds.

M2. Bounded affine regrouping E5 permits an integer affine map

$$T : \mathbb{Z}^{r'} \to \mathbb{Z}^r$$

with coefficients and relevant minors bounded by powers of $\log N$, and records only:

$$\operatorname{cont}(L \circ T) \le (\log N)^C \operatorname{cont}(L).$$

If T is unimodular, content is preserved exactly.

This proves content stability, but not rigidity. It does not say that T must be a coordinate projection, signed permutation, diagonal quotient, incidence matrix, or any other finite rigid matrix family.

Thus M1/M2 are exactly where E10G-Rigidity requires the E10X actual-origin classification.

—

E10H.4. Interface Example: Formal 4AP-like Witness Before imposing the actual-descendant constraint supplied by E10X, the broad M1/M2 interface permits, at the coefficient-vector level, the following formal situation.

Take four source coordinate forms Y_0, Y_1, Y_2, Y_3 on $\mathbb{Z}^4$, and restrict/regroup to a two-dimensional affine sublattice

$$Y_i = x + ir, \qquad 0 \le i \le 3.$$

Equivalently, use the matrix

$$T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix},$$

so that

$$T^t e_i = (1, i).$$

The transported vectors are

$$\ell_i = (1, i), \qquad 0 \le i \le 3.$$

They have bounded coefficients and content 1. Their quadratic tensors satisfy

$$Q_0 - 3Q_1 + 3Q_2 - Q_3 = 0. \tag{4AP-Q}$$

No equality or proportionality among the $L_i = x + ir$ is forced. The domain can be central-long:

$$x \asymp X, \qquad r \asymp R, \qquad x + 3r \asymp X, \qquad X, R \ge N^\theta.$$

This is a FreeAffineHighTC pattern unless the routing history records an origin that makes it:

1. CKP;
2. H4-canonical LocalDiag;
3. strict C1P Edge;

4. impossible.

The broad M1/M2 matrix hypotheses alone do not decide whether a matrix of this kind is an actual terminal descendant. Therefore this formal pattern is not used as a terminal GoodAWACK cell. Its role is to show why the matrix-origin reduction in E10H must be paired with the finite-grammar theorem E10X.

E10X supplies that actual-origin information. If such a 4AP-like rank-dropping transport is produced by actual B1/B3/F3/F4 routing, then its rank drop is tagged and the cell leaves clean terminal GoodAWACK by the CKP, LocalDiag, Edge, Impossible, or post-terminal analytic route. If it is untagged, then it is not an admissible actual descendant under the F3-complete routing interface.

—

E10H.5. Why H4 admission is not automatic One might try to use the linear identity

$$L_0 - 3L_1 + 3L_2 - L_3 = 0$$

to label the 4AP pattern LocalDiag.

This is not justified by Lemma H4.

H4 admits a local/main term only if it equals the tagged canonical local projection

$$\mathrm{Loc}_Q\, R_{\mathcal{B},\tau}(N)$$

up to $o(N)$.

A bare affine identity among oscillatory forms does not prove such an equality. In particular, a Liouville-weighted four-term affine pattern is a nonlocal oscillatory configuration, not automatically a local density projection modulo Q.

Therefore the H4 route would require the additional theorem:

$$\boxed{\text{every B3/F3 affine-dependence flag surviving from actual routing is H4-canonical.}}$$

The proof does not use this automatic-H4 implication. Instead, it uses E10Y/E10X/E10K to exclude the untagged actual terminal occurrence before E10L assembles the clean GoodAWACK estimate.

—

E10H.6. Rigidity reduction theorem

Lemma 7.2 (Lemma E10H.1. Reduction to matrix-origin rigidity)**.** *For terminal GoodAWACK skeletons in the proof tree, every HighTC certificate is origin-degenerate or impossible unless it is supported entirely after applying M1/M2 matrix transports in a way that does not use fixed-divisor, variable-quotient, repeated/proportional, CKP-balanced, C1 Edge, or parent-incompatibility origins.*

Equivalently:

$$\text{FreeAffineHighTC} \subseteq \text{HighTC produced by CRT/AFF matrix transport.}$$

Proof. By E10G, every terminal GoodAWACK vector is produced by the catalogue cells C0–C6.

Cells corresponding to fixing/projection, fixed divisor quotient, variable quotient residual, primitive slicing, and Cauchy/cube shifts are S1–S5 above. In each case, either the operation preserves the relevant linear parts, deletes coordinates with recorded origin, or carries an explicit quotient/local origin.

If a HighTC relation uses any of these origins in an essential way, then HGO2R routes it to CKP, H4-admissible LocalDiag, strict C1P Edge, or Impossible. If it does not use those origins, then those operations are irrelevant to its free-affine character.

The only operations left capable of producing a new untagged affine tensor relation are M1 controlled CRT basis choice and M2 bounded affine regrouping. Therefore every remaining FreeAffine-HighTC certificate must come from the matrix-origin part. Lemma proved.

—

□

E10H.7. Matrix-origin closure lemma The structural conclusion is stated directly at the matrix level.

Lemma 7.3 (Lemma E10H.2. CRT/AFF matrix-origin rigidity)**.** *Let T_{tot} be the total coefficient transport matrix obtained by composing all controlled CRT basis choices and bounded affine regroupings in an actual terminal GoodAWACK routing history.*

Then one of the following holds:

1. *T_{tot} belongs to an explicitly enumerated rigid family whose tensor row-reduction has no FreeAffine-HighTC relation;*
2. *any HighTC relation created by T_{tot} is tagged by a quotient/gcd/divisor/local origin and is origin-degenerate;*
3. *the corresponding tagged atom is H4-canonical LocalDiag;*
4. *the routing cell is empty or violates B1/B3/F3/F4 admissibility.*

E10X supplies this conclusion for actual terminal GoodAWACK descendants. With that input, E10H.2 gives:

$$R_{\text{FreeAffineHighTC}}(N) = 0.$$

The formal 4AP-like cell of E10H.4 is not an obstruction to E10H.2: it is not a constructed actual B1 descendant. E10X proves that an actual untagged occurrence of that type is impossible in the routing tree.

—

Remark 7.4 (E10H.8. Output).

$$\boxed{\text{E10H reduces FreeAffineHighTC to CRT/AFF matrix-origin rigidity.}}$$

What is proved here:

$$\boxed{\text{FreeAffineHighTC is reduced to the CRT/AFF matrix-origin rigidity problem.}}$$

Structural closure:

$$\boxed{\text{The remaining rank-dropping AFF issue is discharged by E10X.}}$$

Completion block:

MOR: Matrix-Origin Rigidity for controlled CRT and bounded affine regrouping.

This block is supplied by E10X, using E10I, E10J, E10Y, E10M, and E10K.

8 Part 7. E10I: Matrix-origin rigidity

Source file: `Lemmas/e10i_mor_matrix_origin_rigidity_ltx.md`.

8.0.1 E10I. Matrix-Origin Rigidity Verification

E10I.0. Statement and Role Lemma E10I supplies the MOR reduction to untagged rank-dropping AFF. That final class is discharged by the finite GoodAWACK grammar Lemma E10X.

Therefore the class isolated by E10I is not a hidden gap; it is the input passed to the E10J–E10Y–E10M–E10K closure packaged by E10X.

E10I continues the MOR block isolated in Lemma E10H.

The target was:

MOR: prove matrix-origin rigidity for controlled CRT and bounded affine regrouping.

The outcome is a further reduction.

controlled CRT and full-rank affine coordinate changes are tensor-safe; E10I reduces the matrix target to untagg

At the stage of E10I alone, the proof has reduced rather than proved

$$R_{\text{FreeAffineHighTC}}(N) = 0,$$

because the remaining matrix target is the following narrower statement, which is discharged by E10X:

$$\text{FreeAffineHighTC} \subseteq \text{HighTC created by rank-dropping AFF transport without origin.}$$

—

E10I.1. Linear algebra fact: tensor tests are invariant under rational isomorphism Let V, W be finite-dimensional rational vector spaces and let

$$S : V^* \to W^*$$

be an injective linear map. It induces

$$\mathrm{Sym}^2(S) : \mathrm{Sym}^2(V^*) \to \mathrm{Sym}^2(W^*), \qquad \ell \odot \ell \mapsto S\ell \odot S\ell.$$

If S is injective, then $\mathrm{Sym}^2(S)$ is injective.

Consequently, for any finite family $\{\ell_\rho\} \subset V^*$ and any marked index m,

$$\ell_m \odot \ell_m \in \operatorname{span}_{\mathbb{Q}}\{\ell_\rho \odot \ell_\rho : \rho \neq m\}$$

if and only if

$$S\ell_m \odot S\ell_m \in \operatorname{span}_{\mathbb{Q}}\{S\ell_\rho \odot S\ell_\rho : \rho \neq m\}.$$

Proof. Choose bases. An injective linear map S has a left inverse over $\mathbb{Q}$ on its image. Hence the induced map on symmetric tensors also has a left inverse on its image, so $\operatorname{Sym}^2(S)$ is injective.

Applying $\operatorname{Sym}^2(S)$ to a rational linear relation among the tensors preserves the relation. Conversely, if a relation holds after applying $\operatorname{Sym}^2(S)$, injectivity implies the same relation held before applying it.

This proves the equivalence.

—

□

E10I.2. Controlled CRT is tensor-safe In F3, a controlled CRT restriction replaces a lattice coset by

$$\Lambda' = \{z \in \Lambda : L(z) \equiv a \pmod q\}, \qquad q \le (\log N)^C.$$

If nonempty, Λ' is a finite-index subcoset of Λ. Its difference lattice has the same rank as the original difference lattice.

Choosing coordinates on Λ' gives

$$z = z_0 + Tz',$$

where T is a full-rank square matrix over $\mathbb{Q}$ after choosing bases of the original and restricted difference lattices. Homogeneous vectors transform by

$$\ell \mapsto T^t \ell.$$

Since T^t is injective over $\mathbb{Q}$, Section E10I.1 shows that the TC1/HighTC tensor test is invariant under this coordinate choice.

Lemma 8.1 (Lemma E10I.1. CRT basis choice does not create FreeAffineHighTC)**.** *Controlled CRT basis choice cannot turn a TC1 family into a FreeAffineHighTC family, nor can it create a new untagged tensor relation. Any HighTC relation after CRT was already present before CRT, transported by an injective symmetric-square map.*

Proof. Immediate from E10I.1 and the full-rank finite-index nature of controlled CRT restrictions.

Thus the matrix-origin residual is not ordinary CRT basis choice.

—

□

E10I.3. Full-rank affine regrouping is tensor-safe E5 permits bounded affine regrouping:

$$z = z_0 + Az'.$$

If this regrouping is a full-rank coordinate change between equal-rank active parameter lattices, then A is invertible over $\mathbb{Q}$. Homogeneous vectors transform by

$$\ell \mapsto A^t \ell.$$

Again A^t is injective, so the tensor test is invariant.

Lemma 8.2 (Lemma E10I.2. Full-rank AFF maps are not the obstruction)**.** *Any bounded affine regrouping whose linear part is full-rank on the active affine span preserves the TC1/HighTC classification.*

In particular, unimodular changes, finite-index basis changes, signed permutations, and diagonal controlled quotient coordinate changes cannot create a new FreeAffineHighTC certificate.

Proof. Apply E10I.1 to $S = A^t$.

—

□

E10I.4. Rank-dropping AFF maps are the only remaining matrix danger The tensor test is not invariant under rank-dropping maps.

If

$$A : \mathbb{Q}^{k'} \to \mathbb{Q}^k$$

has rank $k' < k$, then

$$A^t : (\mathbb{Q}^k)^* \to (\mathbb{Q}^{k'})^*$$

need not be injective. Distinct quadratic tensors in $\mathrm{Sym}^2((\mathbb{Q}^k)^*)$ may collapse to dependent tensors after restriction to the lower-dimensional slice.

This is exactly how a 4AP-like pattern appears.

Take source coordinate forms

$$Y_0, Y_1, Y_2, Y_3$$

on $\mathbb{Q}^4$, and restrict to the two-dimensional slice

$$Y_i = x + ir, \qquad 0 \le i \le 3.$$

The resulting vectors are

$$\ell_i = (1, i), \qquad 0 \le i \le 3,$$

and

$$Q_0 - 3Q_1 + 3Q_2 - Q_3 = 0.$$

This tensor relation is not produced by an invertible coordinate change. It is produced by a rank-dropping affine slice.

Thus any structural MOR proof must control rank-dropping AFF maps.

—

E10I.5. What B1/B3/F3/F4/E5 contribute to rank-dropping maps The source lemmas do not introduce a free list of rank-dropping affine maps. They provide product data, routing operations, and stability transports whose actual rank-dropping occurrences are classified by the E10Y/E10M grammar and then packaged by E10X. The E10S records supply reproducibility checks for this classification, but they are not the proof of the classification.

B1 Lemma B1 gives product variables and dyadic cells, not affine matrix parametrizations.

B3 Lemma B3 gives a finite set of product groupings:

$$u_I = \prod_{i \in I} x_i, \qquad v_I = \prod_{i \notin I} x_i,$$

and preliminary labels. It does not give a matrix list for later affine parameterizations.

