Final Modular Assembly Theorem Package

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Abstract

This package gives an alternative final assembly of the proof. Unlike the full manuscript, it is organized as a modular theorem-package closure: four standalone theorem packages supply the main technical bricks, the base B1/B3/F3/F4 source layer supplies exact decomposition and routing, and the final I1/G2/G1/G0H layer converts the weighted asymptotic into the sufficiently-large binary Goldbach statement.

Contents

1 The Modular Inputs

The assembly uses four theorem packages.

1.1 H4 Local Algebra

The H4 package proves that all admitted local/main terms are canonical tagged Λ_Q-local projections inside the original B1/F3 routing cells, that no local term is double counted, and that the finite CRT factors reconstruct the Goldbach singular series:

$$M_{\text{local}}(N) = \mathfrak{S}(N)N + o(N). \tag{1.1}$$

1.2 TNG/TC1 No Rogue Short Interval

The TNG/TC1 package proves that every actual B1-origin TC1 coarea test reaching the Liouville/Fourier input is near-global:

$$H \geq X(\log X)^{-B},$$

or else is routed to Edge, LongAP/Local, CKP, LocalDiag, empty support, or a finite grammar branch before the Liouville input is invoked. Thus the proof does not use pointwise cancellation on arbitrary shifted short intervals.

1.3 GoodAWACK Finite Grammar

The GoodAWACK finite grammar package proves that actual terminal GoodAWACK skeletons are generated by the finite B1/B3/F3/F4/E5-clean routing grammar. The grammar preserves the no-untagged-rank-drop invariant and excludes untagged rank-dropping affine regrouping. Hence the HighTC-GoodAWACK residual is empty or routed to already handled terminal classes.

1.4 CKP/X10/X16 Analytic Package

The CKP/X10/X16 package proves two analytic outputs:

$$\text{central CKP nonzero frequencies} = o(N),$$

after DFI/X10 is applied to the actual two-variable CKP smooth weight, and

$$\text{X16BRS carrier-slice estimate holds},$$

after reduction to the X16C Shiu/AP product-carrier estimate.

2 Base Exact Decomposition and Routing

The proof starts from the ordered von Mangoldt convolution

$$R_\Lambda(N) = \sum_{\substack{n_1+n_2=N \\ n_1,n_2\geq 1}} \Lambda(n_1)\Lambda(n_2). \tag{2.1}$$

B1 gives an exact fixed-depth typed Heath–Brown decomposition

$$R_\Lambda(N) = \sum_{\mathcal{B}\in\mathfrak{B}_{J_0}} c_{\mathcal{B}} R_{\mathcal{B}}(N). \tag{2.2}$$

No error term is introduced in (2.2).

B3/F3/F3T/F4 give an exact tagged terminal partition of every parent block:

$$R_{\mathcal{B}}(N) = \sum_{\tau \in \mathcal{T}(\mathcal{B})} R_{\mathcal{B},\tau}(N). \tag{2.3}$$

The terminal classes are

Edge, LongAP/Local, CKP, GoodAWACK, LocalDiag.

They are disjoint at the routing-history level and exhaust all descendants. Therefore

$$\begin{aligned} R_{\Lambda}(N) =& R_{\mathrm{Edge}}(N) + R_{\mathrm{LongAP}}(N) + R_{\mathrm{CKP}}(N) \\ &+ R_{\mathrm{GoodAWACK}}(N) + R_{\mathrm{LocalDiag}}(N). \end{aligned} \tag{2.4}$$

3 Terminal Estimates

3.1 Edge

C1A verifies that every terminal Edge input satisfies a strict C1P predicate. C1 then gives

$$R_{\mathrm{Edge}}(N) = o(N). \tag{3.1}$$

3.2 LongAP/Local

F3P gives the positive local-coefficient predicate for LongAP/Local atoms, and D1 expands the resulting LongAP/local algebra into an LPI-admitted local projection:

$$R_{\mathrm{LongAP}}(N) = M_{\mathrm{LongAP}}(N) + o(N). \tag{3.2}$$

3.3 CKP

For CKP, the zero-frequency term is a canonical LPI-admitted local projection later assembled by H4. The central nonzero-frequency terms are $o(N)$ by the CKP/X10/X16 analytic package, and all noncentral CKP ranges are routed by X10ER and C1P/C1A/C1 before X10 is invoked. Hence

$$R_{\mathrm{CKP}}(N) = M_{\mathrm{CKP}}(N) + o(N). \tag{3.3}$$

3.4 GoodAWACK

The GoodAWACK branch is split into TC1 and HighTC alternatives.

The TC1 alternative is controlled by the TNG/TC1 package, together with the near-global X9L/Davenport input. The BRS carrier-slice estimates used inside the TC1 chain are supplied by the X16BRS part of the CKP/X10/X16 analytic package.