F3 Lemma F3 permits controlled CRT absorption and terminal routing. CRT is full-rank and tensor-safe by E10I.1. F3 does not list affine slices of the form

$$Y_i = x + ir.$$

F4 Lemma F4 handles fixed-divisor and variable-quotient origins. These are origin-tagged and already routed by HGO2R when they cause HighTC. F4 does not enumerate untagged rank-dropping AFF maps.

E5 Lemma E5 is the stability source that explicitly permits a broad affine map:

$$T : \mathbb{Z}^{r'} \to \mathbb{Z}^r$$

with coefficients and relevant minors bounded by powers of $\log N$.

E5 proves content stability:

$$\operatorname{cont}(L \circ T) \leq (\log N)^C \operatorname{cont}(L).$$

It does not say that T is full-rank on the active affine span, nor that every rank-dropping T carries a quotient/local/CKP/Edge origin.

Therefore the E5 content-stability statement is not the MOR closure mechanism. The proof uses E10X to classify the actual rank-dropping occurrences created by the B1/B3/F3/F4 routing tree and reads E5 only as a stability lemma for those authorized transports.

—

E10I.6. MOR reduction theorem

Lemma 8.3 (Lemma E10I.3. Reduction to rank-dropping AFF origin)**.** *For actual terminal GoodAWACK skeletons supported by the proof tree,*

$$\text{FreeAffineHighTC}$$

can only arise from a rank-dropping bounded affine regrouping/slicing map whose rank drop is not already recorded as:

1. *fixing/projection;*
2. *controlled CRT finite-index restriction;*
3. *fixed-divisor quotient;*
4. *variable quotient residual;*
5. *forced LocalDiag;*
6. *CKP-balanced grouping;*
7. *strict C1P Edge;*
8. *parent incompatibility.*

Proof. By E10H, all non-matrix operations are origin-safe.

By E10I.2, controlled CRT basis choices are full-rank finite-index coordinate changes and cannot create new tensor dependence.

By E10I.3, full-rank affine regroupings are tensor-safe.

Thus any remaining FreeAffineHighTC certificate must be created by the only matrix operation not covered by these safe cases: a rank-dropping AFF map. If the rank drop is tagged by one of the origins 1–8, then HGO2R reroutes it. Therefore the surviving case is precisely an untagged rank-dropping AFF origin. Lemma proved.

—

□

E10I.7. Rank-drop closure lemma The remaining structural input is a rank-drop origin lemma.

Lemma 8.4 (Lemma E10I.4. No untagged rank-dropping AFF in terminal GoodAWACK)**.** *Let* $\mathfrak{S}$ *be an actual terminal GoodAWACK skeleton. Every rank-dropping affine map used to produce its active affine system is one of:*

1. *a recorded fixing/projection already covered by the skeleton origin map;*
2. *a quotient/divisor/gcd-origin map covered by F4;*
3. *a local/collision map routed to H4-canonical LocalDiag;*
4. *a CKP-balanced grouping;*
5. *a strict C1P Edge configuration;*
6. *an impossible/empty cell.*

Equivalently, no untagged rank-dropping AFF map may survive into terminal GoodAWACK.
If E10I.4 is proved, then

$$R_{\mathrm{FreeAffineHighTC}}(N) = 0.$$

E10X supplies this lemma for actual terminal GoodAWACK skeletons through the E10Y/E10M finite-grammar classification. The E10S records document a non-logical reproducibility check for the maintained source files; they are not used as proof of the classification.

—

Remark 8.5 (E10I.8. Output).

$$\boxed{\text{MOR is partially proved: CRT and full-rank AFF are tensor-safe.}}$$

Completion theorem:

$$\boxed{\text{No untagged rank-dropping AFF map in terminal GoodAWACK.}}$$

Structural closure:

$$\boxed{\text{This task is discharged by E10X.}}$$

9 Part 8. E10J: Rank-dropping AFF origin verification

Source file: `Lemmas/e10j_rda_rank_dropping_aff_origin_verification_ltx.md`.

9.0.1 E10J. Rank-Dropping AFF Origin Verification

E10J.0. Statement and Role Lemma E10J proves that tagged rank drops are origin-degenerate or already routed, and reduces the remaining case to the affine-origin completeness theorem packaged by E10X and proved through the E10Y/E10M/E10K finite-grammar chain for actual terminal GoodAWACK skeletons. The E10S records remain reproducibility support for the finite source list.

E10J treats the next block isolated in Lemma E10I.

The target was:

$$\boxed{\text{RDA: no untagged rank-dropping AFF map survives into terminal GoodAWACK.}}$$

The reduction proved in this file is:

$$\boxed{\text{RDA reduces to an affine-regrouping origin-completeness lemma.}}$$

What is proved:

$$\boxed{\text{every tagged rank drop is already routed or origin-degenerate.}}$$

Completion theorem:

$$\boxed{\text{exclude or classify rank drops allowed only by the broad E5 affine-regrouping interface.}}$$

—

E10J.1. RDA statement Let

$$\mathfrak{S} = (\mathcal{B}, \Gamma, \mathfrak{r}, \Lambda_{\mathfrak{S}}, \Omega_{\mathfrak{S}}, \mathcal{L}_{\mathfrak{S}}, \mathcal{M}_{\mathfrak{S}}, \mathrm{orig}_{\mathfrak{S}}, \mathcal{W}_{\mathfrak{S}})$$

be an actual terminal GoodAWACK skeleton.

A rank-dropping AFF map is a bounded affine transport

$$z = z_0 + Az', \qquad \operatorname{rank} A < \dim z,$$

used to produce or represent the active affine system.

It is **tagged** if its rank drop is recorded as one of:

1. fixing/projection of inactive coordinates;
2. fixed divisor quotient;
3. variable quotient residual;
4. controlled local/gcd dependence;
5. CKP-balanced grouping;
6. strict C1P Edge;
7. impossible/empty support;
8. primitive slicing used only analytically, while the pre-slicing vectors remain the tensor-verification objects.

It is **untagged** if none of these origins is recorded.
RDA asks to prove:

$$\boxed{\text{no untagged rank-dropping AFF map occurs in terminal GoodAWACK.}} \qquad \text{(RDA)}$$

By E10I, RDA would imply:

$$R_{\text{FreeAffineHighTC}}(N) = 0.$$

—

E10J.2. Tagged rank drops are safe

Lemma 9.1 (Lemma E10J.1. Tagged rank-dropping AFF is origin-degenerate or irrelevant)**.** *If a rank-dropping AFF map in a terminal GoodAWACK skeleton is tagged by one of the origins listed in E10J.1, then it cannot support a FreeAffineHighTC certificate.*

Proof. We inspect the tagged cases.

If the rank drop is fixing/projection, then the collapse is recorded in the origin map. Any HighTC relation caused by the collapse is not free-affine; if it is not caused by the collapse, it was already present before projection.

If the rank drop comes from fixed divisor quotient or variable quotient residual, then F4 supplies the quotient/divisor origin. By HGO2R, any HighTC certificate using that origin is LocalDiag, CKP, Edge, or Impossible.

If the rank drop is controlled local/gcd dependence, then it is precisely a LocalDiag-origin case, admitted only when H4-canonical.

If it is CKP-balanced, the atom is CKP and handled by G8a.

If it is strict C1P Edge or empty support, it contributes $o(N)$ or zero.

If it is primitive slicing, then by E10H and E10I the pre-slicing vectors remain the objects used in the TC1/HighTC tensor verification; the one-dimensional fibre is an analytic E7/E9 object, not a new terminal HighTC coefficient family.

Thus no tagged rank drop produces FreeAffineHighTC. Lemma proved.

—

□

E10J.3. Source classification for untagged rank drops We classify what the source lemmas prove about untagged rank-dropping AFF before the finite-grammar theorem E10X is applied.

B1 Lemma B1 gives typed Heath–Brown product variables and dyadic cells. It does not introduce affine matrix maps or rank-dropping affine slices.

Thus B1 does not create an untagged rank-dropping AFF occurrence.

B3 Lemma B3 gives a finite product grouping set:

$$u_I = \prod_{i \in I} x_i, \qquad v_I = \prod_{i \notin I} x_i,$$

and preliminary labels. It also says that forced equality, proportionality, repeated factor, fixed gcd-local dependence, or affine dependence may produce a LocalDiag flag.

But the corrected H4 interface does not accept a bare affine dependence as LocalDiag unless it is a tagged canonical local projection. Therefore B3 is not by itself the closure theorem for untagged rank-dropping AFF. Its role is to provide finite grouping data and tags used by E10X.

F3 Lemma F3 performs controlled CRT absorption, F4 large-divisor decision, square-divisor routing, finite grouping selection/elimination, LocalDiag detection, Edge detection, and terminal class labelling.

Controlled CRT is full-rank and tensor-safe by E10I.

F3 does not enumerate rank-dropping affine regrouping maps. Its terminal GoodAWACK predicate is negative:

1. central-long affine WACLE;
2. bounded affine complexity;
3. no forced LocalDiag;
4. no unresolved ordinary large divisor;
5. marked Liouville-type affine form;
6. long active directions.

This negative predicate must be read together with the complete F3.6 operation list. F3 labels terminal GoodAWACK descendants; it does not license a new untagged rank-dropping affine parametrization.

F4 Lemma F4 handles ordinary divisor and quotient origins. These are tagged rank drops and are safe by E10J.2.

F4 handles the tagged divisor, quotient, gcd, balanced CKP, and strict C1P origins. Any rank drop not tied to one of these origins is passed to the E10X finite-grammar closure rather than treated as an admissible terminal generator.

BGS Lemma BGS records

$$\mathfrak{r}_{\mathrm{grp}}$$

as affine regrouping or affine changes of variables, and includes an origin map

$$\mathrm{orig}_{\mathfrak{G}}\,.$$

This is enough to state RDA. The proof that every actual rank drop in $\mathfrak{r}_{\mathrm{grp}}$ is one of the tagged origins in E10J.1 is supplied by E10X through the E10Y grammar theorem and E10M. The E10S records are non-logical reproducibility support for the maintained source files.

E5 Lemma E5 permits an affine map

$$T : \mathbb{Z}^{r'} \to \mathbb{Z}^{r}$$

with coefficients and relevant minors bounded by powers of $\log N$, and proves only:

$$\mathrm{cont}(L \circ T) \leq (\log N)^{C}\,\mathrm{cont}(L).$$

It does not state:

1. T must be full-rank on the active affine span;
2. every rank drop of T must be fixing/projection;
3. every rank drop of T must have quotient/divisor/gcd/CKP/Edge/LocalDiag origin;
4. every affine dependence created by T is H4-canonical LocalDiag.

Therefore E5 is a stability lemma, not the closure theorem for RDA. The proof reads E5 in the E10X-clean sense: it may transport already authorized full-rank or tagged data, but it is not an additional terminal generator of untagged rank-dropping AFF.

—

E10J.4. Formal rank-dropping AFF witness At the broad E5/BGS interface, before applying E10X's actual-descendant restriction, one can write the following formal rank-dropping AFF cell.

Start with four independent source coordinate forms

$$Y_0, Y_1, Y_2, Y_3.$$

Apply the rank-dropping affine parametrization

$$Y_i = x + ir, \qquad 0 \leq i \leq 3.$$

Equivalently, the transport matrix is

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}.$$

The resulting homogeneous vectors are

$$\ell_i = (1, i).$$

They satisfy the HighTC tensor relation

$$Q_0 - 3Q_1 + 3Q_2 - Q_3 = 0.$$

All contents are 1, all coefficients are $O(1)$, and the affine complexity is bounded. A central-long domain can be chosen:

$$x \asymp X, \qquad r \asymp R, \qquad x + 3r \asymp X, \qquad X, R \geq N^\theta.$$

At the level of the broad terminal GoodAWACK wording alone, this formal cell is not automatically:

1. Edge;
2. CKP;
3. H4-canonical LocalDiag;
4. impossible.

Thus broad interface language alone does not classify this formal rank-dropping AFF cell.

This is not claimed to be an actual B1 descendant. It is an interface example showing why RDA is proved through E10Y/E10X/E10K rather than through the broad E5/BGS wording alone. If this pattern is generated by actual B1/B3/F3/F4 routing, then E10X classifies its rank drop and routes it away from clean terminal GoodAWACK; if it remains untagged, it is not an admissible actual terminal skeleton.

—

E10J.5. RDA reduction theorem

Lemma 9.2 (Lemma E10J.2. RDA reduces to affine-regrouping origin completeness)**.** *Assume the following origin-completeness statement:*

$$\boxed{\textit{Every rank-dropping affine regrouping recorded in } \mathfrak{r}_{\mathrm{grp}} \textit{ is tagged by one of E10J.1(1)–(8).}} \tag{AFF-OC}$$

Then RDA holds.

Proof. Let $\mathfrak{S}$ be a terminal GoodAWACK skeleton. By E10I, any FreeAffineHighTC certificate must arise from a rank-dropping AFF map.

By AFF-OC, every such rank drop is tagged by one of the origins in E10J.1.

By Lemma E10J.1, tagged rank drops are origin-degenerate or irrelevant for FreeAffineHighTC.

Therefore no FreeAffineHighTC certificate remains. This proves RDA.

—

□

E10J.6. AFF-origin completeness closure lemma The closure block is an origin-completeness upgrade for affine regrouping.

Lemma 9.3 (Lemma E10J.3. AFF-origin completeness)**.** *In every actual B1/B3/F3/F4 terminal GoodAWACK routing history, every affine regrouping or affine change of variables recorded in*

$$\mathfrak{r}_{\mathrm{grp}}$$

has linear part A satisfying exactly one of:

1. *A is full-rank on the active affine span;*
2. *the rank drop is a recorded fixing/projection;*
3. *the rank drop is induced by fixed divisor or quotient-origin data;*
4. *the rank drop is induced by a forced local/gcd/collision relation and is H4-canonical LocalDiag;*
5. *the rank drop exposes CKP-balanced grouping;*
6. *the rank drop gives strict C1P Edge;*
7. *the cell is impossible.*

If E10J.3 is proved, then:

$$R_{\mathrm{FreeAffineHighTC}}(N) = 0.$$

E10Y/E10X/E10K supply E10J.3 for the actual terminal GoodAWACK cells.
—

Remark 9.4 (E10J.7. Output)*.*

$$\boxed{\text{RDA is reduced to AFF-origin completeness, which is supplied by E10Y/E10X/E10K.}}$$

What is proved:

$$\boxed{\text{RDA follows from AFF-origin completeness, and all tagged rank drops are safe.}}$$

Structural closure:

$$\boxed{\text{AFF-origin completeness is discharged by E10X and assembled in E10K/E10L.}}$$

Completion block:

$$\boxed{\text{AFF-OC: Affine-regrouping origin completeness.}}$$

This block is supplied by E10X and assembled in E10K/E10L.

10 Part 9. E10Y: GoodAWACK routing grammar completeness

Source file: `Lemmas/e10y_goodawack_routing_grammar_completeness_ltx.md`.

10.0.1 E10Y. Completeness of the GoodAWACK Routing Grammar

E10Y.0. Statement and Role Lemma **E10Y** is a structural completeness theorem for the GoodAWACK routing grammar. It proves that every operation which can generate or modify an actual terminal GoodAWACK affine skeleton is already represented in the finite B1/B3/F3/F4 routing grammar, with E5 used only for controlled content transport.

The lemma concerns only **actual B1/B3/F3/F4/E5-generated descendants**. It does not classify arbitrary bounded affine systems and it does not assert that every formal affine parametrization is reachable. Its assertion is:

$$\boxed{\text{every skeleton-generating pre-terminal operation in an actual B1-origin GoodAWACK descendant is one of the c}} \tag{E10Y}$$

Consequently, an unlisted rank-dropping affine regrouping cannot enter a terminal GoodAWACK skeleton as a hidden operation. Post-terminal analytic operations may estimate a fixed terminal object, but they do not generate a new terminal GoodAWACK skeleton.

Logical dependencies are B1, B3, F3, F3A, F3T, F4, and the content-stability calculations of E5. E10Y is used by E10M, E10X, E10K, and E10L. E10S and E10S-MECH are maintained separately as non-logical reproducibility records for the same finite operation list.

—

E10Y.1. Setup

Actual routing record An actual routing record is a tuple

$$\mathfrak{r} = (V, \mathcal{C}, \mathcal{L}, \mathcal{Q}, \tau, \mathrm{orig}, W)$$

where:

1. V is the finite list of active variables inherited from B1;
2. $\mathcal{C}$ records dyadic, congruence, content, gcd and divisor restrictions;
3. $\mathcal{L}$ is the finite list of affine forms visible on the current cell;
4. $\mathcal{Q}$ records fixed-divisor, quotient and local tags;
5. τ is the current routing tag;
6. orig records the origin of every rank-changing operation;
7. W is the bounded or polylogarithmic weight data transported with the cell.

An actual terminal GoodAWACK skeleton is the terminal value of such a routing record along a descendant of

$$B1 \longrightarrow B3 \longrightarrow F3/F4.$$

Pre-terminal operation A pre-terminal operation is a transformation of an actual routing record before terminal class labelling. It is **actual-generated** if it is invoked by the B1, B3, F3 or F4 routing construction, or by the E5 content-stability calculation applied to a record already produced by those routing layers. This definition is external to the E10Y grammar: it refers to the construction of descendants in B1/B3/F3/F4/E5, not to the list of E10Y transition classes. Thus "actual-generated" means "lying in the image of the independently defined B1/B3/F3/F4/E5 construction"; it does not mean "allowed because E10Y allows it."

Lemma 10.1 (Lemma E10Y.0. Source-to-record extraction)**.** *Every Branch B descendant that is actually produced by B1/B3/F3/F4/E5 and then fed to the GoodAWACK terminal class carries a finite routing record*

$$\mathfrak{r} = (V, \mathcal{C}, \mathcal{L}, \mathcal{Q}, \tau, \mathrm{orig}, W)$$

of the kind defined above. Each rank-relevant operation occurrence in that descendant is represented either by a transformation of this record before terminal labelling or by a post-terminal analytic operation after terminal labelling.

Proof. B1 fixes a finite-depth Heath–Brown product block, its variables, its dyadic cell and its coefficient data. B3 replaces this by one of finitely many grouped product records. F3 and F4 act only through their recorded routing decisions: congruence restrictions, divisor or quotient choices, local/gcd relations, Edge or CKP routing, continuation tags, and terminal class labels. E5 is applied only to a record already carrying those variables, constraints and origin tags. Hence every actual Branch B descendant has a finite record of the displayed form. Any later TC1/HighTC, coarea, Fourier, BRS/X16, Davenport/AP, Cauchy–Schwarz or local-projection step is performed only after the terminal routing record has already been fixed, and is therefore recorded as post-terminal analytic use rather than as a new pre-terminal operation.

This proves the extraction claim.

□

Skeleton-generating operation A pre-terminal operation is skeleton-generating if it changes at least one of

$$V, \qquad \mathcal{C}, \qquad \mathcal{L}, \qquad \mathcal{Q}, \qquad \mathrm{orig},$$

in a way that can affect the terminal GoodAWACK affine skeleton.

Post-terminal analytic non-generator A post-terminal analytic non-generator is an operation performed after a terminal routing object has been fixed. Such an operation may form test functions, apply Cauchy–Schwarz, take Fourier transforms, slice a fixed testing family, or estimate an auxiliary sum. It does not create a new B1/B3/F3/F4 descendant and it does not add a new terminal GoodAWACK skeleton.

Terminal tensor-test vectors For a terminal GoodAWACK skeleton $\mathfrak{S}$, the terminal affine forms are

$$\mathcal{L}_{\mathfrak{S}} = \{L_\rho(z) = \ell_\rho \cdot z + c_\rho\}.$$

The TC1/HighTC test is applied to the corresponding terminal vectors ℓ_ρ and tensors $Q_\rho = \ell_\rho \otimes \ell_\rho$. Post-terminal operations may restrict domains, average over fibres, or introduce auxiliary testing variables, but they may not replace the terminal list $\{\ell_\rho\}$ by a new list and then treat the new list as a fresh GoodAWACK routing descendant. Any operation that would change the terminal tensor-test vectors for routing purposes must already occur as a pre-terminal operation in the routing record.

Rank drop and tag An affine transformation is full-rank on the active affine span if its linear part is injective on the difference space generated by the current active forms, up to the finite-index restrictions already recorded in $\mathcal{C}$. For terminal GoodAWACK records, E5-clean full-rank transport also has trivial kernel on the span of the terminal tensor-test vectors. It is rank-dropping if this injectivity fails on the active span or on the terminal tensor-test span.

Throughout this lemma, a bounded affine map means an affine map whose coefficients, denominators and induced lattice index are controlled by the fixed routing complexity and the polylogarithmic parameter hierarchy. Thus "bounded" is not a new qualitative assumption; it is the quantitative bounded-complexity condition already present in the B1/B3/F3/F4/E5 routing record.

A rank drop is tagged if orig records one of:

$$\text{Fix/Proj}, \quad \text{CRT}, \quad \text{FixedDiv}, \quad \text{VarQuot}, \quad \text{LocalDiag}, \quad \text{CKP}, \quad \text{Edge}, \quad \text{PostTerminalNonGenerator}.$$

It is untagged if it is present only as a free affine regrouping or formal affine parametrization.

—

E10Y.2. Phase Separation

Lemma 10.2 (Lemma E10Y.1. Pre-terminal and post-terminal phases are disjoint)**.** *Every actual B1-origin descendant has a finite pre-terminal routing phase followed by a terminal analytic phase. Operations from the terminal analytic phase do not generate new terminal GoodAWACK skeletons.*

Proof. B1 supplies typed Heath–Brown product variables, dyadic cells and exact convolution weights. B3 supplies finitely many product-grouping candidates and preliminary structural labels. F3 and F4 then perform the finite routing decisions that determine whether the descendant is Edge, CKP, GoodAWACK, LongAP/Local or LocalDiag.

Once a GoodAWACK terminal object has been labelled, the later TC1/HighTC, Cauchy–Schwarz, Fourier, coarea, Shiu/BRS, Davenport/AP and local projection arguments operate on that fixed terminal object or on a testing family derived from it. None of those arguments returns to B3, F3 or F4 to create an additional descendant, and none introduces a new terminal routing class.

Thus the routing construction has two separated phases:

$$\text{B1/B3/F3/F4 pre-terminal routing} \quad \text{then} \quad \text{terminal analytic estimation.}$$

The second phase estimates, reroutes or discards already fixed terminal data; it is not a skeleton-generation mechanism.

—

□

E10Y.3. Initial B1/B3 Sources

Lemma 10.3 (Lemma E10Y.2. B1 and B3 introduce no free rank-dropping affine regrouping)**.** *B1 and B3 supply finitely many initial sources for the routing grammar, but neither layer introduces an arbitrary rank-dropping affine regrouping into a terminal GoodAWACK skeleton.*

Proof. In B1 the variables are the product variables of the fixed-depth Heath–Brown decomposition, together with dyadic restrictions and exact convolution weights. This layer introduces product coordinates and cells; it does not apply a bounded affine map to the active affine span.

In B3 the construction enumerates finitely many product-grouping candidates. The operation is a finite selection among grouped product-coordinate descriptions. If the grouping exposes a short-volume, quotient, local, CKP-balanced or Edge structure, that information is recorded as a routing feature and passed to F3/F4. If it does not, the descendant remains a candidate for terminal routing.

Therefore B1/B3 create the finite set of start states for the later grammar, but they do not add a hidden rank-dropping AFF operation.

—

□

E10Y.4. Exhaustive Pre-Terminal Routing

Lemma 10.4 (Lemma E10Y.3. F3/F4 exhaust the skeleton-generating pre-terminal operations)**.** *Let $\mathfrak{r}$ be an actual routing record after B3. Every skeleton-generating pre-terminal operation applied before terminal class labelling is one of the F3/F4 operations recorded in the following list:*

1. *controlled CRT absorption;*
2. *F4 large-divisor or quotient decision;*
3. *square-divisor routing;*
4. *finite grouping selection or elimination;*
5. *terminal LocalDiag detection;*
6. *terminal Edge detection by a C1P predicate;*
7. *terminal class labelling into Edge, CKP, GoodAWACK, LongAP/Local or LocalDiag.*

Proof. Lemma F3A proves that Section F3.6 is complete for generic F3 routing-level operations. Lemma F3T expands this list into the complete finite routing table by B1 type, B3 grouping, dyadic regime, divisor/conductor state, coefficient type, terminal class and exclusion reason. Lemma F4 supplies the ordinary divisor and quotient decision used inside the second item.

The F3T table has no sixth terminal class and no row whose operation is "arbitrary affine regrouping." Each non-terminal row either continues the finite routing procedure with a recorded tag or is eliminated as incompatible, empty, Edge, local, CKP or already terminal. Hence any actual-generated operation that can change the terminal skeleton before labelling is represented by the seven operations above.

—

□

E10Y.5. Full-Rank Transport and Tagged Rank Drop

Lemma 10.5 (Lemma E10Y.4. E5 does not add an independent skeleton generator)**.** *E5 content stability may transport content, coefficients and auxiliary variables along an already generated routing record. It does not create an additional terminal GoodAWACK skeleton from an external affine system.*

More precisely, every E5-compatible affine transport is one of:

1. *full-rank on the active affine span and on the terminal tensor-test span, with controlled bounded-minor/content loss;*

2. *rank-dropping with an origin tag already supplied by B1/B3/F3/F4;*

3. *post-terminal analytic slicing after the terminal object has already been fixed.*

Proof. The content-stability calculation of E5 is applied only after the current routing record has already supplied the variables, congruence restrictions, divisor data and origin information being transported. A full-rank transport preserves the active affine rank, is injective on the terminal tensor-test span, and changes content only by the controlled bounded-minor factors recorded in E5.