The HighTC alternative is closed by the GoodAWACK finite grammar package. Therefore

$$R_{\mathrm{GoodAWACK}}(N) = o(N). \tag{3.4}$$

3.5 LocalDiag

LocalDiag terms are not error terms. They are canonical local/main terms admitted by H4. Denote their total tagged local contribution by

$$M_{\mathrm{LocalDiag}}(N). \tag{3.5}$$

4 Local Main Term Assembly

Collect all LPI-admitted local terms assembled by H4:

$$M_{\mathrm{local}}(N) = M_{\mathrm{LongAP}}(N) + M_{\mathrm{CKP}}(N) + M_{\mathrm{LocalDiag}}(N). \tag{4.1}$$

There is no fourth local summand. Auxiliary local-looking terms created by controlled CRT absorption, fixed-divisor quotienting, or primitive local slicing are tagged refinements of one of the three displayed classes, while endpoint and smooth-boundary localizations are C1 Edge errors.

By the H4 local algebra package,

$$M_{\mathrm{local}}(N) = \mathfrak{S}(N)N + o(N). \tag{4.2}$$

The singular series is normalized for ordered Goldbach pairs:

$$\mathfrak{S}(N) = 2C_2 \prod_{\substack{p|N \\ p>2}} \frac{p-1}{p-2}, \qquad C_2 = \prod_{p>2} \left(1 - \frac{1}{(p-1)^2}\right) > 0. \tag{4.3}$$

5 Weighted Assembly

Using the terminal partition and the terminal estimates,

$$R_\Lambda(N) = M_{\mathrm{local}}(N) + o(N). \tag{5.1}$$

The global error budget GEB records that the terminal $o(N)$ terms remain $o(N)$ after the finite and polylogarithmic terminal summations, including Edge savings, CKP derivative losses, TC1 Davenport/AP losses, X16BRS carrier-slice losses, and local boundary terms.

Combining (5.1) with the H4 local algebra identity gives I1:

$$\boxed{R_\Lambda(N) = \mathfrak{S}(N)N + o(N).} \tag{5.2}$$

6 Prime-Power Removal

Define the genuine prime-prime weighted sum

$$R_{pp}(N) = \sum_{\substack{p_1+p_2=N \\ p_1,p_2 \text{ prime}}} \log p_1 \log p_2, \tag{6.1}$$

again over ordered pairs.

G2 proves

$$R_\Lambda(N) - R_{pp}(N) = O(N^{1/2}(\log N)^2) = o(N). \tag{6.2}$$

The difference is exactly the nonnegative contribution of ordered pairs in which at least one coordinate is a nontrivial prime power p^k, $k \geq 2$. There are $O(N^{1/2})$ such possible coordinates up to N, the other coordinate is determined, and each Λ-weight is at most $\log N$.

From (5.2) and (6.2),

$$R_{pp}(N) = \mathfrak{S}(N)N + o(N). \tag{6.3}$$

7 Positivity and Strong Goldbach

For even N, every factor $(p-1)/(p-2)$ in (4.3) is positive and at least one, so

$$\mathfrak{S}(N) \geq 2C_2 > 0. \tag{7.1}$$

Hence, for all sufficiently large even N,

$$R_{pp}(N) \geq C_2 N > 0. \tag{7.2}$$

Every summand in $R_{pp}(N)$ is nonnegative and is strictly positive exactly for a genuine prime representation $N = p_1 + p_2$. Therefore $R_{pp}(N) > 0$ implies that such a representation exists.

Thus

$$\boxed{\text{every sufficiently large even } N \text{ is a sum of two primes.}} \tag{7.3}$$

8 Output Theorem

Theorem 1 (Final modular assembly)**.** *Assume the base exact decomposition and routing lemmas B1/B3/F3/F3T/F4, the Edge and LongAP/Local lemmas C1P/C1A/C1 and D1, the global parameter and error-budget lemmas PAR/GEB, and the four standalone theorem packages:*

1. *H4 Local Algebra;*
2. *TNG/TC1 No Rogue Short Interval;*
3. *GoodAWACK Finite Grammar;*
4. *CKP/X10/X16 Analytic.*

Then I1 gives

$$R_\Lambda(N) = \mathfrak{S}(N)N + o(N).$$

Together with G2 and G1/G0H this proves that every sufficiently large even integer is a sum of two primes.

9 Status of This Assembly

This document is an independent final assembly module. It is not the long full manuscript and not a replacement for the source md proofs. Its purpose is to show the complete logical closure once the four technical theorem packages are accepted.

The full manuscript remains available in `manuscript_full_md/` and `manuscript_latex/`. Both assembly variants use the same proof-source md layer.