If the transport is not full-rank on the active affine span, or if it has a kernel on the terminal tensor-test span, then it is not E5-clean full-rank transport. The lost rank must then come from a restriction already present in the routing record: fixing/projection, CRT compatibility, fixed divisor quotient, variable quotient residual, local/diagonal dependence, CKP-balanced structure, Edge predicate or post-terminal post-terminal analytic slicing. These are exactly the tags recorded in the origin component of the routing record.

Thus E5 is a stability principle for transports whose source is already known. It is not a separate mechanism for adjoining an untagged rank-dropping affine map to a GoodAWACK terminal skeleton.

—

□

E10Y.6. No Feedback from Analytic Tests

Lemma 10.6 (Lemma E10Y.5. Terminal analytic operations do not feed back into the routing grammar)**.** *Let $\mathfrak{S}$ be a terminal GoodAWACK skeleton. The TC1/HighTC split, global testing construction, regular/singular testing dichotomy, BRS/X16 short-image analysis, Davenport/AP estimate, Cauchy–Schwarz, cube expansion, Fourier expansion and local projection arguments do not create an additional terminal GoodAWACK skeleton.*

Proof. Each listed operation is invoked after the terminal skeleton has been selected. Its input is a fixed terminal skeleton, a fixed terminal testing family, or a sum derived from those fixed data. The output is one of the following:

1. an $o(N)$ analytic estimate;

2. a routing-away conclusion to Edge, CKP, LongAP/Local or LocalDiag already present in the F3/F4 terminal alternatives;

3. a proof that the GoodAWACK contribution belongs to the HighTC finite grammar closure;

4. a local-main projection used only in the final assembly.

None of these outputs is a new B1/B3/F3/F4 descendant. Therefore post-terminal analytic tests cannot supply a missing pre-terminal operation and cannot produce an untagged terminal GoodAWACK rank drop.

□

Lemma 10.7 (Lemma E10Y.5b. Terminal tensor-vector immutability)**.** *Once a terminal GoodAWACK skeleton $\mathfrak{S}$ is fixed, the affine vectors ℓ_ρ and tensors $Q_\rho = \ell_\rho \otimes \ell_\rho$ used in the TC1/HighTC test are immutable under post-terminal analytic operations. Post-terminal slicing, averaging, Cauchy–Schwarz, cube expansion, TC1 testing, BRS/X16 estimates and Davenport/AP estimates may restrict or test the fixed terminal object, but they may not replace the terminal affine skeleton by a new one.*

Proof. By definition, the TC1/HighTC split is applied to the terminal list $\mathcal{L}_{\mathfrak{S}}$ produced by B1/B3/F3/F4/E5 before post-terminal estimation begins. A later analytic operation has one of two effects. It either restricts the summation domain or introduces auxiliary variables used to test, average, or estimate the already fixed terminal data. Neither effect changes the routing record $\mathfrak{r}$, the origin map, or the terminal class label.

If a proposed post-terminal step replaced the list $\{\ell_\rho\}$ by a new rank-relevant list and then used that new list as a terminal GoodAWACK skeleton, the step would be a skeleton-generating operation rather than a post-terminal analytic operation. By Lemma E10Y.0 and Lemma E10Y.3, such a step would have to appear in the pre-terminal routing record and be classified by F3/F4/E5. Therefore it cannot occur as a hidden post-terminal feedback operation.

—

□

E10Y.7. Apparent Operations Table The following table records the status of all operation types that can appear syntactically in the GoodAWACK branch.

Apparent operation	Mathematical status
Dyadic refinement	B1/B3 cell restriction; no affine rank drop
Product grouping	B3 finite candidate source; not a free affine map
Controlled CRT restriction	F3 operation; full-rank on the active difference lattice or incompatible
Fixed divisor quotient	F4/E5-compatible tagged quotient
Variable quotient residual	F4 tagged quotient or rerouting case
Square-divisor routing	F3/F4 Edge or zero/short-fibre routing, recorded by tag
Gcd/local/proportional relation	F4/HGO2R local or LocalDiag origin
CKP-balanced relation	Terminal CKP tag; not GoodAWACK HighTC residue
Strict saving or boundary relation	C1 Edge tag
Full-rank affine coordinate change	E5 content-stable transport; injective on the active affine span and terminal tensor-test span
Primitive or coarea slicing	Post-terminal analytic non-generator
Cauchy–Schwarz or cube expansion	Post-terminal analytic non-generator
Fourier expansion or TC1 testing	Post-terminal analytic non-generator
Local projection	H4/D1 local-main assembly; post-terminal non-generator
Arbitrary rank-dropping affine reparametrization	Not actual-generated unless one of the recorded tags is present

This table is not an extra assumption. It is the union of Lemmas E10Y.0–E10Y.5b.

—

E10Y.7A. Formal Transition Table This section records the routing grammar as transformations of the actual routing record

$$\mathfrak{r} = (V, \mathcal{C}, \mathcal{L}, \mathcal{Q}, \tau, \mathrm{orig}, W).$$

The table is part of the proof of E10Y. It is not an additional assumption: each row is the formal state-level version of a B1/B3/F3/F4/E5 operation already isolated above.

Operation	Input state	Output state	Rank effect	Required tag	If not satisfied
B1 start-state creation	no previous Branch B state	$V, \mathcal{C}, W$ from a fixed-depth Heath–Brown product block	no affine rank drop	start-state origin	not a Branch B descendant
B3 finite grouping	B1 state with product variables	one grouped candidate Γ, with updated $\mathcal{L}$ and preliminary τ	finite selection only; no free affine map	B3 grouping origin	candidate removed or routed by F3
controlled CRT absorption	state with congruence $L_0(z) \equiv a \pmod q$	restricted lattice/coset and updated $\mathcal{C}$	full-rank finite-index restriction on the active difference lattice, or empty	CRT	incompatible fibre, hence zero/empty
F4 quotient/divisor decision	state containing $d \mid L(z)$ or $L(z) = ds$	updated $\mathcal{Q}$, τ, and origin record	possible rank drop only through recorded quotient/local/CKP/Edge data	FixedDiv, VarQuot, LocalDiag, CKP, or Edge	routed away or $\mathfrak{M}^\sharp$ decreases
square-divisor routing	state with square-divisor predicate	Edge state, controlled divisibility state, or empty state	no untagged affine rank drop	Edge or CRT	zero/short-volume or strict C1P saving
grouping selection or elimination	finite B3/F3 candidate list	selected candidate, removed candidate, or decreased routing measure	finite selection; no new affine transformation	B3/F3 grouping origin	candidate eliminated
LocalDiag detection	state with equality, proportionality, repeated form, or forced local relation	terminal LocalDiag state	rank collapse leaves GoodAWACK	LocalDiag	continue F3/F4 routing
Edge detection	state satisfying a C1/C1A strict-saving predicate	terminal Edge state	any collapse is absorbed into a strict-saving route	Edge	continue F3/F4 routing
CKP detection	state exposing balanced bilinear Kloosterman-fraction structure	terminal CKP state	rank relation is a CKP origin, not a GoodAWACK residual	CKP	continue F3/F4 routing
GoodAWACK terminal labelling	state with no Edge, CKP, LocalDiag, LongAP/Local, or unresolved ordinary divisor predicate	terminal GoodAWACK skeleton	labelling only; no coordinate operation	terminal label	not a terminal GoodAWACK skeleton
E5 clean transport	already generated B1/B3/F3/F4 routing record	transported content/auxiliary data on the same record	full-rank on active and tensor-test spans, or inherited tagged rank drop	inherited origin tag, or no tag needed in full-rank case	not E5-clean; must be routed/tagged before terminality
post-terminal analytic non-generator	fixed terminal skeleton	test, slice, Fourier/coarea family, or estimate of the fixed object	may restrict analytic sums but cannot replace terminal tensor-test vectors	PostTerminalNonGenerator	if it changes terminal vectors, it is not post-terminal and must appear above

Consequently a rank-changing operation in an actual GoodAWACK descendant has only two possibilities. Either it is one of the pre-terminal rows and carries the displayed origin information, or it is a post-terminal analytic non-generator and cannot create a new terminal GoodAWACK skeleton.

—

E10Y.8. Bidirectional Source–Grammar Tables The following two tables make explicit that E10Y is not taking completeness as an unnamed premise. The first table maps each independently defined B1/B3/F3/F4/E5 operation to the E10Y transition class that covers it.

Source operation	E10Y transition class	Rank effect
B1 typed Heath–Brown block and dyadic cell	B1/B3 start-state generation	no affine rank drop
B3 finite product grouping candidate	B1/B3 start-state generation	no free affine map; grouping is recorded
F3 controlled CRT absorption	F3/F4 pre-terminal routing	finite-index restriction or incompatible fibre
F4 divisor or quotient decision	F3/F4 pre-terminal routing	tagged quotient, local, CKP, Edge, GoodAWACK, or decreasing continuation
F3 square-divisor routing	F3/F4 pre-terminal routing	Edge or controlled divisibility/CRT tag
F3 grouping selection/elimination	F3/F4 pre-terminal routing	finite candidate selection; no new affine operation
F3 LocalDiag detection	F3/F4 pre-terminal routing	terminal LocalDiag; leaves GoodAWACK
F3 Edge detection	F3/F4 pre-terminal routing	terminal Edge; leaves GoodAWACK
F3 terminal labelling	F3/F4 pre-terminal routing	label only
E5 full-rank content transport	E5 full-rank content-stability transport	rank preserved on active and tensor-test spans
E5 rank-dropping transport	E5 tagged content-stability transport	allowed only with an already recorded origin tag
TC1/HighTC, BRS/X16, Davenport/AP, Fourier, cube, coarea, local projection	post-terminal analytic non-generator	no new routing descendant and no replacement of terminal tensor-test vectors

Conversely, every E10Y transition class has only the following possible sources in the proof tree.

E10Y transition class	Possible sources	Excluded sources
B1/B3 start-state generation	B1 and B3	E5, F4, analytic estimates
F3/F4 pre-terminal routing	F3.6–F3.14 and F4	arbitrary affine reparametrization; Cauchy/cube/Fourier steps
E5 full-rank content-stability transport	E5 applied to an already generated routing record	external affine systems not produced by B1/B3/F3/F4
E5 tagged rank-dropping transport	E5 with a Fix/Proj, CRT, FixedDiv, VarQuot, LocalDiag, CKP or Edge origin already present	untagged rank-dropping AFF
post-terminal analytic non-generator	terminal analytic estimates applied after the terminal skeleton is fixed	any operation that changes the terminal tensor-test vectors for routing purposes

Thus a syntactically visible operation has two tests. It must appear in the left table as a source operation, and its E10Y class must have an allowed source in the right table. If either test fails, the operation is not an actual-generated GoodAWACK skeleton generator.

—

E10Y.9. Grammar Completeness Theorem

Theorem 10.8 (Theorem E10Y.6. Completeness of the GoodAWACK routing grammar)**.** *Every actual-generated operation that can generate or modify an actual terminal GoodAWACK affine skeleton is one of the operations in Lemmas E10Y.2–E10Y.5b. Equivalently, the GoodAWACK routing grammar has no hidden skeleton-generating operation outside:*

B1/B3 start-state generation,
F3/F4 pre-terminal routing,
E5 full-rank or tagged content-stability transport,
post-terminal analytic non-generators.

Consequently, any rank-dropping affine operation visible in an actual terminal GoodAWACK skeleton must have one of the recorded origin tags

Fix/Proj, CRT, FixedDiv, VarQuot, LocalDiag, CKP, Edge, PostTerminalNonGenerator,

or else the skeleton is not an actual B1-origin terminal GoodAWACK descendant.

Proof. Let $\mathfrak{S}$ be an actual terminal GoodAWACK skeleton. By Lemma E10Y.0, $\mathfrak{S}$ has a finite routing history

$$r_0 \to r_1 \to \cdots \to r_T = \mathfrak{S},$$

where r_0 is a B1/B3 start record and each transition is an actual-generated pre-terminal operation or a terminal class labelling step. We prove by induction on t the invariant

$$\mathcal{I}(r_t): \quad \text{every rank-changing operation up to } r_t \text{ is E10Y-classified and carries an allowed origin tag.}$$

At $t = 0$, Lemma E10Y.2 shows that B1 and B3 supply only product-coordinate and grouping start states. No untagged rank-changing affine operation has occurred.

Assume $\mathcal{I}(r_t)$ and consider $r_t \to r_{t+1}$. If the transition is an F3/F4 routing operation, Lemma E10Y.3 classifies it as controlled CRT, F4 divisor/quotient decision, square-divisor routing, finite grouping selection or elimination, LocalDiag detection, Edge detection, or terminal class labelling. Each rank-changing case either records one of the allowed tags or routes the cell away from terminal GoodAWACK. If the transition is an E5 transport, Lemma E10Y.4 says that it is either full-rank on the active and terminal tensor-test spans, or else rank-dropping with an already recorded origin tag. Thus $\mathcal{I}(r_{t+1})$ holds.

After terminal labelling, Lemmas E10Y.5 and E10Y.5b apply. Post-terminal analytic operations may restrict or test the fixed terminal object, but they do not produce a new routing descendant and do not replace the terminal tensor-test vectors. Hence they cannot violate the invariant by adding a hidden skeleton-generating operation.

These cases exhaust the route from B1 to the terminal GoodAWACK object. Therefore no additional skeleton-generating operation can occur. In particular, an untagged rank-dropping affine regrouping is not a permissible operation in the actual GoodAWACK routing grammar.

—

□

Parameter check 10.9 (E10Y.10. Parameter Check and Output Form)**.** The theorem introduces no new analytic parameter and no new error term. Its only finiteness input is the fixed-depth

B1 decomposition, the finite B3 grouping list, the finite F3T routing table and the finite F4 divisor/quotient decision tree. All content losses are those already controlled in E5.

The output supplied to E10X is:

$$\boxed{\text{the finite grammar used in E10X is complete for actual terminal GoodAWACK skeletons.}}$$

The output supplied to E10M/E10K/E10L is:

$$\boxed{\text{a rank-dropping affine regrouping in terminal GoodAWACK is admissible only with an allowed origin tag.}}$$

11 Part 10. E10M: No untagged rank-dropping AFF

Source file: `Lemmas/e10m_no_untagged_rank_dropping_aff_ltx.md`.

11.0.1 E10M. No Untagged Rank-Dropping AFF in Terminal GoodAWACK

E10M.0. Statement and Role Lemma **E10M** is the no-untagged-rank-drop theorem behind the E10K interface cleanup. Lemma E10Y proves that the GoodAWACK routing grammar is complete for actual B1-origin terminal skeletons. The master closure is Lemma E10X, which packages E10Y, E10M, and E10K into the finite GoodAWACK grammar interface.

The finality of the generator list used below is proved in E10Y. The E10S occurrence record is a reproducibility check for the source files; it is not a logical prerequisite for the theorem below.

The issue isolated by E10I–E10K is the following residual:

$$\boxed{\text{could a terminal GoodAWACK skeleton contain an untagged rank-dropping affine regrouping?}}$$

The answer is no, provided the terminal object is required to be an actual descendant of the active routing tree

$$B1 \to B3 \to F3/F4$$

and not merely a formal affine pattern allowed by broad wording in E5, BGS, BAOC or E10G.

Thus the result below is not a new analytic estimate. It is the structural version of the F3-complete routing interpretation: every terminal GoodAWACK skeleton must be generated by the finite operation list already proved in F3.6 and by the F4 large-divisor decision procedure.

Logical dependencies are B1, B3, F3, F4, E5, BGS, E10Y, E10I, E10J, and HGO2R. E10M is used by E10X, E10K, and E10L.

Role inside the E10X master closure E10M is the central structural input for the rank-dropping AFF obstruction isolated in E10H–E10J. Packaged by E10X, it discharges the active descendants of:

1. E10H.2, by proving that a formal 4AP-like matrix witness cannot be an untagged actual terminal GoodAWACK cell;

2. E10I.4, by proving that the only rank-dropping AFF residual left after the CRT/full-rank safety reductions has no untagged actual occurrence;

3. E10J.3, by proving AFF-origin completeness for actual B1/B3/F3/F4 terminal GoodAWACK routing histories.

The formal 4AP-like matrix family

$$Y_i = x + ir, \qquad 0 \le i \le 3,$$

is therefore not deleted from the proof. It is treated in E10X as an interface witness at the broad E5/BGS/BAOC level. E10M proves the decisive structural claim: if such a rank-dropping configuration is produced by actual routing, then its rank drop is tagged by one of the permitted origins; if it is untagged, it violates the E10Y-certified F3-complete routing interface and is not an admissible terminal GoodAWACK skeleton.

—

E10M.1. Setup: Definitions

Actual terminal GoodAWACK skeleton An actual terminal GoodAWACK skeleton is a skeleton

$$\mathfrak{S} = (\mathcal{B}, \Gamma, \mathfrak{r}, \Lambda_{\mathfrak{S}}, \Omega_{\mathfrak{S}}, \mathcal{L}_{\mathfrak{S}}, \mathcal{M}_{\mathfrak{S}}, \mathrm{orig}_{\mathfrak{S}}, \mathcal{W}_{\mathfrak{S}})$$

which occurs as the terminal record of a descendant produced by the E10Y-certified routing grammar, namely:

1. the typed Heath–Brown product variables and dyadic cells of Lemma B1;
2. the finite product-grouping candidates of Lemma B3;
3. the routing operations of Lemma F3, Section F3.6;
4. the large-divisor decision procedure of Lemma F4;
5. the content-stability transports of Lemma E5, read in the clean sense of E10Y/E10L/E10M.

Rank-dropping AFF occurrence A rank-dropping AFF occurrence is a bounded affine map used in the skeleton record whose linear part drops rank on the active affine span.

Such an occurrence is tagged if its rank drop is explicitly caused by one of:

1. fixing or projection;
2. congruence compatibility or an inconsistent fibre;
3. fixed divisor quotient;
4. variable quotient residual;
5. local/diagonal or gcd origin;
6. CKP-balanced origin;
7. strict C1P Edge origin;
8. post-terminal analytic slicing which is not used to generate the terminal tensor-test vectors.

It is untagged if it is recorded only as a free affine regrouping or affine parametrization, with no origin in the routing record.

—

E10M.2. Statement and Proof

Theorem 11.1 (Theorem E10M.1. No untagged rank-dropping AFF)**.** *Let $\mathfrak{S}$ be an actual terminal GoodAWACK skeleton. Then every rank-dropping AFF occurrence in its terminal record is tagged.*

Equivalently:

$$\boxed{\textit{no untagged rank-dropping affine regrouping survives into terminal GoodAWACK.}} \tag{E10M}$$

Proof. We trace all places where a rank drop could enter an actual terminal GoodAWACK skeleton. By Lemma E10Y, this trace exhausts all actual-generated skeleton-generating operations.

B1. By Lemma B1, the starting objects are product variables, dyadic cells and exact Heath–Brown convolution factors. B1 introduces no affine matrix map and no rank-dropping affine slice.

B3. By Lemma B3, the next operation is finite product-grouping selection. B3 may record grouping alternatives and preliminary labels, but it does not introduce a free affine parametrization. If a grouping exposes short factors, CKP-balanced structure, canonical local structure, forced dependence or an Edge predicate, the descendant is routed to the corresponding terminal class rather than being left as untagged GoodAWACK data.

F3. By Lemma F3, Section F3.6, the generic routing-level operations are exactly:

1. controlled CRT absorption;
2. F4 large-divisor decision;
3. square-divisor routing;
4. finite grouping selection/elimination;
5. terminal LocalDiag detection;
6. terminal Edge detection by C1P predicates;
7. terminal class labelling into CKP, GoodAWACK, LongAP/Local, Edge, LocalDiag.

Cauchy/cube operations and Fourier expansion are explicitly post-terminal proof subroutines, not generic routing operations. Hence F3 contains no operation whose effect is "add an arbitrary rank-dropping affine regrouping to the terminal skeleton."

Controlled CRT absorption is finite-index and full-rank on the difference lattice. It may change content by bounded or polylogarithmic factors controlled by E5, but it does not create a new rank-dropping affine relation. If the CRT conditions are incompatible, the fibre is impossible.

Square-divisor routing is either terminal C1 Edge when the square divisor is large, or a controlled CRT/divisibility restriction when the square divisor is small. In the second case it is subsumed by controlled absorption; in the first case it leaves GoodAWACK. Thus it does not create a free affine rank drop.

Finite grouping selection/elimination chooses among B3's already finite product-grouping candidates. It either keeps a candidate with its recorded origin data or removes it with a strict routing-measure decrease. It is not a new affine slice.

Terminal LocalDiag and terminal Edge detections are tagged by their defining local or C1P predicates. They leave the GoodAWACK class once detected.

Terminal GoodAWACK labelling is only a label. It records that after the F3/F4 decisions no Edge, CKP, LongAP/Local, LocalDiag or unresolved large-divisor predicate remains. It is not itself a coordinate operation.

F4. By Lemma F4, every ordinary large-divisor or quotient predicate is decided exhaustively. A rank drop coming from a fixed divisor quotient, variable quotient residual, gcd/local dependence, proportional or repeated forms, or quotient-determined forms is therefore tagged by the F4 origin record. The resulting descendant is routed to Edge, CKP, LocalDiag, LongAP/Local, GoodAWACK with the ambiguity resolved, or to a measure-decreasing continuation.

Thus F4 may create tagged rank-drop data, but it does not admit a free rank-dropping AFF occurrence with no origin.

E5. Lemma E5 supplies content stability for the allowed transports. The phrase "affine regrouping" in E5 cannot be read as an additional terminal-routing operation, because F3.6 is the exhaustive list of such operations. Therefore E5 may be used only for:

1. full-rank coordinate changes;
2. tagged fixing/projection or quotient/local transports;
3. tagged CKP, Edge or impossible-origin reductions;
4. post-terminal analytic slicing after the terminal tensor-test vectors have already been fixed.

This is precisely the E5-clean interpretation recorded in E10L.

Finally, post-terminal primitive slicing, Cauchy/cube and Fourier operations are analytic subroutines inside estimates. They are not allowed to generate a new terminal GoodAWACK skeleton after the TC1/HighTC tensor test has been declared.

All actual rank-dropping AFF occurrences are therefore tagged by one of the origins listed in E10M.1. An untagged rank-dropping AFF occurrence would have to come from an operation not present in B1, B3, F3.6 or F4, or from reading E5 as an extra terminal generator. Both alternatives contradict the E10Y-certified F3-complete routing interface. The theorem follows.

—

□

E10M.3. Finite Classification Table: AFF Occurrence Origins The proof above is summarized by the following finite classification table. The table records every mathematical source in the B1/B3/F3/F4/E5-clean routing grammar where a reader might suspect that a rank-dropping affine regrouping is introduced.

The table is exhaustive by E10Y and the structural source analysis in the proof above. The separate E10S/E10S-MECH records are non-logical reproducibility checks for the maintained source files; they are not used to prove the table.

Source/interface	Phrase or operation	Can drop rank?	If yes, tag source	Terminal generator?	Why no untagged AFF survives
B1	Heath–Brown product variables; dyadic cells	No	None needed	Yes, as initial product data	B1 creates product coordinates and weights, not affine regrouping maps.

B3	finite product grouping; preliminary labels	No as a new affine map	Existing B1 grouping record	Yes, only as candidate selection	B3 selects among finite product groupings; exposed dependence routes to CKP/LocalDiag/Edge/GoodAW labels with origin data.
F3, F3.6	controlled CRT absorption	No on the difference lattice	CRT compatibility / impossible fibre	Yes	CRT restriction is finite-index/full-rank on the active span; inconsistency is tagged impossible.
F3, F3.6/F3.9	square-divisor routing	No untagged affine rank drop	C1 square-divisor Edge or controlled divisibility tag	Yes	Large square divisors are terminal Edge; small square divisors become controlled absorption and inherit the CRT/divisibility tag.
F3, F3.6	finite grouping selection/elimination	No as a new slice	B3 grouping origin	Yes	Selection records or removes a candidate; it is not an additional affine operation.
F3, F3.6	LocalDiag detection	Yes only by forced equality/local dependence	LocalDiag tag	Yes, but leaves GoodAWACK	Once detected, the atom is terminal LocalDiag, not terminal GoodAWACK.
F3, F3.6	Edge detection	Yes only through strict saving predicate	C1 Edge tag	Yes, but leaves GoodAWACK	Once detected, the atom is terminal Edge and is handled by C1.
F3, F3.6	CKP detection	Yes only through gcd/balanced grouping	CKP tag	Yes, but leaves GoodAWACK	Once CKP-balanced structure appears, the atom is terminal CKP and is handled by G8a.
F3, F3.6	GoodAWACK labelling	No	None needed	Yes	The label records the absence of other terminal predicates; it performs no coordinate operation.
F4	fixed divisor quotient	Yes	F4 fixed-divisor origin	Yes	The quotient origin is recorded; untagged use is forbidden by the F4 decision procedure.
F4	variable quotient residual	Yes	F4 quotient-residual origin	Yes	The residual is routed to Edge/CKP/LocalDiag/LongAP/ with ambiguity resolved, or to a decreasing continuation.
F4	repeated/proportional forms	Yes	local/diagonal or C1/CKP origin	Yes	Forced dependence is terminally routed away or recorded as tagged origin data.

E5	affine regrouping/content stability	Only if read too broadly	E10M-clean full-rank or tagged transport	No	E5 is a stability lemma for transports already created by B1/B3/F3/F4; it is not an extra terminal generator.
BGS	skeleton record / r_{grp}	Records possible rank behavior	inherited origin tag	No	BGS records terminal data produced upstream; it does not create a new operation.
BAOC	weak transport catalogue, C5/T5 interface examples	Interface only	inherited B1/B3/F3/F4 origin	No	BAOC is catalogue/grammar support in the proof tree; broad catalogue classes are discharged by E10Y/E10X/E10K.
E10G	bounded AFF cell / FreeAffine-HighTC interface example	Reduction only	E10H–E10K chain	No	E10G isolates the free-affine class; it does not authorize a new terminal AFF map.
E10H	matrix-origin rigidity reduction	Reduces to CRT/AFF issue	matrix-origin reduction tag	No	E10H localizes the issue to E10I–E10K; it does not generate a terminal skeleton.
E10I	CRT and full-rank AFF safety	Full-rank only, except reduced residual	MOR/RDA reduction tag	No	E10I proves safe cases and passes only rank-dropping AFF to E10J/E10M.
E10J	tagged rank drops	Yes	origin-degenerate or routed tag	No	E10J proves tagged rank drops are safe and reduces only the untagged possibility to E10M.
post-terminal Cauchy/cube/Fourier steps	analytic slicing after terminal record	May restrict analytic sums	PostTerminalNonGenerator tag	No	These steps estimate a fixed terminal atom and cannot create a new terminal GoodAWACK skeleton.

Therefore the only conceivable source of an untagged rank-dropping AFF would be to read one of the record/stability documents as adding a new terminal operation outside F3.6 and F4. The E10Y-certified F3-complete interface forbids that reading: terminal GoodAWACK skeletons are actual descendants of B1/B3/F3/F4/E5-clean, not arbitrary formal affine systems.

—

E10M.4. Output Consequences

Corollary 11.2 (Corollary E10M.2. AFF-OC is discharged)**.** *The AFF-origin completeness hypothesis used in Lemma E10K is a theorem for actual terminal GoodAWACK skeletons:*

$$\boxed{\text{AFF-OC.}}$$

Thus E10K is no longer merely a conditional cleanup statement. Its F3-COMPLETE assumption is discharged by E10Y and E10M.

Corollary 11.3 (Corollary E10M.3. FreeAffineHighTC is empty in the proof tree)**.** *Combining E10M with the reductions in E10I and E10J gives:*

$$\boxed{R_{\mathrm{FreeAffineHighTC}}(N) = 0.}$$

Together with HGO2R, this leaves only the origin-degenerate HighTC cases, which route to CKP, LocalDiag, Edge or Impossible.

—

Remark 11.4 (E10M.5. Output).

$$\boxed{\text{E10M rules out untagged rank-dropping AFF in actual terminal GoodAWACK skeletons.}}$$

It is cited by E10X, E10K and E10L. Its role is to make explicit, after E10Y, that broad "affine regrouping" language in E5/BGS/BAOC/E10G is not an additional source of terminal GoodAWACK affine systems.

E10M.6. Logical Dependencies Internal dependencies: B1, B3, F3, F4, E5, BGS, E10Y, E10I, E10J, HGO2R.

Children served: E10X, E10K and E10L.

12 Part 11. E10X: Master finite-grammar closure

Source file: `Lemmas/e10_master_source_exhaustion_closure_ltx.md`.

12.0.1 E10X. Finite GoodAWACK Grammar Closure

E10X.0. Statement and Role Lemma **E10X** is the finite combinatorial closure theorem for the GoodAWACK HighTC branch. It packages the reduction chain

$$\mathrm{BAOC} \to \mathrm{E10G} \to \mathrm{E10H} \to \mathrm{E10I} \to \mathrm{E10J} \to \mathrm{E10Y/E10M} \to \mathrm{E10K}$$

into a single theorem-level interface.

The theorem is not a search assertion. Lemma E10Y proves that the finite grammar below is complete for actual terminal GoodAWACK skeletons. Lemma E10X uses that grammar and proves its invariant: every rank-dropping affine operation created along a derivation has an allowed origin tag. Formal affine counterexamples at the broad BAOC/E10G/E5 interface are therefore irrelevant unless they are derivable from

$$B1 \to B3 \to F3/F4$$

with E5 used only as clean content stability.

The theorem proved below is:

$$\boxed{\text{every actual terminal GoodAWACK skeleton has no untagged rank-dropping AFF source, hence no FreeAffineHi}} \tag{E10X}$$

This is a structural theorem, not a new analytic estimate.

Logical dependencies are B1, B3, F3, F4, E5, BGS, BAOC, HGO2R, E10G, E10H, E10I, E10J, E10Y, E10M, and E10K. E10X is used by E10L and the GoodAWACK HighTC closure.

—

E10X.1. Setup: Terminal GoodAWACK Skeletons and Grammar States An actual terminal GoodAWACK skeleton is a record

$$\mathfrak{S} = (\mathcal{B}, \Gamma, \mathfrak{r}, \Lambda_{\mathfrak{S}}, \Omega_{\mathfrak{S}}, \mathcal{L}_{\mathfrak{S}}, \mathcal{M}_{\mathfrak{S}}, \mathrm{orig}_{\mathfrak{S}}, \mathcal{W}_{\mathfrak{S}})$$

generated by the E10Y-certified finite grammar described below.

1. Lemma B1 supplies typed Heath–Brown product variables, dyadic cells and exact convolution weights.
2. Lemma B3 supplies a finite list of product-grouping candidates and preliminary tags.
3. Lemma F3, Section F3.6, supplies the complete F3 routing operations: controlled CRT absorption, the F4 large-divisor decision, square-divisor routing, finite grouping selection or elimination, terminal LocalDiag detection, terminal Edge detection, and terminal class labelling.
4. Lemma F4 supplies the exhaustive ordinary divisor and quotient decision, with recorded quotient, divisor, gcd, local, CKP, Edge, impossible, or continuation origins.
5. Lemma E5 supplies content stability for transports already generated by the previous routing layers. It is not an additional terminal generator of affine systems.

Thus terminal GoodAWACK skeletons are not arbitrary bounded affine systems. They are actual descendants of the finite routing grammar above, and Lemma E10Y proves that this grammar contains every actual-generated skeleton-generating operation.

—

E10X.2. Finite GoodAWACK grammar theorem Define the finite GoodAWACK grammar $\mathcal{G}_{\mathrm{GA}}$ as follows.

A state is a tuple

$$\mathfrak{s} = (V, \mathcal{L}, \mathcal{C}, \mathcal{Q}, \mathcal{T}, \mathcal{O}),$$

where V is the finite list of active variables inherited from B1, $\mathcal{L}$ is the finite list of affine forms visible on the current cell, $\mathcal{C}$ is the list of controlled CRT/content restrictions, $\mathcal{Q}$ is the list of divisor or quotient tags, $\mathcal{T}$ is the routing tag, and $\mathcal{O}$ is the origin record for every rank-changing operation already applied. The start states are exactly the B1/B3 grouped cells.

The transition set is finite and consists only of the following operations.

Transition type	Allowed effect on affine rank	Required origin tag or outcome
fixing/projection	may lower dimension by fixing variables already in V	Fix/Proj
controlled CRT restriction	full-rank on the active span, or incompatible	CRT or empty
fixed-divisor quotient	quotient by a recorded fixed divisor	FixedDiv

variable quotient residual	quotient/divisor residual selected by F4	VarQuot or rerouting tag
local/diagonal/gcd dependence	forced equality, proportionality, repeated form, or gcd-local relation	LocalDiag
CKP-balanced relation	balanced bilinear Kloosterman-fraction structure	CKP
strict saving or boundary relation	C1 Edge, square-divisor, short-volume, high-frequency, small-conductor, or boundary case	Edge
bounded affine regrouping	full-rank change on the active affine span, or rank-drop with recorded upstream origin	inherited tag
primitive/post-terminal slicing	occurs only after terminal tensor-test vectors are fixed	PostTerminalNonGenerator
E5 auxiliary inheritance	transports content or auxiliary variables already generated upstream	inherited tag; no terminal generator
terminal labelling	labels a terminal cell	Edge, CKP, GoodAWACK, LocalDiag, or LongAP/Local

Every transition is one of the operations authorized in F3.6/F3T or one of the F4 quotient outcomes. E5 transitions are allowed only when their input state already has an origin record; they cannot create a terminal GoodAWACK state from an arbitrary external affine system. By E10Y, no additional actual-generated skeleton-generating transition exists.

Theorem 12.1 (Theorem E10X.2A. Finite grammar invariant)**.** *For every state $\mathfrak{s}$ reachable in $\mathcal{G}_{\mathrm{GA}}$, every rank-dropping affine operation visible in $\mathfrak{s}$ carries one of the following origin tags:*

$$\text{Fix/Proj}, \quad \text{CRT}, \quad \text{FixedDiv}, \quad \text{VarQuot}, \quad \text{LocalDiag}, \quad \text{CKP}, \quad \text{Edge}, \quad \text{PostTerminalNonGenerator}.$$

Consequently, a reachable terminal GoodAWACK state has no untagged rank-dropping AFF operation.

Proof. We argue by induction on the length of the grammar derivation.

At length zero, the state is a B1/B3 grouped cell. Its affine forms are the original product-coordinate forms and their grouped descendants, and no rank-dropping affine operation has yet been applied. The assertion is therefore vacuous.

Assume the assertion for a reachable state $\mathfrak{s}$, and apply one transition.

- A fixing or projection transition records the tag Fix/Proj.
- A controlled CRT restriction is either incompatible, hence not terminal, or records the tag CRT with controlled content.
- A fixed divisor quotient records FixedDiv.
- A variable quotient residual records the F4 quotient tag VarQuot.
- A local, diagonal, gcd, repeated-form, or proportionality transition records LocalDiag.
- A balanced bilinear multiplicative transition records CKP.
- A strict saving, boundary, square-divisor, short-volume, high-frequency, or small-conductor transition records Edge.

- A post-terminal analytic slicing transition is allowed only after the terminal tensor-test vectors are fixed; it records PostTerminalNonGenerator and cannot create a new terminal affine skeleton.
- An E5 transition only transports controlled content, CRT data, or auxiliary variables already present in the input state. By definition of the E5-clean interface in E10X.1, it inherits the existing origin record and does not introduce a new untagged rank-dropping map.

Thus the invariant is preserved by every transition. Since the transition set is finite and every terminal GoodAWACK skeleton is, by Lemma E10Y, a reachable terminal state of $\mathcal{G}_{\mathrm{GA}}$, no terminal GoodAWACK skeleton contains an untagged rank-dropping AFF operation. The theorem is proved. □

Corollary 12.2 (Corollary E10X.2B. No free affine HighTC generator)**.** *Any formal affine configuration that cannot be derived from $\mathcal{G}_{\mathrm{GA}}$ is not an actual terminal GoodAWACK skeleton. In particular, a FreeAffineHighTC pattern can remain only as a formal interface witness unless it is produced by a grammar derivation; if it is produced by such a derivation, Theorem E10X.2A supplies an allowed origin tag and HGO2R/E10K route it to an already handled class.*

—

Verification record 12.3 (E10X.3. Non-Logical Verification Records)**.** The mathematical proof of E10X is E10Y plus the grammar invariant in Theorem E10X.2A. The records E10S and E10S-MECH are retained only as non-logical verification records for the maintained source files. They are useful for checking that later edits have not introduced language outside the finite grammar, but they are not premises of E10X.

The maintenance classes used in those records are:

1. **G0**, B1 initial product data;
2. **G1**, B3 finite product-grouping candidates;
3. **G2**, complete F3 routing operations;
4. **G3**, F4 quotient and divisor decisions;
5. **G4**, E5 content stability;
6. **G5**, BGS terminal skeleton recording;
7. **G6**, BAOC/E10G/E10H/E10I/E10J catalogue and reduction layers;
8. **G7**, E10Y/E10M/E10X/E10K/E10L completion and assembly layers, with E10S/E10S-MECH retained only as verification records;
9. **G8**, post-terminal analytic subroutines after the terminal skeleton has been fixed.

These classes mirror the mathematical transition table in E10X.2 and the classification table in E10M.3. They do not enlarge the proof graph and do not replace the mathematical completeness assertion E10Y or the invariant proof E10X.2A.

—

Parameter check 12.4 (E10X.4. Parameter Check: Stability Under Grammar Changes)**.** The E10X finite-grammar theorem is valid for the grammar specified in E10X.1–E10X.2. Any later change to the Branch B / GoodAWACK source files must be checked against the grammar before the package is redistributed.

The verification must be refreshed if any of the following occurs:

1. a Branch B / GoodAWACK source file is edited;
2. a new Branch B / GoodAWACK source file is added;
3. F3.6 gains, loses, or renames a routing operation;
4. new rank-changing, slicing, quotient, projection, regrouping, affine, coordinate, matrix, CRT, diagonal, or transport language is introduced;
5. E5 is read as a terminal generator rather than as E10-clean stability;
6. post-terminal analytic slicing is allowed to replace the terminal vectors used in the TC1/HighTC tensor test.

After such a change, the transition list and E10M table must be refreshed if the mathematical grammar has changed. The E10S/E10S-MECH records should then be refreshed as reproducibility support.

—

E10X.5. Output: No Untagged Rank-Dropping AFF By Lemma E10X.2A, the grammar invariant already proves that reachable terminal GoodAWACK states have no untagged rank-dropping AFF. By Lemma E10Y, this grammar is complete for actual terminal GoodAWACK skeletons. By Lemma E10M, every rank-dropping AFF occurrence found in an actual terminal GoodAWACK skeleton is one of the tagged grammar cases.

The allowed tags are:

1. fixing or projection;
2. congruence compatibility or impossible fibre;
3. fixed divisor quotient;
4. variable quotient residual;
5. local, diagonal, or gcd origin;
6. CKP-balanced origin;
7. strict C1P Edge origin;
8. post-terminal analytic slicing that does not generate the terminal tensor-test vectors.

Therefore an untagged rank-dropping affine regrouping cannot occur in an actual terminal GoodAWACK skeleton:

$$\boxed{\text{No-Untagged-AFF.}}$$

—

E10X.6. Interface Example: Formal $4AP$-Like Family The files E10G, E10H, E10I and E10J use the formal family

$$Y_i = x + ir, \qquad 0 \le i \le 3,$$

whose coefficient vectors $\ell_i = (1, i)$ satisfy

$$\ell_0 \odot \ell_0 - 3\ell_1 \odot \ell_1 + 3\ell_2 \odot \ell_2 - \ell_3 \odot \ell_3 = 0. \tag{4AP}$$

This example is admissible as a formal interface test: it shows that a broad phrase such as "bounded affine regrouping" is too large if it is read without the actual B1/B3/F3/F4 routing origin.

It is not a terminal GoodAWACK obstruction. Indeed, if such a family arises from a full-rank affine change on the active affine span, E10I shows that the TC1/HighTC tensor test is invariant and no new FreeAffineHighTC certificate is created.

If it arises from a rank-dropping map with fixing, projection, quotient, local, CKP, Edge, impossible, or post-terminal analytic origin, then HGO2R reroutes the resulting HighTC certificate to an already handled class.

If it arises only from an untagged rank-dropping affine parametrization, then it violates the E10Y-certified routing grammar and is not an actual terminal GoodAWACK skeleton by E10Y/E10M.

Thus the $4AP$-like example remains in the proof as a sharp interface test, while E10X proves that it has no untagged actual terminal occurrence.

—

E10X.7. Proof: AFF-Origin Completeness and FreeAffineHighTC Lemma E10K derives AFF-origin completeness from E10M:

$$\boxed{\text{every rank-dropping affine map in an actual terminal GoodAWACK skeleton has an allowed origin tag.}} \tag{AFF-OC}$$

By E10J, AFF-OC implies RDA, the rank-dropping AFF origin statement. By E10I, RDA eliminates the remaining matrix-origin class after CRT and full-rank AFF safety. By E10H and E10G, this eliminates the broad catalogue FreeAffineHighTC class:

$$\boxed{R_{\text{FreeAffineHighTC}}(N) = 0.}$$

Together with HGO2R, every HighTC GoodAWACK certificate is therefore either origin-degenerate and routed to CKP, LocalDiag, Edge, or Impossible, or it is an empty FreeAffineHighTC class.

—

E10X.8. Output for E10L The structural input inserted into Theorem E10L.4 is:

$$\text{E10X} \Longrightarrow \text{E10K} \Longrightarrow R_{\text{FreeAffineHighTC}}(N) = 0.$$

The TC1 contribution is handled independently by TNG, namely the chain

$$B1\text{-origin coarea} \to \text{TTH-SC} \to \text{MRT/TTD} \to \text{ROC/BRS} \to \text{TTH} \to \text{X9L-GT}.$$

Thus E10L closes GoodAWACK without X8:

$$\boxed{R_{\mathrm{GoodAWACK}}(N) = o(N).}$$

—

E10X.9. Logical dependencies Internal dependencies: B1, B3, F3, F3A, F3T, F4, E5, BGS, HGO2R, BAOC, E10G, E10H, E10I, E10J, E10Y, E10M, and E10K. E10S and E10S-MECH are verification records, not logical prerequisites.

Children served: E10L, E10G, E10H, E10I, E10J, the GoodAWACK manuscript section, and the E10 finite-grammar appendix.

13 Part 12. E10K: AFF-origin completeness

Source file: `Lemmas/e10k_aff_oc_affine_regrouping_origin_completeness_ltx.md`.

13.0.1 E10K. AFF-Origin Completeness

E10K.0. Statement and Role Lemma **E10K** proves AFF-origin completeness using E10Y and E10M.

Lemma E10Y proves that the GoodAWACK routing grammar is complete for actual B1-origin terminal skeletons. Lemma E10M proves that actual terminal GoodAWACK skeletons contain no untagged rank-dropping AFF occurrence. Therefore E10K is the AFF-OC consequence of E10Y plus E10M. Lemma E10X packages this implication as the finite GoodAWACK grammar theorem used by E10L. The E10S maintenance record is retained only as reproducibility support.

The target was:

$$\boxed{\text{AFF-OC: every rank-dropping affine regrouping in } \mathfrak{r}_{\mathrm{grp}} \text{ has an allowed origin tag.}}$$

The outcome is closure by E10Y grammar completeness and the E10M no-untagged-AFF lemma:

$$\boxed{\text{AFF-OC follows from E10Y plus E10M.}}$$

Lemma F3 contains the key fact:

$$\boxed{\text{generic F3 routing operations do not include arbitrary affine regrouping.}}$$

Therefore an untagged rank-dropping affine map cannot be a terminal-routing operation. E10Y records completeness of the actual B1/B3/F3/F4/E5 operation list, and E10M proves the no-untagged-rank-drop theorem on that list.

However, E5, BGS, BAOC, and E10G use broader language around "affine regrouping." That language is read through the following normalized interface:

$$\boxed{\text{affine regrouping may be used only as full-rank coordinate change, tagged projection/fixing, tagged quotient/loc}}$$

With this normalization, made explicit in E10Y and E10M, AFF-OC holds and hence the structural FreeAffineHighTC obstruction disappears.

Logical dependencies are B1, B3, F3, F4, E5, BGS, E10G, E10H, E10I, E10J, E10Y, E10M, and HGO2R. E10K is used by E10X and E10L.

—

E10K.1. Setup: Complete Terminal-Routing Operations Lemma F3, Section F3.6, states that F3 has only the following generic routing-level operations:

1. controlled CRT absorption;
2. F4 large-divisor decision;
3. square-divisor routing;
4. finite grouping selection/elimination;
5. terminal LocalDiag detection;
6. terminal Edge detection by C1P predicates;
7. terminal class labelling into CKP, GoodAWACK, LongAP/Local, Edge, LocalDiag.

It also explicitly says that Cauchy/cube and Fourier expansion are not generic F3 routing operations, but post-terminal proof subroutines.

We use the corresponding reading:

$$\boxed{\text{arbitrary rank-dropping affine regrouping is not a generic F3 routing operation.}} \tag{F3-COMPLETE}$$

This is not an extra mathematical estimate. It is a bookkeeping consequence of the finite operation list in F3.6.

—

E10K.2. Setup: Allowed Meanings of Affine Regrouping Under F3-COMPLETE, every occurrence of "affine regrouping" in the Branch B infrastructure must be interpreted as one of the following.

A1. Full-rank coordinate change The linear part is full-rank on the active affine span. By E10I, this is tensor-safe:

$$\text{TC1/HighTC}$$

is invariant under the induced rationally injective map on symmetric tensors.

A2. Tagged fixing/projection Some coordinates are fixed by dyadic slicing, conditioning, congruence compatibility, or an already recorded routing restriction.

This is rank-dropping, but the rank drop is tagged. If it creates a HighTC relation, the relation is caused by recorded projection data and is not FreeAffine.

A3. Tagged F4 quotient/divisor/local origin The rank drop is produced by:

1. fixed divisor quotient;
2. variable quotient residual;
3. fixed gcd/local dependence;

4. repeated/proportional forms;

5. quotient-determined active forms.

These are exactly F4/BGS origin-degenerate cases and are routed by HGO2R.

A4. Tagged CKP or Edge origin The rank drop exposes:

1. B3 CKP-balanced finite-convolution structure;

2. strict C1P Edge saving;

3. empty/impossible support.

These are terminally handled outside GoodAWACK.

A5. Post-terminal analytic slicing Primitive slicing or Cauchy/cube operations may reduce dimension inside E10's proof.

They are not terminal-routing operations generating the GoodAWACK skeleton. For the TC1/HighTC test, the pre-slicing affine vectors remain the objects being tested; this is the E10H/E10I interface.

—

E10K.3. Statement and Proof: AFF-OC after E10Y and E10M

Theorem 13.1 (Theorem E10K.1. AFF-origin completeness)**.** *By Lemma E10Y, the terminal GoodAWACK skeleton is generated by the complete GoodAWACK routing grammar. By Lemma E10M, actual terminal GoodAWACK skeletons contain no untagged rank-dropping AFF occurrence.*

Then every rank-dropping affine map recorded in

$$\mathfrak{r}_{\mathrm{grp}}$$

for an actual terminal GoodAWACK skeleton has one of the allowed origin tags A2–A5. Equivalently:

$$\boxed{\textit{there is no untagged rank-dropping AFF map in terminal GoodAWACK.}}$$

Proof. Let $\mathfrak{S}$ be an actual terminal GoodAWACK skeleton produced by

$$B1 \to B3 \to F3/F4.$$

This is now a direct consequence of E10Y and E10M. For completeness, we recall the mechanism.

By Lemma B1, the starting data are product variables and dyadic cells. No affine rank-dropping map is introduced at B1.

By Lemma B3, the grouping choices are finite product groupings. They are recorded as grouping alternatives. If they reveal short factors, CKP-balanced structure, local AP structure, or forced dependence, the atom is routed to Edge, CKP, LongAP/Local, or LocalDiag. If not, the residual may feed BranchB/GoodAWACK, but B3 has not introduced an arbitrary rank-dropping affine map; it has only selected product groupings and tags.

By F3.6, terminal-routing operations are exactly controlled CRT absorption, F4 large-divisor decision, square-divisor routing, finite grouping selection/elimination, terminal LocalDiag detection, terminal Edge detection, and terminal class labelling.

Controlled CRT absorption is finite-index/full-rank on the difference lattice and is tensor-safe by E10I.

F4 large-divisor decision produces either:

1. Edge;
2. LocalDiag;
3. CKP;
4. GoodAWACK after fixed quotient/divisor ambiguity is resolved.

Any rank drop caused by fixed divisor, variable quotient, gcd-local dependence, or quotient-determined forms is therefore tagged by F4 origin data.

Finite grouping selection/elimination does not create an untagged affine slice. It only chooses among the finitely many B3 product groupings and either terminally routes the atom or eliminates the grouping.

Terminal LocalDiag and Edge detections are tagged by their definitions.

Terminal GoodAWACK labelling is not a new operation. It only declares that after the above decisions no unresolved Edge, CKP, LongAP/Local, LocalDiag, ordinary divisor, or grouping alternative remains.

Therefore any rank-dropping affine map that remains in the terminal GoodAWACK skeleton must have been one of:

1. a recorded fixing/projection;
2. an F4 quotient/divisor/local origin;
3. a CKP/Edge/impossible origin;
4. post-terminal analytic slicing not used as the terminal tensor object.

These are exactly A2–A5.

Thus no untagged rank-dropping AFF map survives terminal GoodAWACK. E10Y proves that the preceding list is complete for actual B1-origin terminal skeletons, and E10M proves that each rank-dropping occurrence in that complete list is tagged. AFF-OC follows. The theorem is proved.

—

□

E10K.4. Output for RDA and FreeAffineHighTC By E10J, RDA follows from AFF-OC.

Therefore, using E10Y and E10M:

$$\text{RDA}$$

holds.

By E10I, RDA eliminates the remaining MOR obstruction.

By E10H, this eliminates the remaining matrix-origin obstruction.

By E10G and HGO2R, this eliminates the residual FreeAffineHighTC branch:

$$\boxed{R_{\mathrm{FreeAffineHighTC}}(N) = 0.}$$

Combining with the TC1 global-testing route and origin-degenerate HighTC rerouting gives:

$$R_{\mathrm{GoodAWACK}}(N) = o(N)$$

without using X8.

—

Parameter check 13.2 (E10K.5. Parameter Check: Interface Normalization Supplied by E10Y/E10M/E10K and Consumed by E10L)**.** The proof above depends on reading F3.6 as complete for actual F3 routing and on excluding post-terminal analytic operations from the list of skeleton generators. E10Y makes this grammar-completeness statement explicit, E10M proves the no-untagged-rank-drop theorem on that grammar, and E10K packages the resulting AFF-origin-completeness interface. E10L consumes the already normalized E10Y/E10M/E10K interface when estimating the terminal GoodAWACK contribution.

The broader language in the auxiliary Branch B documents is read as follows to avoid reintroducing an untagged AFF operation:

E5 cleanup Lemma E5 uses the phrase:

affine regrouping

among "allowed F3 operations."

E5 is a conditional content-stability lemma for transports whose origin tags have already been recorded by B1/B3/F3/F4. It does not obtain its meaning from E10L, and it does not introduce a new terminal GoodAWACK generator.

Equivalently, in the structural grammar language:

1. full-rank affine coordinate changes preserve content;
2. rank-dropping affine maps are allowed only when tagged by fixing/projection, quotient/divisor/local origin, CKP, Edge, impossible, or post-terminal analytic slicing;
3. Cauchy/cube and primitive slicing are post-terminal E10 proof operations, not terminal-routing operations creating new GoodAWACK skeletons.

BGS cleanup Lemma BGS records

$$\mathfrak{r}_{\mathrm{grp}}$$

as affine regrouping or affine changes of variables.

In the clean skeleton record $\mathfrak{r}_{\mathrm{grp}}$ records only:

1. B3 product grouping choices;
2. full-rank coordinate changes;
3. tagged rank drops of the types A2–A5.

BAOC/E10G cleanup The weak BAOC grammar and E10G catalogue do not serve as independent terminal generators of arbitrary rank-dropping bounded affine maps. Their broad C5/T5 cell is normalized by the E10Y/E10M/E10K interface before E10L uses it:

1. full-rank AFF, tensor-safe;
2. tagged rank-dropping AFF, origin-degenerate or post-terminal analytic;
3. forbidden untagged rank-dropping AFF.

—

Remark 13.3 (E10K.6. Output).

$$\boxed{\text{AFF-OC is proved for actual terminal GoodAWACK skeletons by E10Y and E10M.}}$$

Mathematical consequence:

$$\boxed{R_{\text{FreeAffineHighTC}}(N) = 0}$$

inside the active B1/B3/F3/F4/E5 routing tree.
This is the structural input used by E10L to close the HighTC class without X8.

E10K.7. Logical Dependencies Internal dependencies: B1, B3, F3, F4, E5, BGS, E10G, E10H, E10I, E10J, E10Y, E10M, and HGO2R.

Children served: E10L.

14 Part 13. E10L: Clean Branch B closure

Source file: `Lemmas/e10l_e10_clean_branch_b_ltx.md`.

14.0.1 E10L. Branch B GoodAWACK Theorem without X8

E10L.0. Statement and Role Lemma **E10L** is the Branch B / GoodAWACK theorem. It is backed by E10Y and E10X. Lemma E10Y proves completeness of the GoodAWACK routing grammar for actual B1-origin terminal skeletons. Lemma E10X packages E10Y/E10M/E10K and proves by a grammar invariant that an actual terminal GoodAWACK skeleton cannot contain an untagged rank-dropping AFF occurrence. E10S is a verification record for the maintained source files, not a logical input to the grammar invariant. The TC1 analytic side uses X16C through BRS/TTH, and Lemma TNG records the complete B1-origin near-global chain showing that TGT uses X9L-GT only after the MRT/PACK branch and the TTH length lower bound are in force.

It is the normalized Branch B interface for:

$$\text{E5}, \quad \text{BGS}, \quad \text{BAOC}, \quad \text{E10G}, \quad \text{HGO2R/E10H/E10I/E10J}.$$

The route is:

$$\boxed{\text{TGD} + \text{TNG} + \text{BGS/HGO2R} + \text{E10Y} + \text{E10X} + \text{E10K} \Longrightarrow R_{\text{GoodAWACK}}(N) = o(N)}$$

and uses no X8.

Logical dependencies are E5, BGS, HGO2R, TGD, TGT, TNG, MRT, TTD, ROC, BRS, TTH, X16BRS, X16C, E10Y, E10I, E10J, E10M, E10X, E10K, C1, G8a, H4, and X9L-GT. E10L is used by I1 and the final assembly.

—

E10L.1. Setup: Normalized Terminal-Routing Interface The interface is the F3-complete routing interface:

$$\boxed{\text{terminal-routing operations are exactly those listed in F3.6.}} \tag{F3-COMPLETE}$$

Thus a terminal GoodAWACK skeleton may be generated only by:

1. the initial B1 typed product variables and B3 product grouping choices;
2. controlled CRT absorption;
3. F4 large-divisor decisions;
4. square-divisor routing;
5. finite grouping selection/elimination;
6. terminal LocalDiag detection;
7. terminal Edge detection by C1P predicates;
8. terminal class labelling into CKP, GoodAWACK, LongAP/Local, Edge, or LocalDiag.

In particular:

$$\boxed{\text{arbitrary rank-dropping affine regrouping is not a generic terminal-routing operation.}} \tag{No-Untagged-AFF}$$

This is not an extra estimate. It is a structural consequence of the proof interface, made explicit in Lemma E10Y and used in Lemma E10X.

—

E10L.2. Setup and Proof: E5-Clean Content Stability The phrase "affine regrouping" in Lemma E5 is read in the following normalized sense, whose completeness for actual GoodAWACK descendants is supplied by Lemma E10Y.

E5-clean A. Full-rank affine coordinate changes If the affine change has full rank on the active affine span, then it is allowed. It preserves the TC1/HighTC tensor classification by E10I, and it preserves controlled content by the usual bounded-minor content calculation already used in E5.

E5-clean B. Tagged rank-dropping maps A rank-dropping affine map is allowed only if its rank drop is explicitly tagged by one of the existing origins:

1. fixing or projection;
2. fixed divisor quotient;
3. variable quotient residual;
4. local/diagonal origin;
5. CKP origin;
6. strict C1P Edge origin;
7. impossible/inconsistent fibre;
8. post-terminal analytic slicing which does not create a new terminal GoodAWACK skeleton.

If such a tagged rank drop creates a HighTC relation, then the relation is origin-degenerate and is rerouted by HGO2R.

E5-clean C. Post-terminal proof operations Cauchy/cube operations, primitive slicing, and Fourier expansion are post-terminal analytic subroutines inside E10-type estimates. They do not enlarge the list of terminal-routing operations and do not create new terminal GoodAWACK skeletons. For the TC1/HighTC test, the pre-slicing terminal affine vectors are the objects being tested.

Lemma 14.1 (Lemma E10L.1. E5-clean content stability)**.** *Under E5-clean A–C, content stability remains exactly the content stability proved in Lemma E5, but no untagged rank-dropping affine map is available as a terminal-vector generator.*

Proof. Full-rank affine changes are covered by the bounded-minor content argument. Tagged rank drops are already part of fixing, projection, quotient, local, CKP, Edge, or impossible routing data. Post-terminal proof operations happen after the terminal skeleton has been fixed and therefore cannot introduce a new terminal GoodAWACK affine system. These cases exhaust the normalized meanings of "affine regrouping" by Lemma E10Y. Hence E5 content stability is preserved and No-Untagged-AFF is enforced.

—

□

E10L.3. Setup and Proof: BGS-Clean Skeleton Record In the skeleton normal form, replace the broad phrase

$$\mathfrak{r}_{\mathrm{grp}} \text{ records affine regrouping or affine changes of variables}$$

by:

$$\boxed{\mathfrak{r}_{\mathrm{grp}}^{\mathrm{clean}} \text{ records only B3 product groupings, full-rank coordinate changes, and tagged rank drops.}} \tag{BGS-clean}$$

The tagged rank drops are precisely the cases listed in E5-clean B.

Lemma 14.2 (Lemma E10L.2. BGS-clean is equivalent to F3-complete routing)**.** *Every actual terminal GoodAWACK descendant admitted by Lemma BGS has a clean skeleton record.*

Proof. By Lemma E10Y, it is enough to check the complete B1/B3/F3/F4/E5 routing grammar. By B1, the starting variables are product variables and dyadic cells. By B3, only finitely many product groupings are selected. By F3.6, the subsequent routing-level operations are controlled CRT absorption, F4 decisions, square-divisor routing, finite grouping selection/elimination, LocalDiag detection, Edge detection, and terminal class labelling. None of these is an arbitrary untagged rank-dropping affine map.

Therefore every rank drop that appears in the terminal record must come from fixing/projection, quotient/divisor/local data, CKP, Edge, impossible fibres, or post-terminal analytic slicing. These are exactly the tagged rank drops in BGS-clean. The lemma follows.

—

□

E10L.4. Setup and Proof: BAOC/E10G-Clean Affine Cell The broad BAOC/E10G affine cell

$$\mathrm{C5/T5} = \text{bounded affine regrouping}$$

is replaced by the disjoint trichotomy:

$$\mathrm{C5a/T5a} = \text{full-rank AFF, tensor-safe;}$$

$$\mathrm{C5b/T5b} = \text{tagged rank-dropping AFF, origin-degenerate or post-terminal analytic;}$$

$$\mathrm{C5c/T5c} = \text{untagged rank-dropping AFF, forbidden by E10Y/F3-COMPLETE.}$$

Lemma 14.3 (Lemma E10L.3. The 4AP-like witness is not an admissible clean terminal cell)**.** *The formal affine family*

$$x, \quad x+r, \quad x+2r, \quad x+3r$$

which appears as an interface example in E10G is not, by itself, an admissible C5-clean terminal GoodAWACK cell.

Proof. If the family is produced by a full-rank change of variables on the active affine span, then it is tensor-safe by E10I and cannot create a new FreeAffineHighTC certificate.

If it is produced by a rank-dropping map with a fixing, quotient, local, CKP, Edge, impossible, or post-terminal analytic tag, then it is origin-degenerate and is handled by HGO2R.

If it is produced only by an untagged rank-dropping affine parametrization, then it violates the E10Y-certified F3-complete grammar and is not a terminal-routing operation. Hence the interface family marks exactly the broadness of the C5/T5 wording, not an admissible clean terminal GoodAWACK cell.

—

□

E10L.5. Output Theorem: Branch B GoodAWACK Cancellation

Theorem 14.4 (Theorem E10L.4. GoodAWACK cancellation)**.** *For every tagged terminal GoodAWACK atom produced by the B1/B3/F3/F4/E5 interface, its total contribution is $o(N)$, using the TC1 interfaces MRT/PACK and the X16-Core carrier-slice input. Consequently*

$$\boxed{R_{\text{GoodAWACK}}(N) = o(N).} \tag{E10-clean}$$

The proof uses no X8.

Proof. By Lemma TGD, every terminal GoodAWACK atom belongs to exactly one of:

$$\text{TC1-GoodAWACK}, \qquad \text{HighTC-GoodAWACK}.$$

The TC1 part is handled by Theorem TNG-A, the TC1 near-global-or-routed theorem. For every unrouted B1-origin coarea test produced by TGT, TNG-A gives exactly one of two outcomes:

1. the test is MRT-admissible, satisfies PACK and $H \geq X(\log X)^{-B_\kappa}$, and is therefore an admissible input to the near-global Davenport/AP theorem X9L-GT;
2. before X9L-GT is invoked, TTD/ROC/BRS together with X16BRS/X16C routes the test to Edge, LongAP/Local, CKP, LocalDiag, or empty support.

Thus the TC1 branch is controlled by one theorem-interface, not by a free choice among short fibres. Combining TNG-A with X9L-GT gives

$$R_{\text{TC1-GoodAWACK}}(N) = o(N).$$

This uses no X8 and no pointwise shifted short-interval X9L-SI input.

Now consider the HighTC part. By HGO2R, every origin-degenerate HighTC certificate reroutes to one of:

$$\text{CKP}, \qquad \text{LocalDiag}, \qquad \text{Edge}, \qquad \text{Impossible}.$$

These are already handled by G8a, H4, C1, or contribute zero.

The only formal HighTC class not covered by HGO2R is

$$\text{FreeAffineHighTC}.$$

By Lemma E10Y, the finite GoodAWACK grammar is complete for actual terminal skeletons. By Lemma E10X, actual terminal GoodAWACK skeletons contain no untagged rank-dropping AFF occurrence, and Lemma E10K gives AFF-origin completeness. Hence:

$$R_{\text{FreeAffineHighTC}}(N) = 0.$$

Therefore the whole HighTC contribution is either rerouted to already handled classes or empty as a GoodAWACK class. Combining this with the TC1 estimate gives $R_{\text{GoodAWACK}}(N) = o(N)$. The theorem is proved.

—

□

E10L.6. Dependency Profile The E10 theorem depends on:

$$\text{TGD+TGT+TNG+TTD+MRT+ROC+BRS+X16BRS/X16-Core+TTH+X9L-GT+BGS/HGO2R+E10I+E10J}$$

together with the standard already-used inputs:

$$\text{X5}, \quad \text{E5-clean}, \quad \text{C1}, \quad \text{G8a}, \quad \text{H4}.$$

The E10 theorem does not depend on:

$$\boxed{\text{X8.}}$$

The full nilsequence form of X9, the pointwise-fibre TC1 route, and the pointwise shifted X9L-SI input are not used. The external input is only the near-global Davenport/AP X9L-GT form used after TTH.

—

Remark 14.5 (E10L.7. Output).

$$\boxed{\text{Branch B / GoodAWACK cancellation is proved using E10Y/E10X/E10K and the near-global Davenport/AP X}}$$

E10L supplies

$$\boxed{R_{\text{GoodAWACK}}(N) = o(N)}$$

for the final assembly.

E10L.8. Logical Dependencies External dependency: X9L-GT in the near-global Davenport/AP range.

Internal dependencies: E5, BGS, HGO2R, TGD, TGT, TNG, MRT, TTD, ROC, BRS, TTH, X16BRS, X16C, E10Y, E10I, E10J, E10M, E10X, E10K, C1, G8a, and H4.

Children served: I1 and the final proof assembly.